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Effect of Intermediate Location and Time-Varying End Mass on the Dynamic Response of a Flexible Robot Manipulator in Tracing Multi-Straight-Line Path

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Abstract. This paper presents the effects of intermediate location and time-varying end mass on the dynamic response of flexible robot manipulator with rotating-prismatic joint in tracing multi-straight-line path. The tip end of the flexible robot manipulator traces a multi-straight-line path under the action of external driving torque and axial force. Flexible arm which consist of a rotating-prismatic joint, is assumed to be an Euler-Bernoulli beam carrying an end mass. The Lagrangian dynamics in conjunction with the assumed modes method is utilized in deriving the equations of motion. Effect of rotary inertia, axial shortening and gravitation has been considered in developing the dynamic model. Equations of motion are numerically solved by using the Runge-Kutta method. Numerical results of computer simulations for tip deflection are presented in graphical form. Physical trends of the obtained numerical results are discussed.

Keywords: Dynamic response, Flexible robot manipulator, Rotating-prismatic joint, Straight-line path, time-varying end mass.

1. Introduction

Robotic manipulators are commonly used to help in dangerous, tedious, and monotonous jobs. Most of the existing robotic manipulators are designed and built in a manner to maximize stiffness in order to minimize the vibration of the end-effector to achieve a good position accuracy. This high stiffness is achieved by using heavy material and bulky design. Hence, the existing heavy rigid manipulators are shown to be inefficient in terms of power consumption or speed with respect to the operating payload. Also, the operation of high precision robotic systems is severely limited by their dynamic deflection, which persists for a period of time after completion of a movement. The settling time required for this residual vibration delays subsequent operations, thus conflicting with the demand of increased productivity [2]. Increasing demands for high-speed performance and low-energy consumption of robotic systems, coupled with needs of limited space applications have necessitated the design of light-weight robot manipulators. Link flexibility is a result of the light-weight constructional feature in manipulator arms that are designed to operate at high-speeds. This would result in an increase in elastic deflections and poor performance due to the effect of mechanical vibration in the links. Considering that the flexible members can be seen in many robot applications, nuclear maintenance, micro-surgical operation, painting or drawing robots and many similar applications. But the biggest disadvantage of these manipulators is the vibration problem due to low stiffness.

Some papers have extensively studied and modelled vibrational analysis and control of flexible manipulators with conventional joints and different types of boundary conditions [1]-[2]. Chang *et al.* [3] used Hamilton's principle and finite element technique based on a Runge-Kutta algorithm in order to validate stability and vibration of an axially moving Rayleigh beam in contrast with Euler beam theory. Yau

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and Fung [4] investigated effect of centrifugal stiffening due to rotation on the dynamic response of single flexible arm that carrying a moving mass. Hamilton's principle and assumed mode method used to obtain the dynamic model. Khadem and Pirmohammadi [5] considered a mathematical model capable of handling a three-dimensional (3-D) flexible-degree of freedom manipulator having both revolute and prismatic joints. In most of practical robot applications the robot arm carries an end mass. This fact was not taken into consideration by some previous investigators [3, 5]. Kalyoncu [6] studied the dynamic response of flexible arm with end mass which would trace straight-line paths passing through prescribed points.

The aim of this study is investigating on the effects of intermediate points location and time varying end mass on dynamic response of a flexible arm in tracing multi-straight-line paths. Flexible arm is assumed to be axially moving Euler-Bernoulli beam carrying end mass that traces path under the action of an external driving torque and force. In addition, the devised model takes into account rotary inertia, axial shortening and gravitation effects. The prismatic joint is treated as rigid. The Lagrangian dynamics in conjunction with the assumed modes method is utilized in deriving the equations of motion and numerically solved by using the Runge-Kutta method. Physical trend of the numerical results are discussed in order to demonstrate the validity and the accuracy of the analysis.

2. Mathematical Model

Fig. 1 shows the physical configuration of the robot arm considered in this work with the following features and assumptions: (a) the elastic arm is uniform and slender at all times and hence the Euler-Bernoulli beam theory is applicable with small deformations. (b) The end mass is assumed to be varying with time. (c) The prismatic joint is assumed to be rigid and the sliding motion is assumed to be frictionless. (d) The length of the beam is denoted as *L* (t) and varying with time; the beam is inextensible. (e) The elastic arm slides in the prismatic joint under the action of force *F*, and torque *T* rotates prismatic joint about *Z* axis.

