

Study upon the Dynamic Answer of Plane Manipulators

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Abstract. The paper illustrates the structure of plane mechanical systems used to manipulators. In the first part are presented some kinematics schemes used to plane manipulators. These mechanisms are used to manipulators, positioning and control systems. In generally these mechanisms have two or three degree of mobility. The purpose of the paper is to study the dynamics of a plane manipulator mechanical system used to manipulate garbage containers. It is presented the kinematics scheme of a plane manipulator, used to this purpose and is presented the mechanism functional description. In the second part is presented the kinematical and dynamical analysis for the plane manipulator mechanism. In the last part of the paper are presented graphical results for the dynamics parameters.

Keywords: manipulator, dynamics, optimization.

1. Introduction

The aim of this research paper is to study the dynamic answer of a parallel manipulator, which is used to containers lifting. Also, is studied the dynamics of the mechanism, in the purpose of the actuation moment minimization.

In the literature, they are some studies concerning the dynamics and optimization of different manipulators.

In generally, the mechanisms with hydraulic actuation are manipulation, positioning and control mechanisms. These mechanisms are in the structure of the harvester machine, industrial robots, and manipulators. The mechanisms mobility correspond with the number of independents parameters, necessary to assure the mechanism functionality. These mechanisms can be characterized by total degree of freedom, partial or fractioned [2, 3]. The actuation of those modules is realized with mechanical – hydraulics systems, type cylinder piston (CH).

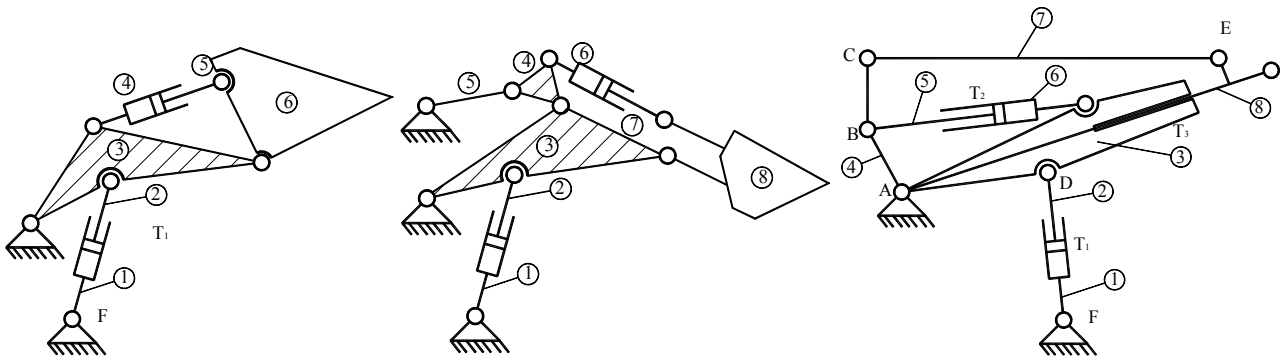


Fig. 1: Kinematics scheme of two degree of mobility mechanisms.

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2. The manipulator structure

The mechanism works sequentially, with a single motor element (the crank A_0A). The kinematics' scheme of the plane mechanism [2, 3, 4], with two degree of mobility, is presented in figure 2.

