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Mechanical Loading of R-R-T Dyade

Dan Ilincioiu [1], Cosmin Mirițoiu [1 +], Juan Pablo Jimenez [1] , Cătălin Roşu[1]

[1] University of Craiova, Faculty of Mechanics, Calea Bucureşti Street, No. 107, Code 200512, Craiova, Romania

Abstract. The handling of bodies which have a larger mass is generating supplementary problems to cope with the kinematic chains of the robots. In these cases, the mechanical stresses and the displacements exerted up on the kinematic elements must be defined. In this paper we present a mathematical model able to calculate the section forces that act upon the elements of the RRT dyade, in order to facilitate the required strength check-ups.

Keywords: mechanics, solid, mechanical stress, mechanisms, RRT dyade.

1. Introduction

The automation and robotics are normal tendencies of actual human activities, the engineering has a main part in this direction. If the aspects regarding the command and control of automatic systems have gained an outstanding evolution, the execution part still presents many problems, leaving room for further studies on mechanical systems. The elements essembly, jointed in different ways, being in well defined movement, is the subject of Mechanisms Theory, technical science that allows a theoretical approach of automation and robotics projects. The mechanical system called mechanism, presented in fig. 1.a, is a routine problem for this science; the reaction forces or moments and kinematical parameters calculus is easily done by using classical algorithms. The specialists from Mechanisms Theory have created kinetostatic calculus programs that allow a relatively simple approach of a mechanism, [1] being such example.

According to [2] and [3], the solving of theoretical automation problems and finding the solution of kinematical scheme problems don't mean the end of designing process. The constructive design and correct analysis of the constructive solutions are hard steps for a robot execution. The authors consider that the final step of the designing process, before manufacturing, is the strength calculus (the stress and strain determination). It is true that, the strength calculus can be made using the numerical calculus softwares, like the finite element method. The computer aided design (CAD-CAM-CAE) allows the general approach of any mechanical system, and thus robot mechanisms. The idea presented in [3] that the computer aided design, with the automatic calculus softwares that have become classical, must be duplicated with usual methods, defined by the *Strength of Materials*, is considered true. The algorithms defined by the *Strength of Materials* are used with the automatic calculus softwares, the mechanical phenomena is analytically approached. That is why, in this paper, the dyade structural element is analyzed, part of a planar mechanism, in order to determine the mechanical loading state.

2. Definitions and conditions

A two body essembly is studied, called ***dyade*** in Mechanisms Theory, the connections between them are two joints and a translation couple (named RRT dyade in fig. 1.b). The RRT dyade is a compound of a planar mechanism; the xOy cartesian system is used for its geometrical define, the couple coordinates are known; the shape of an element is defined by a **y(x)** function that describes the locus of transversal sections centers.

+ Corresponding author. Tel.: 0742801945; fax: 0251416630.
E-mail address: miritoiucosmin@yahoo.com.

The kinematical calculus of the mechanism determines the next parameters for the dyade: joints velocities $\overline{V_1}, \overline{V_2}$; angular velocities ω_1, ω_2; angular accelerations ε_1, ε_2. In kinetostatic calculus, the reaction forces $\overline{R_1}, \overline{R_2}$, from extreme couples, are determined. The function **A(s),** that defines the transversal section area variation along the element (**s** is the section variable position towards the end of the element, measured along the curve), is known; the axial strength modulus of transversal sections is given by the **W(s)** function, there are also given the material's density $\boldsymbol{\rho}$ and admissible stress $\boldsymbol{\sigma_a}$.

