Comparison Performance Between PID and LQR Controllers for 4-leg Voltage-Source Inverters

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Abstract. This article presents a comparative evaluation based on time response specification performance between two controllers for 4-leg Voltage-Source Inverters. 4-leg inverters are adopted in many applications such as UPS systems and satellite earth stations. A Linear-Quadratic-Regulator (LQR) and a Proportional-Integrated-Derivative (PID) method are proposed and compared for controlling the 4-leg Voltage-Source Inverters. Proposed Controllers are designed and simulated using MATLAB/Simulink. Simulation Results show that both the controllers are capable of controlling the time domain response of 4-leg inverters successfully. According to the results, LQR method gives the better performance, such as rise time, settling time, compared to PID controller.

Keywords: LQR Control, 4-leg Voltage-Source Inverters, PID Control, MATLAB/Simulink

1. Introduction

The four-leg inverter (FLI) is utilized in three-phase four-wire power converter due to its superior performance characteristics such as relatively low DC bus voltage and switching loss, and capability to handle unbalanced load currents [1]. Four-leg inverters are used in critical application such as Uninterruptable Power Supplies (UPS) systems, Satellite earth stations, and active power filters [2]. So, sinusoidal waveform is needed in order to achieve balanced voltage for these inverters.

Four-leg inverter model has been studied in the literature [1], [4]. Many Literature Works such as [2], [3] proposed classic controller for 4-leg inverters. On the other hand a modern control strategy such as Pole Placement discussed and proposed strategy was static, adjustable transient state condition and good tracking reference input [4]. Pole placement for controller design based on characteristics of the desired closed loop poles of the system. This is usually difficult to define, particularly for large dimension systems. In addition, with pole placement design there is no consideration given to the actuation effort that gets used during closed loop operation.

The PID controller, which has proportional, integral and derivative elements, is widely applied in feedback control of industrial processes. These controllers are described with their simple structure and principle. PID controllers are also enable provide good performance for various systems. However, PID method in many cases such as parameter variations or disturbances is not appropriate. In this paper a PID controller is designed to improve the behaviour of 4-Leg inverters.

In order to overcome some problems that faced by PID controller, the other type of control methods can be developed such as Linear-Quadratic Regulator (LQR) optimal control. LQR is a control scheme that gives the best possible performance with respect to some given measure of performance. The performance measure is a quadratic function composed of state vector and control input.

This paper is organized as follows. In the next section, the system modeling of the 4-Leg voltage-source inverters are described. Subsequently, the basic concepts of PID and LQR control schemes are explained in the
section 3. In section 4, the results of simulation of the system with each controller are presented. Finally, in the last section, the conclusion of this study is given.

In this study a state feedback controller using the Linear Quadratic Regulator (LQR) design technique and a PID controller for 4-Leg Inverters is designed. By compared the best tuning output from these controllers, it can be investigated which controller will provide a better performance for 4-Leg inverters.

2. Modeling of 4-Leg Inverters

The average model of the three-phase four leg voltage source inverter with LC filter in stationary coordinates is shown in Fig.1.

![Fig.1: Average large signal model of the inverter with LC filter](image)

The inverter output voltage $V_{\text{out}}(i = a, b, c)$ and line current $i_{\text{line}}(i = a, b, c)$ can be expressed as:

$$
[V_{\text{a}} V_{\text{b}} V_{\text{c}}]^T = [d_{\text{a}} d_{\text{b}} d_{\text{c}}]^T V_{\text{dc}}
$$

$$
I_{\text{p}} = [d_{\text{a}} d_{\text{b}} d_{\text{c}}] [i_{\text{a}} i_{\text{b}} i_{\text{c}}]^T
$$

(1)

And, $d_{\text{if}} (i = a, b, c)$ are the line-to-neutral duty cycles controlled in a way so as to produce sinusoidal waveform voltages at output of the filter without considering the load.

In order to generate sinusoidal output voltages and apply control methods a DC operating point is needed so RL load is assumed in rotating coordinates.

The steady state output load voltages are DC quantities as:

$$
[V_{\text{a}} V_{\text{q}} V_{\text{o}}]^T = \begin{bmatrix} V_{\text{m}} \\ 0 \\ 0 \end{bmatrix}
$$

(2)

$V_{\text{m}}$: rated output voltage.

Due to balanced RL load, it can be considered as:

$$
R_{\text{d}} = R_{\text{q}} = R_{\text{o}} = R_{\text{a}} = R_{\text{b}} = R_{\text{c}} = R
$$

$$
L_{\text{d}} = L_{\text{q}} = L_{\text{o}} = L_{\text{a}} = L_{\text{b}} = L_{\text{c}} = L
$$

(3)

$R$: resistance, $L$: the inductance of the load. $R_{\text{a}}, R_{\text{b}}, R_{\text{c}}$ are resistance of the load in phases $(a,b,c)$. $R_{\text{d}}, R_{\text{q}}, R_{\text{o}}$ are equivalent resistance of the load in rotating coordinate $dqo$.