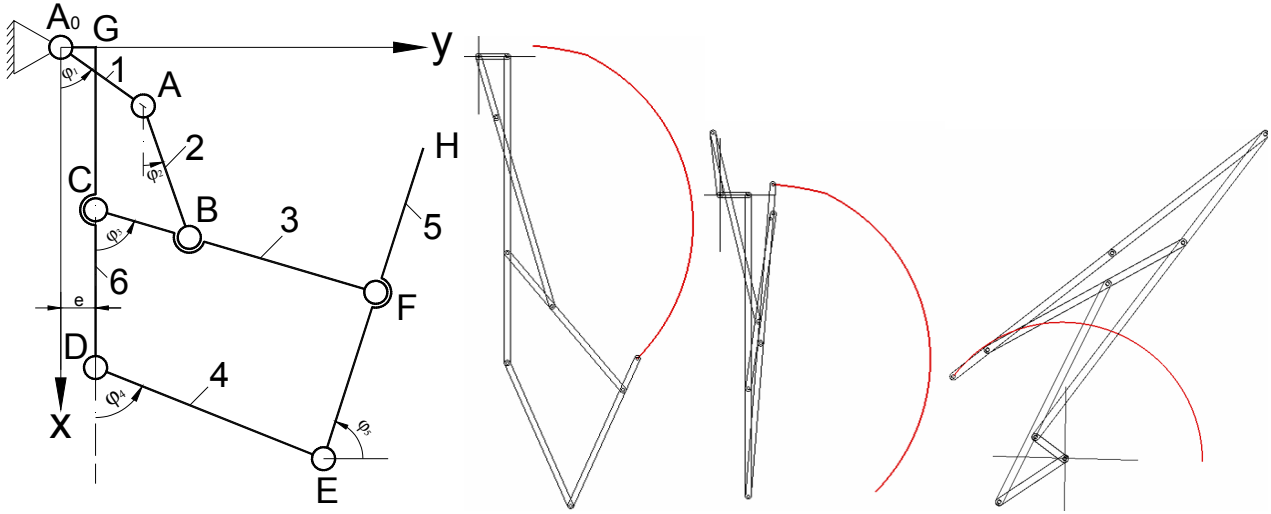


Fig. 2: Kinematics scheme of the manipulator and the trajectory described by the point H.

The characteristics dimensions are, (in conformity with figure 2):

$$\begin{aligned} A_0G = e = 100\text{mm}; \quad CG = l_6 = 680\text{mm}; \quad CD = l'_6 = 380\text{mm}; \\ A_0A = l_1 = 220\text{mm}; \quad AB = l_2 = 680\text{mm}; \quad BC = l_3 = 240\text{mm}; \\ CF = l'_3 = 620\text{mm}; \quad DE = l_4 = 540\text{mm}; \quad EF = l_5 = 440\text{mm}; \quad FH = l'_5 = 120\text{mm}. \end{aligned}$$

The maximum rotation angle of the crank shaft 1 is 260° . In the mechanism functioning we identify two phases [4]:

1. The element 6 (with the points G, C and D) stays fixed until, trough the crank shaft rotation, the point B reach on the vertical part of the element 6, between the points C and G;
2. All the kinematics elements of the mechanism are joining rigid, also continuing the crank shaft rotation until the end position, the mechanism become as a rigid body, which rotate upon the fixed joint A_0 . The trajectories of all mobile joints are circles with the centre in A_0 joint.

3. The dynamic analysis of the manipulator mechanism

For the first kinematics chain ABC, we write the positions equations:

$$\begin{cases} x_A = l_1 \cdot \cos \varphi_1; \\ y_A = l_1 \cdot \sin \varphi_1 \end{cases} \quad \begin{cases} x_B = x_A + l_2 \cdot \cos \varphi_2 \\ y_B = y_A + l_2 \cdot \sin \varphi_2 \end{cases} \quad \text{OR} \quad \begin{cases} x_B = x_C + l_{CB} \cdot \cos \varphi_3 \\ y_B = y_C + l_{CB} \cdot \sin \varphi_3 \end{cases} \quad (1)$$

From equations (1) we obtain:

$$\begin{cases} x_A + l_2 \cdot \cos \varphi_2 = x_C + l_{CB} \cdot \cos \varphi_3 \\ y_A + l_2 \cdot \sin \varphi_2 = x_C + l_{CB} \cdot \cos \varphi_3 \end{cases} \quad (2)$$

After square up of equations (2), and summing, we obtain: $l_2^2 = e_1^2 + e_2^2 + l_{CB}^2 + 2l_{CB} \cdot e_1 \cdot \cos \varphi_3 + 2l_{CB} \cdot e_2 \cdot \sin \varphi_3$
where: $e_1 = x_C - x_A$; $e_2 = y_C - y_A$.