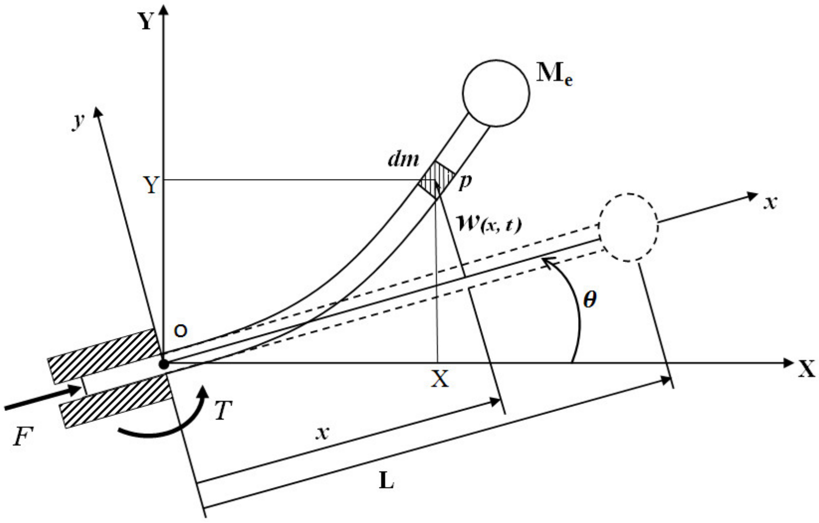

Fig. 1: flexible robot manipulator with rotating-prismatic joint.

3. Kinematic Analysis and Energy Terms

In order to represent energy terms and obtain the position, velocity, and acceleration of flexible robot arm, homogeneous transformation matrices are used in the kinematic analysis. *XYZ* is the global reference frame, while *xyz* is rotating reference frame about a stationary revolute joint O as shown in Fig. 1. The angle between the rotating reference frame *xyz* and the global reference frame *XYZ* is θ. Distance of an infinitesimal mass dm to the origin in *x* direction is *x*(t) and the displacement from the undeformed position in *y* direction of the elastic arm is *w*. The position of an arbitrary material point *p* with respect to the global reference frame *XYZ* can be expressed as:

$$\begin{Bmatrix} X \\ Y \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} x \\ w \end{Bmatrix} \tag{1}$$

By using some trigonometric property and differentiating the expression in (1), translational velocity of typical element at point p can be written as:

$$V^2 = \dot{X}^2 + \dot{Y}^2 = \dot{w}^2 + 2x\dot{w}\dot{\theta} + x^2\dot{\theta}^2 + w^2\dot{\theta}^2 - 2\dot{x}w\dot{\theta} + \dot{x}^2. \tag{2}$$

The kinetic energy consists of two parts which are due to translation and rotation of each typical element, the translational kinetic energy is:

$$T_{trans} = \frac{1}{2}\int_0^L V^2 dm + \frac{1}{2} M_e V_L{}^2. \tag{3}$$

Then, the rotational kinetic energy can be obtained as:

$$T_{rot} = \frac{\rho I}{2}\int_0^L (\dot{\theta} + \dot{w}')^2 \, dm. \tag{4}$$

Let the potential energy due to flexible deformations be written in the following form:

$$U_d = \frac{1}{2}\int_0^L EIw''^2 dx \tag{5}$$

And the gravitational potential energy can be as bellow:

$$U_g = \int_0^L g \sin\theta \, dm + M_e \, gL \sin\theta \tag{6}$$

Components of the acceleration in x direction have to be considered in order to express the strain energy due to axial shortening of the typical element at point p on the beam:

$$a_x = \ddot{L} - x\dot{\theta}^2 - w\ddot{\theta} - 2\dot{w}\dot{\theta}; \quad g_x = -g \sin\theta \tag{7}$$