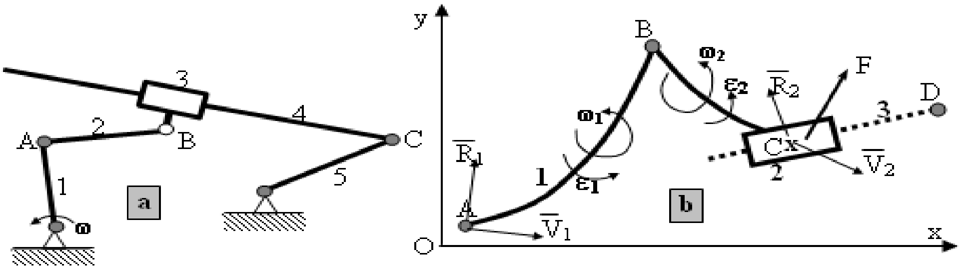


Fig. 1 : a. A simple mechanism; b. Kinetostatic scheme of RRT dyade

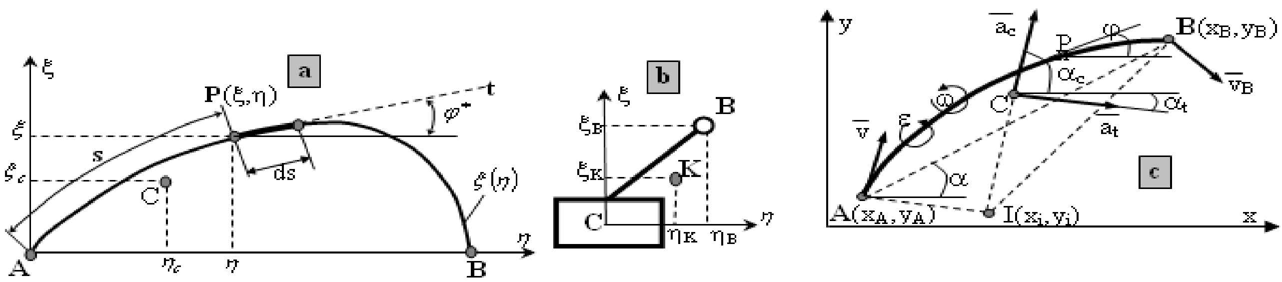


Fig. 2: a,b Dyade RRT elements geometry; c Kinematical parameters for element 1 in the global reference system

In order to analyse the geometry and dynamics of **element 1**, two coordinate systems are used for simplification: **ηOξ** system (intrinsic, according to fig. 2.c), attached to the element, and the global axes system **xOy**. The mechanical loadings are: couple reaction forces and moments (determined with the Mechanism Theory of Mechanics methods and considered known in this study); the inertia loadings due to instant spinning (considered unknown and necessary to be defined); the external loading **F** which acts on the element from the dyade. The strength analysis of the dyade elements are based on the sectional forces (N,T,M) and the mechanical stresses produced by these ones.

3. Dyade geometry

According to fig.3.a., the curve **ξ= ξ(η)** that defines the dyade element one axis is known. The point **P(η,ξ)** is defined on the curve and named in strength calculus as ***current section***. The angle **φ*** of tangent **Pt** is defined in **P**; the variable part of **s** length element (A-P arc), has the centroid in $\mathbf{C(\eta_C, \xi_C)}$. The element section in point P has **A(s)** area and **W(s)** strength modulus (are known as variation laws). The tangent angle in point **P** is:

$$\varphi^* = arctg\left(\frac{d\xi}{d\eta}\right). \tag{1}$$

The coordinates of mass center from A-P segment curve are:

$$\eta_c = \frac{\int_0^\lambda \eta \cdot A(s)\sqrt{1+\left(\frac{d\xi}{d\eta}\right)^2}\, d\eta}{\int_0^\lambda A(s)\sqrt{1+\left(\frac{d\xi}{d\eta}\right)^2}\, d\eta}, \qquad \xi_c = \frac{\int_0^\lambda \xi(\eta)\cdot A(s)\sqrt{1+\left(\frac{d\xi}{d\eta}\right)^2}\, d\eta}{\int_0^\lambda A(s)\sqrt{1+\left(\frac{d\xi}{d\eta}\right)^2}\, d\eta}. \tag{2}$$

The elementary arc **ds** and curve **A-P** length and masses are:

$$dm = \rho \cdot A(s)\cdot ds\,, \quad m = \rho\int_0^\eta A(s)\sqrt{1+\left(\frac{d\xi}{d\eta}\right)}\cdot d\eta. \quad ds = \sqrt{1+\left(\frac{d\xi}{d\eta}\right)^2}\, d\eta\,, \tag{3}$$

The second element of the dyade (the slide) is geometrically defined in figure 3.b. The own coordinate system centered in C, conveniently defined on the sliding direction; the joint is defined by its coordinates B(ξ_B, η_B), the centroid position of element two, point K(ξ_K, η_K), is defined similar.