We obtain a trigonometrically equations with variable coefficients, under the form: $A_1 \sin \varphi_3 + B_1 \cos \varphi_3 + C_1 = 0$, where: $A_1 = -2l_{CB} \cdot e_2$; $B_1 = -2l_{CB} \cdot e_1$; $C_1 = l_2^2 - e_1^2 - e_2^2 - l_{CB}^2$

The angle φ_3 is obtained by solution of the equation $\varphi_3 = 2 \arctg \left(\frac{A_1 \pm \sqrt{A_1^2 + B_1^2 - C_1^2}}{B_1 - C_1} \right)$

The angle φ_2 is determined with a similar relation, updated under the form:

$$\varphi_2 = 2 \arctg \left(\frac{A_2 \pm \sqrt{A_2^2 + B_2^2 - C_2^2}}{B_2 - C_2} \right) \quad (3)$$

where: $e_1 = x_C - x_A$; $e_2 = y_C - y_A$; $A_2 = -2l_2 \cdot e_2$; $B_2 = -2l_2 \cdot e_1$; $C_2 = l_{CB}^2 - e_1^2 - e_2^2 - l_2^2$

For the second kinematics chain DEF we write the equations for positions:

$$\begin{cases} x_F = x_C + l_{CF} \cdot \cos \varphi_3 \\ y_F = y_C + l_{CF} \cdot \sin \varphi_3 \end{cases}, \begin{cases} x_E = x_F + l_{EF} \cdot \sin \varphi_5 \\ y_E = y_F + l_{EF} \cdot \cos \varphi_5 \end{cases}, \text{ or } \begin{cases} x_E = x_D + l_{DE} \cdot \cos \varphi_4 \\ y_E = y_D + l_{DE} \cdot \sin \varphi_4 \end{cases} \quad (4)$$

After square up of equations (4), and summing, we obtain: $l_{EF}^2 = e_3^2 + e_4^2 + l_{DE}^2 + 2l_{DE} \cdot e_3 \cdot \cos \varphi_4 + 2l_{DE} \cdot e_4 \cdot \sin \varphi_4$

That is, an equation under the form: $A_4 \sin \varphi_4 + B_4 \cos \varphi_4 + C_4 = 0$

where: $A_4 = -2l_{DE} \cdot e_4$; $B_4 = -2l_{DE} \cdot e_3$; $C_4 = l_{EF}^2 - e_3^2 - e_4^2 - l_{DE}^2$; $e_3 = x_D - x_F$; $e_4 = y_D - y_F$

The angle φ_4 is determined with the relation: $\varphi_4 = 2 \arctg \left(\frac{A_4 \pm \sqrt{A_4^2 + B_4^2 - C_4^2}}{B_4 - C_4} \right)$ (5)

The angle φ_5 is determined with the same relation, updated under the form:

$$\varphi_5 = 2 \arctg \left(\frac{A_5 \pm \sqrt{A_5^2 + B_5^2 - C_5^2}}{B_5 - C_5} \right)$$

where: $A_5 = -2l_{EF} \cdot e_5$; $B_5 = -2l_{EF} \cdot e_6$; $C_5 = l_{DE}^2 - e_5^2 - e_6^2 - l_{EF}^2$; $e_5 = x_F - x_D$; $e_6 = y_F - y_D$.