Where the axial inertial force can be written as:

$$F_a = \rho A \int_x^L (-a_x + g_x) dx + M_e \, (-a_{x_L} + g_x) \tag{8}$$

And the potential energy due to axial shortening is given by:

$$U_a = \frac{1}{2}\int_0^L F_a w'^2 dx \tag{9}$$

Finally, the total energy term of the system is then given by:

$$\sum T = T_{trans} + T_{rot} \tag{10}$$

4. Equation of Motion

To approximate the elastic motion, the elastic displacements are presented in the form of infinite series:

$$w(x, L, t) = \sum_{i=1}^{\infty} \phi_i(x, L) \, q_i(t) \tag{12}$$

Where $\phi_i(x, L)$ are the assumed shape functions, q_i denotes the time dependent generalized coordinate. In equation (12), the beam length L is a function of time. The eigenfunctions of a stationary cantilever beam will be used, after being normalized as the assumed shape functions in the form:

$$\phi_i(x, L) = 1/\sqrt{\rho AL}\left[\cosh\beta_i - \cos\beta_i - \alpha_i(\sinh\beta_i - \sin\beta_i)\right] \tag{13}$$

Where $\alpha_i = (cosh\lambda_i + cos\lambda_i) / (sinh\lambda_i + sin\lambda_i)$ and $\beta_i = \lambda_i x/L$ [7].

These functions satisfy the geometrical boundary conditions and do not prevent the natural boundary condition from being satisfied. By using the derivative of w given as follows:

$$\dot{w}(x,L,t)=\sum_{i=1}^{\infty}\left[\phi_i\,\dot{q}_i+\frac{d\phi_i}{dt}q_i\right] \tag{14}$$

Since ϕ_i is function of $x(t)$ and $L(t)$, its time derivative is [1]:

$$d\phi_i\,/\,dt=(\dot{L}/L)\left[-\frac{1}{2}\phi_i+\phi_i'(L-x)\right] \tag{15}$$

By taking derivative from (15) to (14) and substituting into (10)-(11), and applying Lagrange's equation of motion (here, L is Lagrangian):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right)-\frac{\partial L}{\partial q}=F \tag{16}$$

It is possible to obtain equations of motion after neglecting higher order terms and considerably lengthy calculations as:

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{q}}\}+[\boldsymbol{G}]\{\dot{\boldsymbol{q}}\}+[\boldsymbol{K}]\{\boldsymbol{q}\}+\{\boldsymbol{Q}\}=\{\boldsymbol{F}\} \tag{17}$$

In the form of ordinary differential equations, where:

$[\boldsymbol{M}]=f(L,\rho,A,M_e,\dot{L},\boldsymbol{h}_{ij},\boldsymbol{S}_{ij},\boldsymbol{P}_{ij},\boldsymbol{O}_{ij},\boldsymbol{o}_i,\boldsymbol{v}_i)$, $[\boldsymbol{G}]=f\left(L,\rho,A,M_e,\dot{M}_e,\dot{L},\boldsymbol{h}_{ij},\boldsymbol{S}_{ij},\boldsymbol{P}_{ij},\boldsymbol{O}_{ij},\boldsymbol{o}_i,\boldsymbol{t}_i,\boldsymbol{f}_i,\boldsymbol{c}_i\right)$,
$[\boldsymbol{K}]=f(L,\rho,A,M_e,\dot{M}_e,\dot{L},\ddot{L},\boldsymbol{h}_{ij},\boldsymbol{S}_{ij},\boldsymbol{P}_{ij},\boldsymbol{O}_{ij},\boldsymbol{o}_i,...)$, $[\boldsymbol{Q}]=f(L,\rho,A,M_e,\dot{M}_e,\dot{L},\ddot{\theta},\dot{\theta},\boldsymbol{t}_i,\boldsymbol{o}_i)$.

And ***h,S,P,O*** denotes a square $\infty\times\infty$ matrix [1].

The dynamic model, as represented by equations (17), reduces to that of reference [1], provide that the effect of gravitational acceleration is neglected from equation (8) and end mass is assumed as constant.