4. Inertia loadings

There is considered the dyade element 1, with **P** current section and **C** centroid of **AP** variable part, according to figure 3. c. The velocities of extreme couples are known, with their axes projections $\overline{V_A}(v_{AX}, v_{AY})$ and $\overline{V_B}(v_{BX}, v_{BY})$. The mass center coordinates, in the global system, are:

$$x_c = \eta_c \cos\alpha - \xi_c \sin\alpha + x_A, \quad y_c = \eta_c \sin\alpha + \xi_c \cos\alpha + y_A, \tag{4}$$

where α is the element position angle.

The instant spin center coordinates of element are determined, point **I**(x_i, y_i), at the intersection of perpendiculars on the couples velocities; the coordinates defining relations are:

$$X_I = \frac{b_2 - b_1}{m_1 - m_2}, \quad Y_I = m_2 \cdot x_i + b_2. \tag{5}$$

The parameters from relation (5) are determined in this way:

$$m_2 = tg\left(\frac{\pi}{2} + arctg\frac{v_{2y}}{v_{2x}}\right), \quad b_2 = y_B - m_2 x_B \cdot \quad m_1 = tg\left(\frac{\pi}{2} + arctg\frac{v_{Ay}}{v_{Ax}}\right), \quad b_1 = y_A - m_1 x_A, \tag{6}$$

There have been made the following notations: vAX, vAY, vBX, vBY the couples velocities projections; xA, yA, xB, yB the couples coordinates. Instant rotation radius CI and its αc angle are:

Tangential acceleration angle is:

$$\overline{CI} = \sqrt{(x_c - x_i)^2 + (y_c - y_i)^2}, \quad \alpha_c = arctg\frac{y_c - y_i}{x_c - x_i}. \tag{7}$$

$$\alpha_t = 0{,}5 \cdot \pi - \alpha_n \tag{8}$$

The mass center accelerations of element portion, defined by the current section **P** are:

$$a_c = \omega^2 \cdot \overline{CI}, \quad a_t = \varepsilon \cdot \overline{CI}. \tag{9}$$

The tangent direction at element axis, in the current section **P,** is:

$$\varphi = \alpha + \varphi^* = arctg\frac{y_B - y_A}{x_B - x_A} + arctg\frac{d\xi}{d\eta}. \tag{10}$$

The coordinates of current section in the global system are:

$$x = \eta\cos\alpha - \xi\sin\alpha + x_A, \qquad y = \eta\sin\alpha + \xi\cos\alpha + y_A \tag{11}$$

5. Sectional forces

The scheme from fig.3. is used. In the current section **P**, the tensile force (N), shear force (T), obtained by the force projection on the tangent direction, and bending moment (M) exist. On the element part A-P, next forces act: reaction force from A couple ($\overline{R_{A1}}$), inertial spin ($\overline{F_c}$) and tangential ($\overline{F_t}$) forces. Being more simple to use the axes projections, the forces components that act are:

$$R_1 = \sqrt{R_{1x}^2 + R_{1y}^2}, \quad F_{cx} = m \cdot a_c \cos\alpha_c, \quad F_{cy} = m \cdot a_c \sin\alpha_c, \quad F_{tx} = m \cdot a_t \cos\alpha_t, \quad F_{ty} = m \cdot a_t \sin\alpha_t. \tag{12}$$

The section forces produced by the ($\overline{R_1}$) reaction are:

$$N_1 = -R_{1x}\cos\varphi - R_{1y}\sin\varphi, \; T_1 = -R_{1x}\sin\varphi + R_{1y}\cos\varphi, \; M_1 = -R_{1x}(y - y_A) + R_{1y}(x - x_A). \tag{13}$$