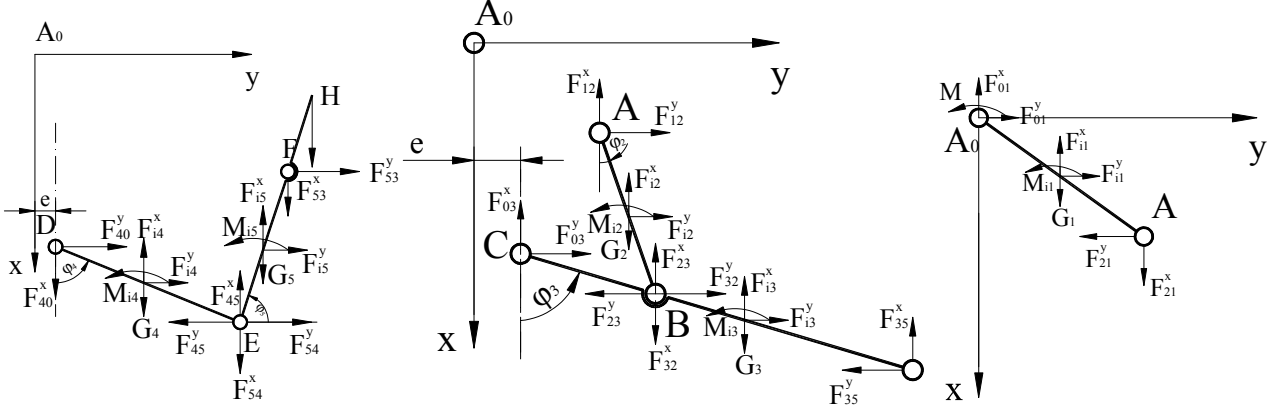


Fig. 3: Forces and moments on the mechanism elements.

We want to establish the joints bound forces: $\vec{F}_{40} = \begin{Bmatrix} F_{40}^x \\ F_{40}^y \end{Bmatrix}$; $\vec{F}_{54} = -\vec{F}_{45} = \begin{Bmatrix} F_{54}^x \\ F_{54}^y \end{Bmatrix}$; $\vec{F}_{53} = \begin{Bmatrix} F_{53}^x \\ F_{53}^y \end{Bmatrix}$

We write the axes projection equations, for the forces which act upon the dyad:

$$\begin{cases} \sum X^{(4+5)} = 0; F_{40}^x - F_{i4}^x + G_4 - F_{i5}^x + G_5 + F_{53}^x + 1000 = 0 \\ \sum Y^{(4+5)} = 0; F_{40}^y - F_{i4}^y + F_{i5}^y + F_{53}^y = 0 \end{cases} \quad (6)$$

We write the moments equations upon the point E, for the elements 4 and 5.

$$\begin{cases} \sum M^{(4)}(E) = 0; F_{40}^y(x_E - x_D) - F_{40}^x(y_E - y_D) - G_4(y_E - y_{C4}) + F_{i4}^y(x_E - x_{C4}) + F_{i4}^x(y_E - y_{C4}) + M_{i4} = 0 \\ \sum M^{(5)}(E) = 0; 1000(y_H - y_E) + F_{53}^y(x_F - x_E) + F_{53}^x(y_F - y_E) + G_5(y_{C5} - y_E) + \\ + F_{i5}^y(x_{C5} - x_E) - F_{i5}^x(y_{C5} - y_E) + M_{i5} = 0 \end{cases} \quad (7)$$

From the relations (6) and (7) we establish the forces: $F_{40}^x, F_{40}^y, F_{53}^x, F_{53}^y$.

We write the axes projection equations for the forces which act upon the element 4 or 5.

$$\begin{cases} \sum X^{(4)} = 0; F_{40}^x - F_{i4}^x + G_4 + F_{54}^x = 0 \\ \sum Y^{(4)} = 0; F_{40}^y + F_{i4}^y + F_{54}^y = 0 \end{cases} \quad (8)$$

From the system of equation (8) we establish the forces, F_{54}^x, F_{54}^y .

We want also to establish the bound forces: $\vec{F}_{03} = \begin{Bmatrix} F_{03}^x \\ F_{03}^y \end{Bmatrix}$; $\vec{F}_{23} = -\vec{F}_{32} = \begin{Bmatrix} F_{23}^x \\ F_{23}^y \end{Bmatrix}$

We write the axes projection equation, for the forces which act upon the dyad:

$$\begin{cases} \sum X^{(2+3)} = 0; -F_{12}^x + G_2 - F_{i2}^x - F_{03}^x + G_3 - F_{i3}^x - F_{35}^x = 0 \\ \sum Y^{(2+3)} = 0; F_{12}^y + F_{i2}^y + F_{03}^y + F_{i3}^y - F_{35}^y = 0 \end{cases} \quad (9)$$