5. Numerical Simulation

In the following example, it is assumed that the RP manipulator requires to move from $p_0(1.8,\ 0)$ to $p_2(0,3)$ passing through intermediate point $p_1(2,2)$ or two straight-line, along the entire path [6]. The physical characteristics of the flexible manipulator are ρ= 4.015 kg/m, A= 0.001471 m^2, I =1.14197$\times$10^{-8} m^4, EI =756.65 Nm^2 [1]. If the beam flexibility is negligible, in order to determinate longitudinal motion according to segment-by-segment straight-line trajectory, mathematical modeling similar to [6] used in the simulation. Each triangular segment involve two adjacent intermediate points on the path, p_{i-1}, p_i, and the reference point, o, to determine the axial displacement $L(t)$ and it is time derivative based on the given time function of rotation $\theta(t)$ and the angular velocity. θ is specified by the uniform cycloidal motion profile as follows:

$$\theta(t)=\theta_{tot}\left[\frac{t}{T_{tot}}-\frac{1}{2\pi}\sin\left(\frac{2\pi t}{T_{tot}}\right)\right] \tag{18}$$

Where θ_{tot} is the total angle of motion, T_{tot} is the total travelling time, t is time. For brevity, only axial velocity and time- derivative are shown below (a detailed derivation is given in [6]):

$$L(t)=L_{i-1}\sin(o\hat{p}_{i-1}p_i)/\sin(\pi-o\hat{p}_{i-1}p_i-\theta(t)+\theta_{i-1}) \tag{19}$$

$$\dot{L}(t)=\dot{\theta}(t)\,L(t)/\tan(\pi-o\hat{p}_{i-1}p_i-\theta(t)+\theta_{i-1}) \tag{20}$$

$$\ddot{L}(t)=\frac{\ddot{\theta}(t)}{\dot{\theta}(t)}\dot{L}(t)+\frac{\dot{L}(t)^2}{L(t)}+\frac{\dot{\theta}(t)^2\,L(t)}{\sin^2(\pi-o\hat{p}_{i-1}p_i-\theta(t)+\theta_{i-1})} \tag{21}$$

It is citable that the location and timing of the intermediate points have a lot of influence on subsequent vibration of the robot arm. Therefore, in the present simulation, four cases with different intermediate location are used. In the case 1 the point p_1 is moved outside in the longitudinal direction to new location (1.95, 1.95), whereas in the case 2 the point p_1 is moved inside to (2.05, 2.05). In the case 3 the point p_1 is moved above on p_1p_2 path to (1.99, 1.9), and finally, in the case 4 the point p_1 is moved lower on p_0p_1 path to (1.96, 2.02). In Fig. 2 and 3 results for tip deflection of flexible manipulator at the new location plotted against primary location of point p_1. In all the cases two elastic mode is used and total travelling time T_{tot} assumed 20s. The tip mass is assumed constant and 2kg. An initial tip deflection of -5mm was given to the free end of the arm.

As seen from Fig. 2 and 3, these slight shifts in the trajectory, lead to an obvious change in the resulting vibration amplitude. These effects can be attributed to the Coriolis acceleration and a severe change in sliding velocity when passing trough intermediate location and show how the effect of intermediate location become more pronounced on the subsequent vibration. This slight shift in the trajectory, was not considered in the results of reference [6].

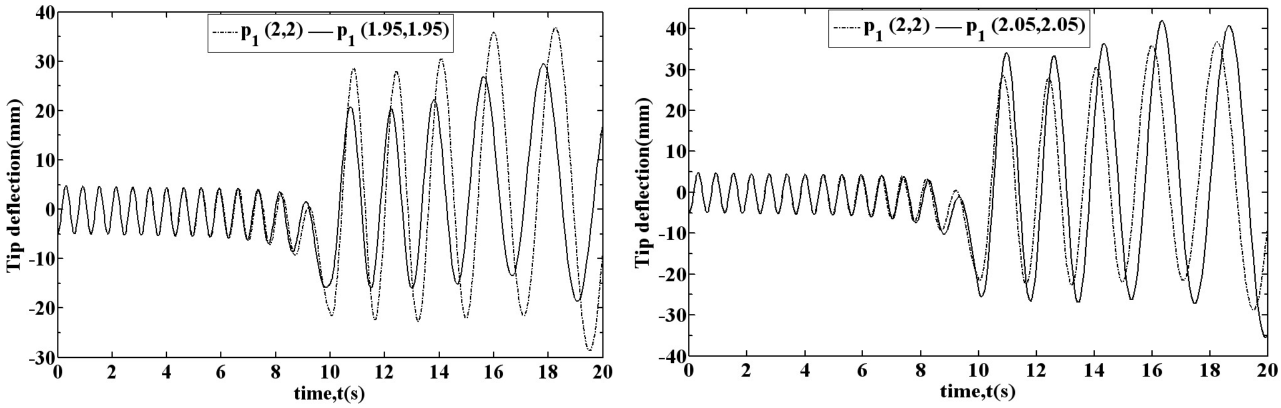

Fig. 2: Amplitude of vibration for shift point p_1 inside and outside in the longitudinal direction to new location.