The section forces produced by the spinning $\mathbf{F_c}$ and tangential $\mathbf{F_t}$ forces are:

$$N_c = -F_{cx}\cos\varphi - F_{cy}\sin\varphi, \; T_c = -F_{cx}\sin\varphi + F_{cy}\cos\varphi, \; M_c = -F_{cx}(y - y_c) + F_{cy}(x - x_c),$$

$$N_t = -F_{tx}\cos\varphi + F_{ty}\sin\varphi, \; T_t = -F_{tx}\sin\varphi - F_{ty}\cos\varphi, \; M_t = -F_{tx}(y - y_c) - R_{ty}(x - x_c). \tag{14}$$

The efforts in current section are obtained by summing:

$$N=N1+Nc+Nt, \qquad T=T1+Tc+Tt, \qquad M=M1+Mc+Mt\,; \tag{15}$$

The relation (16) can be written in function of the compound elements in this way:

$$N = -\left(R_{1x}\cos\varphi + R_{1y}\sin\varphi\right) - m\cdot\overline{CI}\cdot\left[\omega^2\cos(\varphi-\alpha_c) + \varepsilon\cdot\cos(\varphi+\alpha_t)\right], \tag{16}$$
$$T = -\left(R_{1x}\sin\varphi + R_{1y}\cos\varphi\right) - m\cdot\overline{CI}\cdot\left[\omega^2\sin(\varphi-\alpha_c) + \varepsilon\cdot\cos(\varphi-\alpha_t)\right],$$
$$M = -R_{1x}(y-y_1) + R_{1y}(x-x_1) + m\,\overline{CI}\cdot\left\{\omega^2\left[(x-x_c)\sin\alpha_c - (y-y_c)\cos\alpha_c\right] - \varepsilon\left[(x-x_c)\sin\alpha_t + (y-y_c)\cos\alpha_t\right]\right\}$$

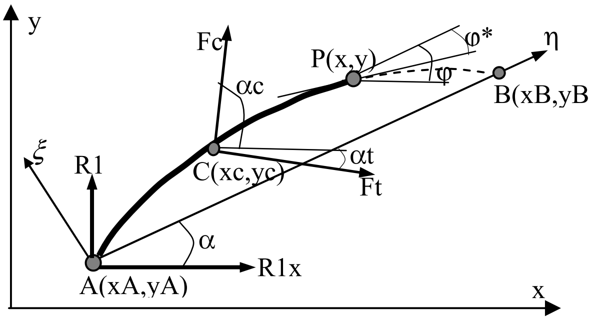


Fig. 3. The statics of element 1 from the dyade

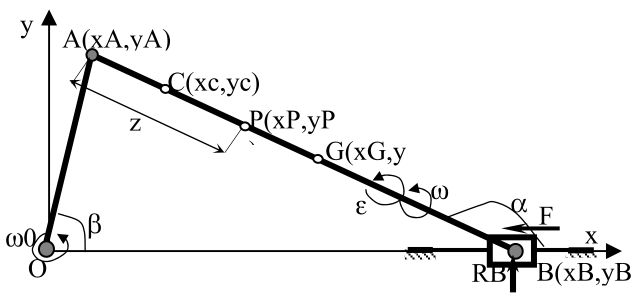


Fig. 4. Connecting rod mechanism

6. Calculus example

In order to present the usage way of the proposed mathematical model, it is analyzed the dyade shown in fig. 4. It is used a simple mechanism in order to decrease the volume of this paper. The fundamental geometrical characteristics are:

- Elements length: OA= r= 0,15 m; AB= l= 0,3 m;
- The kinematics: n= 3 rot/s; ω_0= 6π;
- Section and material: 1x3 cm; ρ=7800 kg/m^3, A=3 cm^2, W=1,5 cm^3, d_b=1 cm, A_b=0,8 cm^2
- Masses, forces: m_1=0,35 kg, m_2=0,7, m_3=0,25 kg, F=50 N.