We write the equation of moments upon the point B, for the element 2 and 3.

$$\begin{cases} \sum M^{(2)}(B) = 0; F_{12}^x(y_B - y_A) + F_{12}^y(x_B - x_A) + F_{i2}^x(x_B - x_{C2}) + F_{i2}^y(y_B - y_{C4}) - G_2(y_B - y_{C4}) + M_{i2} = 0 \\ \sum M^{(3)}(B) = 0; F_{03}^y(x_B - x_C) + F_{03}^x(y_B - y_C) + G_3(y_{C3} - y_B) + F_{i3}^y(x_{C3} - x_B) - \\ - F_{i3}^x(y_{C3} - y_B) - F_{35}^y(y_F - y_B) + F_{35}^x(x_F - x_B) + M_{i3} = 0 \end{cases} \quad (10)$$

From the relations (9) and (10) we establish the forces, $F_{03}^x, F_{03}^y, F_{12}^x, F_{12}^y$.

We write the axes projection equations for the forces which act upon the element 2 and 3.

$$\begin{cases} \sum X^{(2)} = 0; -F_{12}^x + F_{i2}^x - G_2 + F_{32}^x = 0 \\ \sum Y^{(2)} = 0; F_{12}^y + F_{i2}^y + F_{32}^y = 0 \end{cases} \quad (11)$$

We write the axes projection equations for the forces which act upon the element 1.

$$\begin{cases} \sum X^{(1)} = 0; -F_{01}^x - F_{i1}^x + G_1 + F_{21}^x = 0 \\ \sum Y^{(1)} = 0; F_{01}^y + F_{i1}^y - F_{21}^y = 0 \end{cases} \quad (12)$$

We write the equation of moments for the element 1, upon the point A:

$$\sum M^{(1)}(A) = 0; F_{21}^x(y_A - y_{A0}) + F_{21}^y(x_A - x_{A0}) + G_1(x_{C1} - x_{A0}) + F_{i1}^y(x_{C1} - x_{A0}) - F_{i1}^x(y_{C1} - y_{A0}) + M_{i1} - M = 0$$

The reduced moment calculus is made from the condition:

$$\begin{aligned} P_{\text{model}} &= P_{\text{mechanism}} \\ M_{\text{red}} \cdot \omega &= \sum_{i=1}^5 (\vec{F}_i \cdot \vec{v}_{Ci} + \vec{M}_i \cdot \vec{\omega}_i) \end{aligned} \quad (13)$$

where $\vec{\omega}_i = \dot{\varphi}$ is the angular velocity of the element 1.

The establish of the reduced inertia moment is made from the condition: $T_{\text{model}} = T_{\text{mechanism}}$

$$\frac{J_{\text{red}} \cdot \omega^2}{2} = \sum_{i=1}^5 \left(\frac{m_i \cdot v_{Ci}^2}{2} + \frac{J \Delta c_i \cdot \omega_i^2}{2} \right), \quad (14)$$

where $\omega = \dot{\varphi}$ is the angular speed of the element 1.

We apply the kinetic energy theorem:

$$\begin{aligned} dT &= \partial L \\ \begin{cases} d\left(\frac{1}{2} J_{\text{red}} \omega^2\right) = M_{\text{red}} \cdot d\varphi \\ \text{or:} \\ \frac{1}{2} J_{\text{red}} \cdot \omega^2 - \frac{1}{2} J_0 \cdot \omega_0^2 = \int_{\varphi_0}^{\varphi} M_{\text{red}} \cdot d\varphi. \end{cases} \end{aligned} \quad (15)$$

The angular velocity of the motor element is established with the relation:

$$\omega = \sqrt{\frac{2}{J_{\text{red}}(\varphi)} \left[\int_{\varphi_0}^{\varphi} M_{\text{red}} \cdot d\varphi + \frac{1}{2} J_0 \omega_0^2 \right]} \quad (16)$$