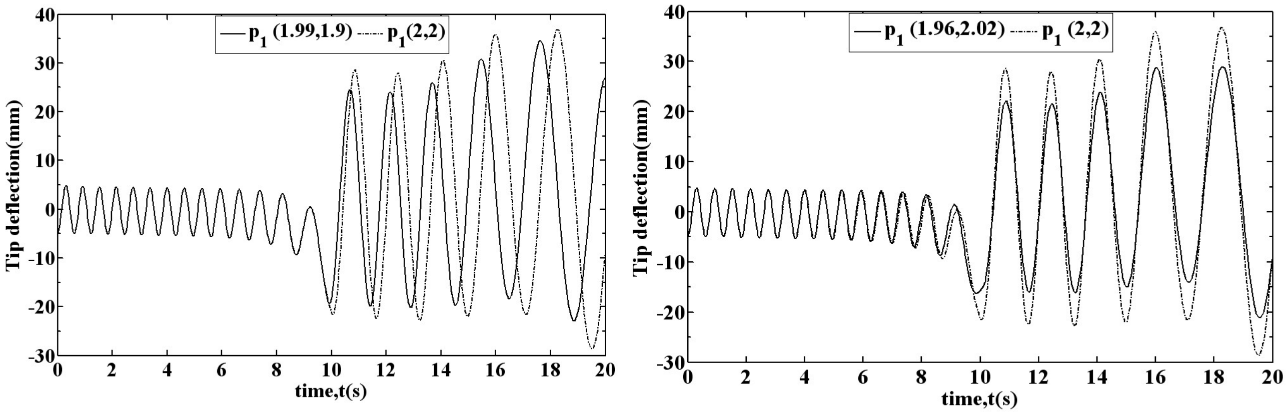

Fig. 3: Amplitude of vibration for shift point p_1 lower on p_0p_1 path and upper on p_1p_2 path.

In order to represent effects of time-varying end mass in the trajectory tracing of straight-line path, the end mass can be assumed changing cycloidally as:

$$M_e(t) = M_{e-tot}[1 - \frac{t}{T_{tot}} + \frac{1}{2\pi}\sin(\frac{2\pi t}{T_{tot}})] \tag{22}$$

Here M_e is assumed varying from 2kg to 0kg and from 0kg to 2kg in time T_{tot} while M_{e-tot} is total end mass. Fig. 4 shows the tip deflection for cases of constant and time-varying end mass. In both cases, increasing in end mass, while robot traces out a straight line between each pair of two neighboring points, decrease both vibration amplitude and vibration frequency and conversely. In fig. 5, corresponding driving axial force and torque values for different values of end mass are plotted versus time. As shown, at the end of the simulation time, the axial force has a value due to neutralize the effects of gravity. It can be seen that sliding direction is suddenly reversed at intermediate point p_1, thus leading to an abrupt change in driving axial force and torque. The corresponding torque is fluctuating around the desired line Proportional to behavior of angular velocity and having initial value at beginning of motion. This behavior was not reported by any previous investigators.