The angles that define the elements position are:

$$\beta(t) = \omega_0\, t, \quad \gamma(t) = \arcsin\left[k\cdot\sin\beta(t)\right], \; \beta(t) = \omega_0\, t, \quad \alpha(t) = \pi - \gamma(t), \quad k = r/l = 0{,}5 \tag{17}$$

Couples and important points (**C,P,G**) coordinates are:

$$x_A(t) = r\cdot\cos\beta, \qquad y_A(t) = r\cdot\sin\beta, \; x_B(t) = r\cdot\cos\beta + l\cdot\cos(\pi-\alpha), \quad y_B = 0. \tag{18}$$
$$x_C(t) = r\cdot\cos\beta + 0{,}5z\cdot\cos(\pi-\alpha), \quad y_C(t) = r\cdot\sin\beta - 0{,}5z\cdot\sin(\pi-\alpha),$$
$$x_P(t) = r\cdot\cos\beta + z\cdot\cos(\pi-\alpha), \quad y_P(t) = r\cdot\sin\beta - z\cdot\sin(\pi-\alpha),$$
$$x_G(t) = r\cdot\cos\beta + 0{,}5l\cdot\cos(\pi-\alpha), \quad y_G(t) = r\cdot\sin\beta - 0{,}5l\cdot\sin(\pi-\alpha).$$

The acceleration of application points from inertia forces are:

$$a_B = -r\,\omega_0^2\cos\beta + l\,\varepsilon\sin(\pi-\alpha) - l\,\omega^2\cos(\pi-\alpha),$$
$$a_{Cx} = -r\,\omega_0^2\cos\beta + 0{,}5z\,\varepsilon\sin(\pi-\alpha) - 0{,}5z\,\omega^2\cos(\pi-\alpha), \tag{19}$$
$$a_{Cy} = -r\,\omega_0^2\sin\beta + 0{,}5z\,\varepsilon\cos(\pi-\alpha) + 0{,}5z\,\omega^2\sin(\pi-\alpha),$$
$$a_{Gx} = -r\,\omega_0^2\cos\beta + 0{,}5l\,\varepsilon\sin(\pi-\alpha) - 0{,}5l\,\omega^2\cos(\pi-\alpha),$$
$$a_{Gy} = -r\,\omega_0^2\sin\beta + 0{,}5l\,\varepsilon\cos(\pi-\alpha) + 0{,}5l\,\omega^2\sin(\pi-\alpha).$$

The angular velocity and acceleration are:

$$\omega(t) = -\frac{3\pi\cos(\omega_0 t)}{\sqrt{1-0{,}25\sin^2(\omega_0 t)}}, \qquad \varepsilon(t) = \frac{18\pi\,\omega_0\sin(\omega_0 t)}{\sqrt{\left[3+\cos^2(\omega_0 t)\right]^3}}. \quad \omega(t) = \frac{d\alpha}{dt}, \qquad \varepsilon(t) = \frac{d\omega}{dt}, \tag{20}$$

From the moments equilibrum condition towards the joint A, results the slide reaction force, in this way:

$$R_B(t) = \frac{1}{x_B(t) - x_A(t)}\cdot\left\{F\cdot y_A(t) + m_2 g\left[x_G(t) - x_A(t)\right] - m_2 a_{Gx}\left[y_A(t) - y_G(t)\right] - m_2 a_{Gy}\left[x_G(t) - x_A(t)\right]\right\}. \tag{21}$$

From the equilibrum conditions of force projections on coordinate axes, for the dyade, the reaction force components from joint A result in this way:

$$R_{Ax}(t) = F - m_2\, a_{Gx}, \qquad R_{Ay}(t) = m_2 g - m_2\, a_{Gy} - R_B(t). \tag{22}$$

The forces that act on the pins of A and B joints are:

$$R_A(t)=\sqrt{R_{Ax}^2+R_{Ay}^2}, \qquad F_B(t)=m_3 a_B(t), \qquad R_B(t)=\sqrt{R_B^2+(F_B-F)^2}. \qquad (23)$$