4. Graphical results

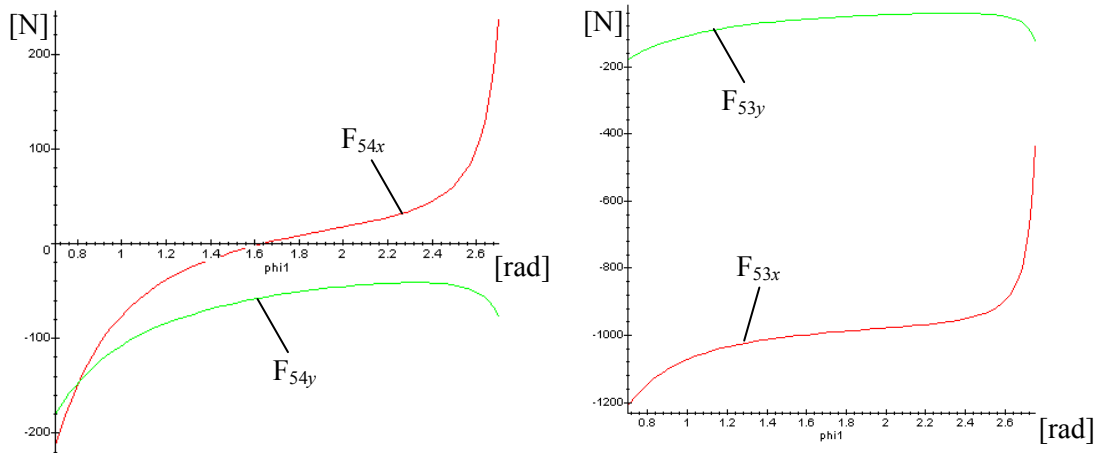


Fig. 4: The graphic for the bound forces F54, F53 variation.

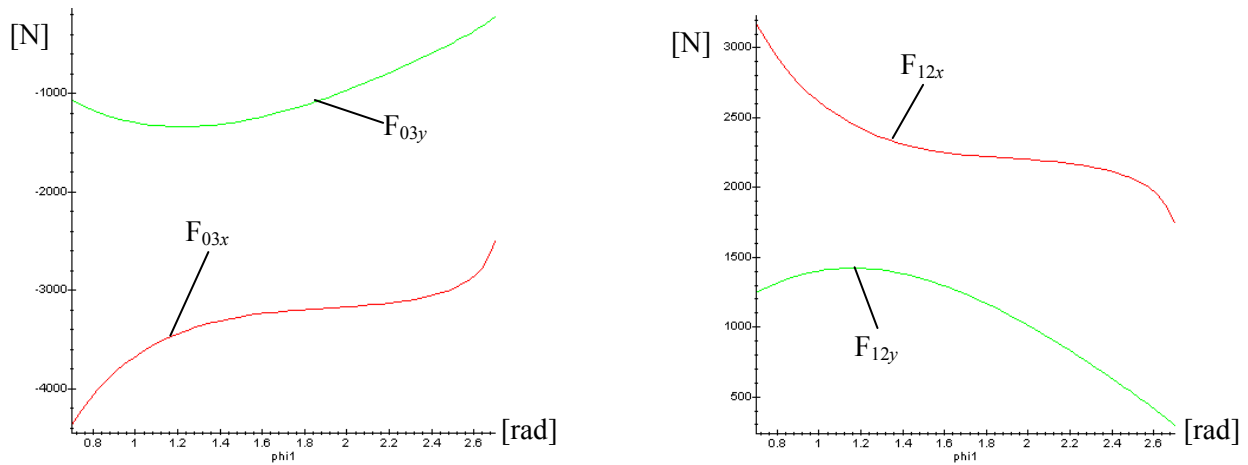


Fig. 5: The graphic for the bound forces F_{03} , F_{12} components variation.