The power stage is a coupled multi-variable multi-loop system. The coupling and present independently control of the system can be neglected as shown in Fig. 4 presents the control structure for load voltage loop control.
Consequently, the state-space equations of the system can be derived for $d$ channel as:

$$\dot{x} = Ax + Bu$$

\[ Y = Cx (4) \]

$$x = \begin{bmatrix} i_{d} \\ V_d \\ i_d \end{bmatrix}, \quad A = \begin{bmatrix} 0 & -\frac{1}{L} & 0 \\ \frac{1}{L} & 0 & -\frac{1}{C} \\ 0 & \frac{1}{L} & \frac{R_d}{L} \end{bmatrix}, \quad B = \begin{bmatrix} V_d \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad (5)$$

3. Control Methods

In this section, two control methods are proposed and explained in detail which is PID controller and Linear Quadratic Regulator (LQR). Furthermore, the following design specifications have been made to evaluate the performance of both control schemes.

1) The percentage of overshoot, less than 5%.
2) The Rise time ($T_r$), less than 3 second.
3) The settling time ($T_s$), less than 5 second.
4) Steady-state error is within 2% of the initial value.

3.1. PID Controller

The Proportional-Integral-Derivative (PID) method is a kind of feedback controller which is generally based on the error ($e$) between desired set point and measured process value. The error is then used to adjust some input to the process in order to its defined set point. Three parameters must be designed in the PID controller and each parameter has an effect on the error.

The transfer function of the PID controller is written as:

$$G_{PID}(s) = K_p + \frac{K_i}{s} + K_d \cdot s \quad (6)$$

Where, $K_p$ is the controller gain, $K_i$ is the integral time and $K_d$ is the derivative time.

It is important to determine appropriate parameters to guarantee stability and system performance. There are several methods for tuning PID parameters [4]. However, in this study the three parameters of PID controller values are computed by trial and error.

3.2. LQR Controller

LQR is a control scheme that provides the best possible performance with respect to some given measure of performance. The LQR design problem is to design a statefeedback controller $K$ such that the objective function $J$ (equation 8) is minimized. In this method a feedback gain matrix is designed which minimizes the objective function in order to achieve some compromise between the use of control effort, the magnitude, and the speed of response that will guarantee a stable system.

For a continuous-time linear system described by

$$\dot{x} = Ax + Bu \quad (7)$$

With a cost functional defined as

$$J = \int_0^\infty (x^TQx + u^TRu)dt \quad (8)$$

Where $Q$ and $R$ are the weight matrices, $Q$ is required to be positive definite or positive semi-definite symmetry matrix; $R$ is required to be positive definite symmetry matrix. One practical method is to $Q$ and $R$ to be diagonal matrix. The value of the elements in $Q$ and $R$ is related to its contribution to the cost function $J$.

The feedback control law that minimizes the value of the cost is

$$u = -Kx (9)$$

$K$ is given by

$$K = R^{-1}B^TP (10)$$

And $P$ can be found by solving the continuous time algebraic Riccati equation

$$A^TP + PA - PB^{-1}B^TP + Q = 0 \quad (11)$$

4. Simulation and Results

The considered 4-Leg voltage-source inverter model parameters are based on the system described in [ ].

Where:

$L = 333 \ \mu H, \ C = 100 \ \mu F, \ R_d = R_q = R_o = 100 \ \Omega, L_d = L_q = L_o = 200 \ \mu H, V_g = 800 \ \nu, V_m = 400 \ \nu$
The PID controller designed and simulated based on the following control parameters:

\[ K_p = 300, K_i = 3, K_d = 185 \]

Also the best LQR controller parameters are

\[ Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 0 \end{bmatrix}, R = 0.0000001 \]

Fig.4 demonstrates the inapplicable and unstable behavior of the open loop 4-Leg inverter system. The time response of the closed loop system with the simulated PID controller and LQR controller are shown in Fig.5 and Fig.6 respectively. In order to investigate and evaluation the performance of theses controllers easily, the time response of closed-loop system with both controllers is shown in Fig.7.

From below figures 5 and 6 it can be realized that both of these controllers are suitable to utilize to control the 4-Leg inverters due to both can give zero steady-state error, fast response and no overshoots at the transient response. However, the results proven that the LQR method acts better than PID controller in terms of its faster response.

![Fig.4: Time response of the open loop system](image1)

![Fig.5: Time response of the closed-loop system with proposed LQR controller](image2)

![Fig.6: Time response of the closed-loop system with proposed PID controller](image3)

![Fig.7: Time response comparison between proposed LQR and PID controller](image4)
5. Conclusion

In this study, based on the mathematical model of 4-leg inverters, two controllers, PID and LQR, are designed and compared to investigate a more appropriate control method. The simulation results demonstrate that both of these controllers are effective and suitable for improving the time domain characteristics of system response, such as settling time and overshoots. According to the results, LQR method give the better performance compared to PID controller. However, as a method the determination of PID parameters are easier to obtain using LQR method.

Various control methods can be implemented for 4-Leg inverters such as Fuzzy control, Fuzzy PID control, Adaptive control, Sliding Mode Control, etc. In addition, for LQR design procedure the weighting matrices can be determined by evolutionary algorithms, such as PSO and GA.

6. References