The components of inertia forces from the variable part that act in the centroid C are:

$$F_{Cx}(t,z)=A\cdot\rho\cdot z\cdot a_{Cx}(t,z), \qquad F_{Cy}(t,z)=A\cdot\rho\cdot z\cdot a_{Cy}(t,z), \qquad G_C(z)=A\rho z. \qquad (24)$$

The section forces in the current rod section (point P) are:

$$N(t,z)=R_{Ay}(t)\cdot\sin[\gamma(t)]-R_{Ax}(t)\cdot\cos[\gamma(t)]+F_{Cy}(t,z)\cdot\sin[\gamma(t)]-G_C(z)\cdot\sin[\gamma(t)]-F_{Cx}(t,z)\cdot\cos[\gamma(t)], \qquad (25)$$

$$T(t,z)=R_{Ay}(t)\cdot\cos[\gamma(t)]+R_{Ax}(t)\cdot\sin[\gamma(t)]+F_{Cy}(t,z)\cdot\cos[\gamma(t)]-G_C(z)\cdot\cos[\gamma(t)]+F_{Cx}(t,z)\cdot\sin[\gamma(t)],$$

$$M(t,z)=R_{Ay}(x_A-x_P)+R_{Ax}(y_A-y_P)+(F_{Cy}-G_C)(x_C-x_P)+F_{Cx}(y_C-x_P). \qquad (26)$$

It is neglected the effect of shear force, the normal stress in the current section P is (according to [4]):

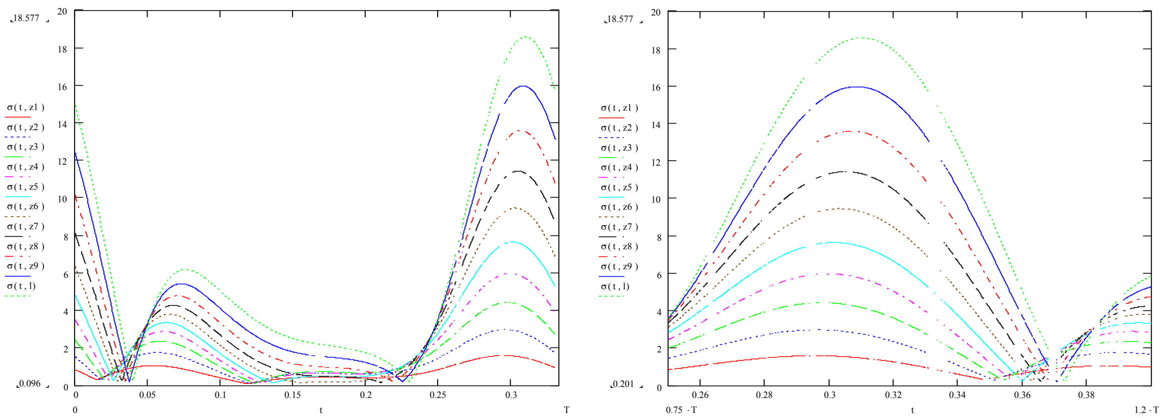
$$\sigma(t,z)=\frac{|N(t,z)|}{A}+\frac{|M(t,z)|}{W} \qquad (27)$$

The shear stresses in the joints pins are:

$$\tau_A(t)=\frac{R_A(t)}{A_b}, \quad \tau_B(t)=\frac{R_B(t)}{A_b}. \qquad (28)$$

The defined mathematical model allows different parameters determining, specific for the mechanism. For example, the motion moment that acts in joint O is calculated:

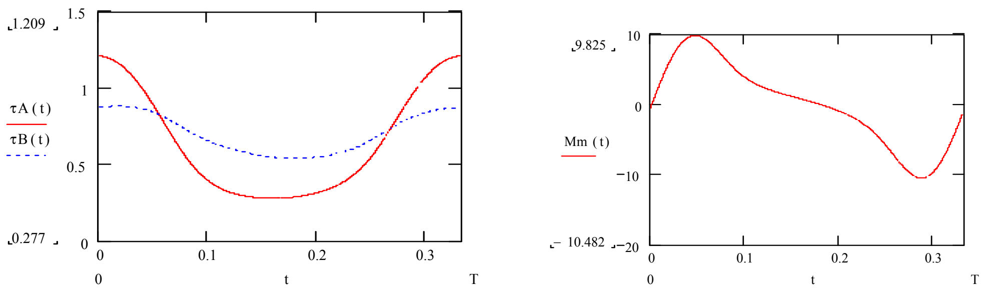
$$M_0(t)=R_{Ax}(t)\cdot r\cdot\sin(\omega_0 t)+R_{Ay}(t)\cdot r\cdot\cos(\omega_0 t). \qquad (29)$$



a. b.

Fig.5: The normal stress in the rod (the time variation of a T period and in element length , z=0, 1k·l k=1, 2, 3, … , 10)
a- stress variation in the time gap 0 – T , b- the time variation in the time gap 0.75T- 1,2T

The variation of normal stresses in the rod is given in fig.5.a, in ten consecutive sections spaced with 3 cm; the calculus time is a period of T= 1/n= 0,333s, corresponding to a complete spin of the crank. It is observed that the maximum stress is 18,6 MPa, at the coupling edge of the slide, at a moment very close of T period. In fig.5.b, it is detailed this period (0,75T- 1,2T). It is observed that the shear force loading is very small (according to fig.6.a). The driving moment variation in a period time is shown in fig.6.b.



a b

Fig.6: The stress in pins and driving part (towards the point O); a- stress variation in pins, b- driving moment variation

7. Conclusions

For the structural group RRT dyade, compound of a certain planar mechanism, there is defined the mathematical model that determines the mechanical stresses in any transversal section of components. The bar hypothesis, straight or curved (with high radius), used at modelling corresponds to the most concrete cases and can be generalized. If this hypothesis is not valid, the results error grade is established in that situation. Even if the calculus outlines underlying the definition of the mathematical model are relatively simple, the model application at a concrete situation, hard mathematical equations will be obtained and hard to be used, so the mathematical model must be used with a programming software.

The exemplification of the usage of defined model proves its utility and correctness. The calculus program used for stress distribution is not presented in this paper in order to decrease its volume. The calculus program was simple to be made because of the defined algorithm clearness. The module of automatic stress analysis in the elements of a mechanism that contain RRT dyades is necessary to be coupled with an automatic calculus module for kinematic and kinetostatic elements of the mechanism. Such a comprehensive program (mechanism analysis, stress analysis in the mechanism elements) will allow a complete study, being a powerful means of investigation.

From the calculus model we observe moderate stresses, which show a good dyade elements dimensioning. Having the calculus program, we can verify various sections, until we obtain the desired stress level. There can also be made a program of iterative calculus that allows mechanisms elements section dimensioning. We must also say that, in order to narrow the volume study, there have not been taken into account all the mechanisms kinetostatic elements (this paper studies only their mechanical strength).

The authors consider that the calculus model presented in this paper is original and cannot be found in any paper or book from the *Mechanisms* or *Strength of Materials* field.

As a further research in this field, we would like to obtain the strains and see how these influence the mechanism functionality. With the defined stresses formulas and by using energy methods, we can research the aspects regarding the geometrical precision of the mechanism.

8. Acknowledgements

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9. References

[1] P. Antonescu, C. Ocnărescu, O. Antonescu*, Mechanisms*, Printech, Bucharest, 2008

[2] D. Ilincioiu, V. Roşca*, RRR Dyade Strength, 10th International Symposium on Experimental Stress and Material Testing*, Sibiu, 2004, pp. 5.55-5.60

[3] D. Ilincioiu, A. Margine, *Stress Analysis in RRT Dyade*, *2nd International Conference "Mechatronics, Microtechnologies and new materials"*, Targoviste, 2004, pp. V42-V48

[4] D. Ilincioiu, C. Miriţoiu, A. Pădeanu, *Strength of Materials*, Universitaria, Craiova, 2010