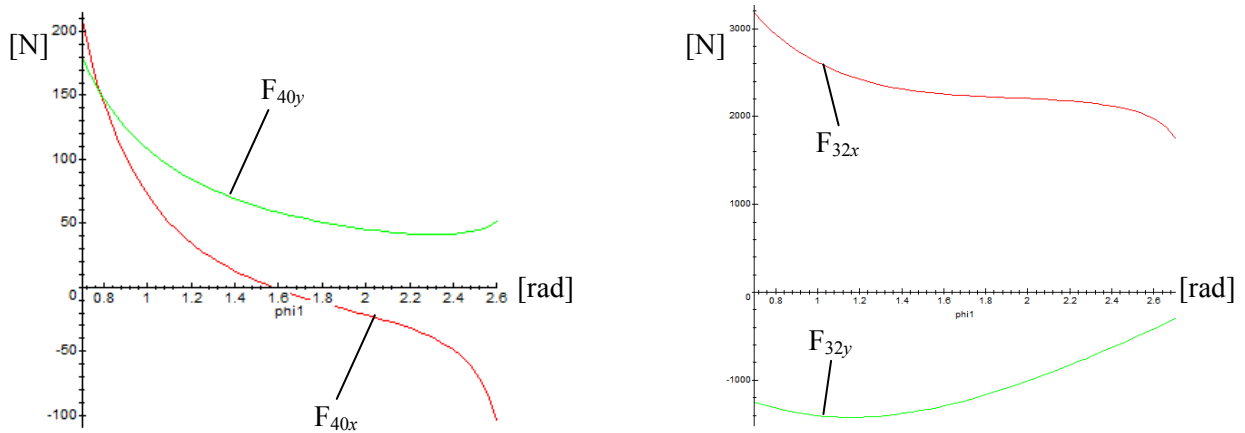


Fig. 6: The graphic for the bound force F_{40} , F_{32} components variation.

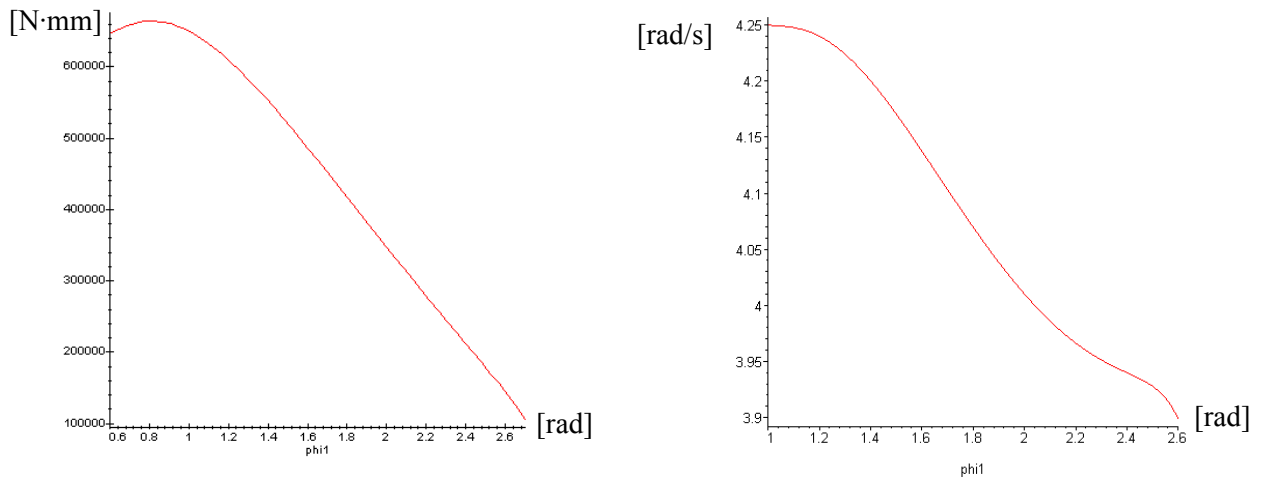


Fig. 7: The law of variation of the torque motor and motor element angular speed.

5. References

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