

Decision Making with Multiple Criteria and Uncertainty

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Abstract. The paper deals with discrete decision problems, in which a decision maker is to choose one out of a finite number of alternatives. The outcomes of such a choice are assumed to be probabilistic. We describe a technique that allows decision makers to visualize stochastic multicriteria decision problems. Rather than working with the original weight space as suggested in the literature, we employ a two-dimensional space that measures expected payoff and uncertainty and allows comparisons among the outcomes of alternatives as well as sensitivity analyses.

Keywords: Multicriteria decision analysis, decision making under uncertainty, domains, visualization

1. Introduction

For time immemorial, decision makers have made decisions, in which a number of criteria had to be considered and in which some features are uncertain. Obvious examples are decisions concerning possible expansions onto other continents (with short-term profit, long-term viability and market share, and cost-considerations as criteria, and the demand and foreign supplies being uncertain), the contracting out of services (with profit and customer satisfaction as criteria, and the service level and its availability being uncertain), and the design of a machine (with its overall reliability, costs, and expected profit as criteria, and its performance being uncertain).

The problem under consideration is of the type that is unique to *selection problems*. These problems comprise a finite number of decision alternatives, exactly one of which is to be chosen. With a single objective, the prototypical knapsack problem results. However, the inclusion of multiple criteria (or objectives) complicates issues. As long as the measures that assess the outcome of a decision given a criterion are deterministic, the main issue concerns the tradeoffs between criteria. In other words, questions such as “how many dollars of profit are worth one unit on the customer satisfaction scale” are to be dealt with. Some of the difficulties with this issue are highlighted in the contributions by Kahneman and Tversky (1979).

A generic multicriteria method is summarized by Eiselt and Sandblom (2004). Specific version of multicriteria decision models are reference point methods (see, e.g., Zeleny, 1986, or the Topsis method by Hwang and Yoon, 1981), data envelopment analysis (dating back to Charnes *et al.* 1978), preference cones (for a good description, see Olson, 1996), multiattribute value functions (see the pioneering work by Keeney and Raiffa 1976 and 1993), outranking methods (see Benayoun *et al.*, 1966, Roy, 1971, and Brans and Vincke, 1985), and methods allowing inconsistent estimates (the most prominent of which being Saaty’s 1980 analytic hierarchy process).

The model described in this presentation provides a visualization technique for a nondeterministic version of the basic model. In particular, evaluating a decision on a specific criterion, the outcome may be uncertain, following any (known) probability distribution. Problems of this nature were first addressed by

Lahdelma *et al.* (1998). Our technique is an extension of their work. It presents a very flexible tool that allows users to visualize some of the main aspects of and differences between the decisions.

2. The Model

Consider a scenario, in which a decision maker has m decisions d_1, d_2, \dots, d_m , which are to be evaluated on n criteria c_1, c_2, \dots, c_n . If the decision maker chooses decision d_i , the evaluation of this decision on criterion c_j is a_{ij} , where a_{ij} is a random variable that follows a known probability distribution. Each outcome a_{ij} may follow its own probability distribution. However, we do make the assumption that all these distributions are independent of each other. In addition, we also define the *lowest acceptable target value* t_j as well as an *ideal value* \bar{t}_j for each criterion c_j . In this presentation we ignore the possibility that the outcomes may or may not have been normalized. The generic method will then assign weights to each of the criteria and aggregate the weighted outcomes (average, worst, or any other measure of the distribution) in some way, mostly additive or multiplicative. The weights, are of course, a contentious issue, since they are fairly arbitrary and decision makers cannot be expected to have specify them exactly.

We can now define the *unrestricted acceptability space* by first assigning a dimension to each weight. Clearly, we will have as many dimensions as we have criteria. The unrestricted acceptability space is then defined as the set of all feasible weight combinations is the set of point in the simplex that is specified by the constraints $\sum w_j = 1, w_j \geq 0 \forall j=1, \dots, n$. These constraints define a set with extreme points $(1, 0, 0, \dots, 0), (0, 1, 0, 0, \dots, 0), \dots, (0, \dots, 0, 1, 0),$ and $(0, \dots, 0, 0, 1)$. Starr (1962) used the same concept in his analysis of games against nature. He then tessellated the unrestricted acceptability space into *domains*, so that the domain of a decision d_i is defined as the set of all weight combinations, for which decision d_i is better than any of the other decisions. The concept of domains was pioneered by Starr (1962) in his analysis of games against nature. However, the main problem related to his approach is the *curse of dimensionality*, which makes the concept unwieldy. We will take a different route to address the problem.

Rather than using the unrestricted acceptability space as indicated above, we may use the *restricted acceptability space* instead, which includes additional constraints that the decision maker may impose. For instance, if the decision maker decides that criterion c_2 is at least as important as criteria c_1 and c_3 together, we will write $w_2 \geq w_1 + w_3$. This restricts the number of possible weight combinations.

Similar to typical analyses in financial portfolio planning, where the focus is on the expected payoff and the risk of the portfolio, the main idea is to concentrate not on all criteria, but on the aggregate utility and the aggregate likelihood to achieve the lowest acceptable target value. In order to do so, we first compute all extreme points of the restricted acceptability space. Each such extreme point corresponds to a specific combination of weights. For each of these combinations, we compute the aggregate utility and the aggregate likelihood for all decisions. We can then plot these points in a two-dimensional (aggregate payoff, aggregate likelihood) diagram. This process is repeated for all weight combinations that are defined as extreme points of the acceptability space that is used for the model. The convex hull of all such points that belong to one decision is referred to as the domain $D(d_i)$ of a decision d_i . We can now construct domains for all decisions.

Even a cursory glance can now help the decision maker to evaluate and compare the decisions. Clearly, the best points are those in the northeast corner of the diagram. This gives rise to two main issues. On the one hand, we can delineate sufficient conditions for dominance based on the relative location of the domains of the decisions. Secondly, we can use the ideas of the TOPSIS method to make a recommendation to the decision maker regarding the quality of the decisions. In order to do so, we take an ideal point and measure the distance between that point and each of the domains. Again, there are various ways to do this: we could determine the average point-to-set distance, the shortest, the longest, or any other. A good compromise appears to be the distance between the ideal point and the center-of-gravity of each of the domains. These distances, put in nondecreasing order, will provide a ranking of the decisions. The use of “distances” again allows a great deal of flexibility: any distance function can be used, even though most decision makers will be most comfortable with Minkowski distances, such as the Euclidean or rectilinear distance measures.

Another possible approach will measure the mass of a decisions domain that is with a given distance of the ideal point. This can be plotted in a two-dimensional (distance to ideal point, relative mass) diagram. Any

decision, whose domain has consistently a higher proportion of its volume at given distances to the ideal point will be preferred by the decision maker. This can easily be determined by the relative location of the curves.

We conclude our analysis with a number of possible extensions. Among them are the determination of the metric based on questions similar to those that are to be answered in multiattribute value functions and the possible inclusion of multiple decision makers. In the latter case, voting procedures or concepts such as approval voting may be employed.

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4. References

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