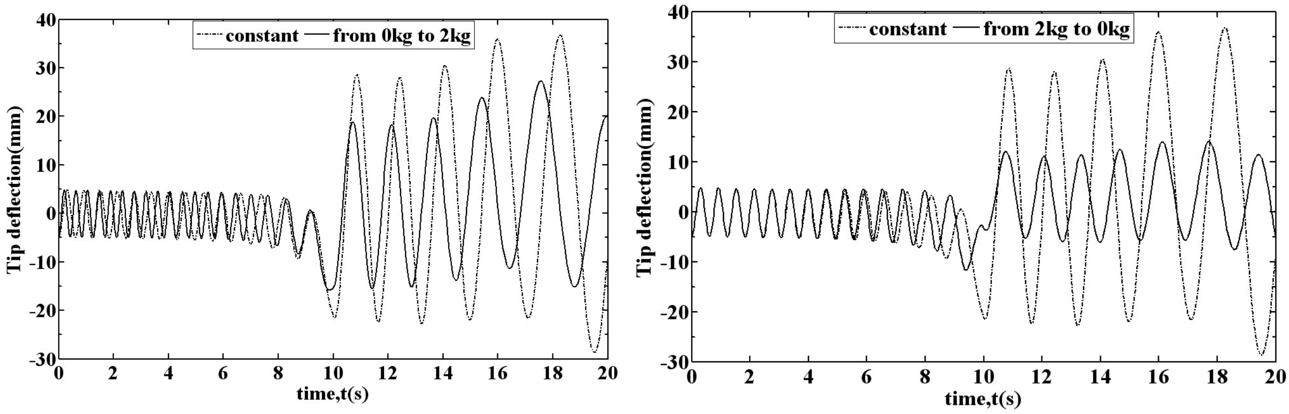

Fig. 4: Amplitude of vibration for change end mass from 0kg to 2kg and 2kg to 0kg.

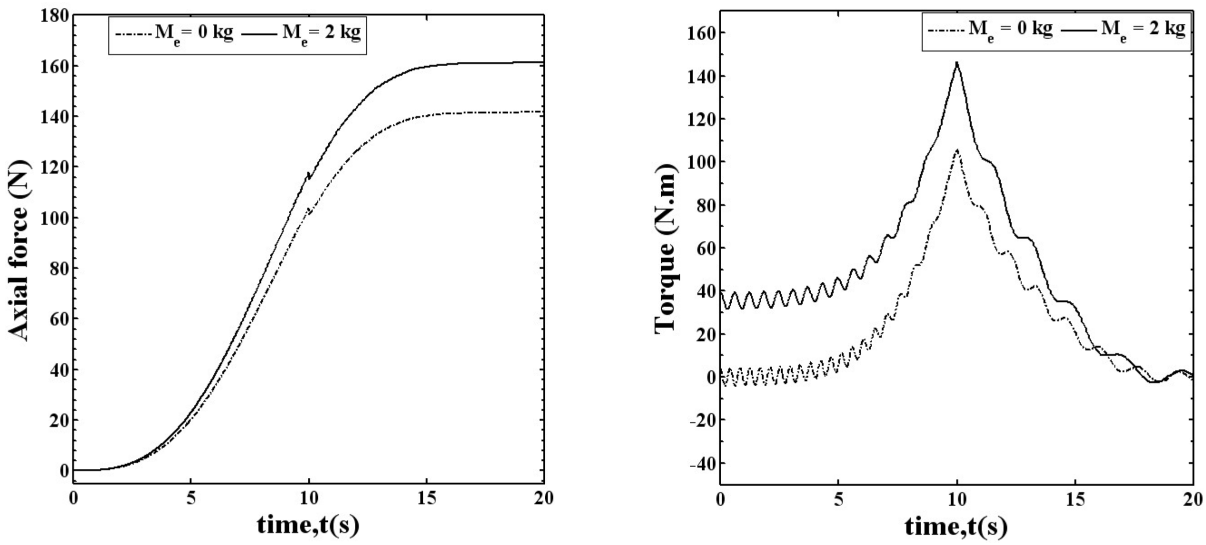

Fig. 5: Corresponding driving axial force and torque obtained by inverse dynamics during trajectory tracing.

6. Conclusion

The dynamic model developed in this paper investigates effects of intermediate points location and time varying end mass on dynamic response of a flexible arm in tracing multi-straight-line paths. The equation of motion of the flexible robot arm are obtained by using Lagrange's equation of motion and solved by using Runge-Kutta method. We showed that slight shifts in the trajectory, lead to an obvious change in the resulting vibration amplitude on the subsequent operations. Moreover, continuously increasing in end mass, when robot traces out a straight line between each pair of two neighboring points, decrease both vibration amplitude and vibration frequency and conversely. At the end of the simulation time, the axial force has a value due to neutralize the effects of gravity and sliding direction is suddenly reversed at intermediate point p_1, thus leading to an abrupt change in driving axial force and torque. The corresponding torque is fluctuating around the desired line Proportional to behavior of angular velocity and having initial value at beginning of motion.

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