2011 International Conference on Information and Electronics Engineering
IPCSIT vol.6 (2011) © (2011) IACSIT Press, Singapore

Smith Compensator Using Modified IMC for Unstable Plant with Time Delay

Hiroki Shibasaki, Manato Ono, Naohiro Ban and Kazusa Matsumoto,

Kazuhiro Sasaki, Junichi Endo and Yoshihisa Ishida

Department of Science and Technology

Meiji University

Kanagawa, JAPAN

Abstract. In this paper, a design of smith compensator using modified IMC for an unstable plant with time delay is proposed. An unstable plant with time delay is controlled by the method of a predicted-state feedback. However, when we introduce an input-side disturbance in the system, it has large influence response for the disturbance to control an unstable plant with time delay. In addition, we introduce the disturbance compensator to overcome the problem in the predicted-state feedback. We applied the predicted-state feedback with disturbance compensator. Furthermore, we introduce this method to the modified smith predictor. In these simulation studies, it is shown that the proposed method has a superior performance in an input side disturbance of the plant response. The nominal system can reduce large influence for disturbance. Furthermore, the robust system was confirmed high robustness.

Keywords: Unstable Plant, Time delay, Internal Model Control, Smith compensator

1. Introduction

Smith Predictor [1] and Internal Model Control (IMC) [2] are effective methods to control a plant with time delay. However, these methods cause a steady–state error by an input side disturbance for the plants with an integrator. Astrom *et al*. [3] proposed new Smith predictor with a superior performance. However, this method has the problem that parameter adjustments are complicated. Authors [4] proposed the discrete modified IMC. This method does not cause a steady-state error by an input-side disturbance for plants with an integrator.

However, Smith Predictor and Internal Model Control do not control an unstable plant with time-delay. In recent year, there are many methods [proposed against the problem. We introduce some papers. De Paor et al. [5] [6] proposed a modified Smith predictor which has a constraint on ratio of time delay to the time constant. Watanabe et al. [7] method is based on the output prediction of a plant. Furukawa et al. [8] proposed a control strategy based on a predicted-state feedback technique with an observer. K.K. Tan's method [9] is based on Generalized Predictive Control approach. Basilio del-Muro-Cuellar et al. [10] used an observer-based predictor with partitions of time delay to stabilize an unstable plant. Above all, authors [11] proposed the modified smith predictor which can control an unstable plant with time delay and eliminates a steady-state error cause by an input-side disturbance.

In this paper, we propose the predicted-state feedback system with the disturbance compensator to eliminate large influence for the input-side disturbance. Furthermore, we introduce the predicted-state feedback system with disturbance compensator to the modified smith predictor [11]. Our proposed method has high robustness against an input-side disturbance.

2. Plant Predictor

+ Hiroki Shibasaki. Tel.: + 81(0)44-934-7307
.*E-mail address*: meiji.shibasaki@gmail.com.

We state the design method of a plant predictor. State space equations which are a controllable and an observable controlled plant with time delay express (1) and (2).

$$\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{b}u(k-d) \tag{1}$$

$$y(k)=\boldsymbol{c}\boldsymbol{x}(k) \tag{2}$$

Then we put $k \to k+1$ in (1). We get 2 sample predicted state of the plant $\tilde{\boldsymbol{x}}(k+2)$.

$$\begin{aligned}\tilde{\boldsymbol{x}}(k+2)&=\boldsymbol{A}\boldsymbol{x}(k+1)+\boldsymbol{b}u(k-d+1)\\&=\boldsymbol{A}^2\boldsymbol{x}(k)+\boldsymbol{A}\boldsymbol{b}^{d-1}u(k-d)+\boldsymbol{b}u(k-d+1)\end{aligned} \tag{3}$$

Repeating the same operation, we get a predicted state of the plant $\tilde{\boldsymbol{x}}(k+d)$.

$$\begin{aligned}\tilde{\boldsymbol{x}}(k+d)&=\boldsymbol{A}^d\boldsymbol{x}(k)+\boldsymbol{A}^{d-1}\boldsymbol{b}u(k-d)+\cdots+\boldsymbol{b}u(k-1)\\&=\boldsymbol{A}^d\boldsymbol{x}(k)+\sum_{i=1}^{d}\boldsymbol{A}^{d-i}\boldsymbol{b}u(k-d+i-1)\end{aligned} \tag{4}$$

Then, a predicted output of the plant $\tilde{y}(k+d)$ is

$$\tilde{y}(k+d)=\boldsymbol{c}\tilde{\boldsymbol{x}}(k+d) \tag{5}$$

The above predicted state equations are not observed directly, then, an observer which estimates states of the plant is used.

$$\hat{\boldsymbol{x}}(k+1)=\boldsymbol{A}\hat{\boldsymbol{x}}(k)+\boldsymbol{b}u(k-d)+L\{y(k)-\hat{y}(k)\} \tag{6}$$

$$\hat{y}(k+d)=\boldsymbol{c}\hat{\boldsymbol{x}}(k+d) \tag{7}$$

Where, L is observer gain. However, the above observer has time delay d in the input u (k-d).

3. Predicted-State Feedback with Disturbance Compensator

We proposed the plant predictor to a state feedback control with a disturbance compensator M to eliminate large influence for an input- side disturbance in this system for a plant with time delay. Fig.1 is a block diagram of the predicted-state feedback system with the disturbance compensator.

A plant predictor is connected by the observer in this section. For this reason, (4) is written by (8).

$$\hat{\boldsymbol{x}}(k+d)=\boldsymbol{A}^d\hat{\boldsymbol{x}}(k)+\sum_{i=1}^{d}\boldsymbol{A}^{d-i}\boldsymbol{b}u(k-d+i-1) \tag{8}$$

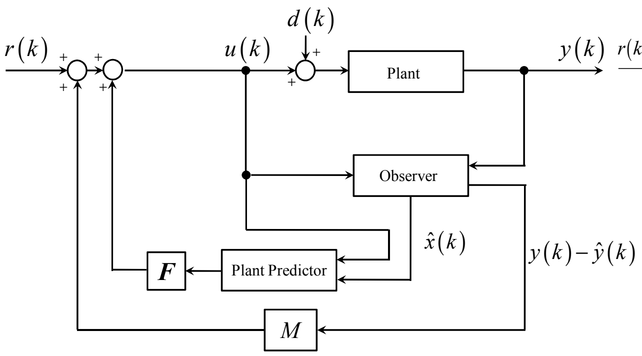

Fig.1 Predicted-State Feedback System with Disturbance Compensator

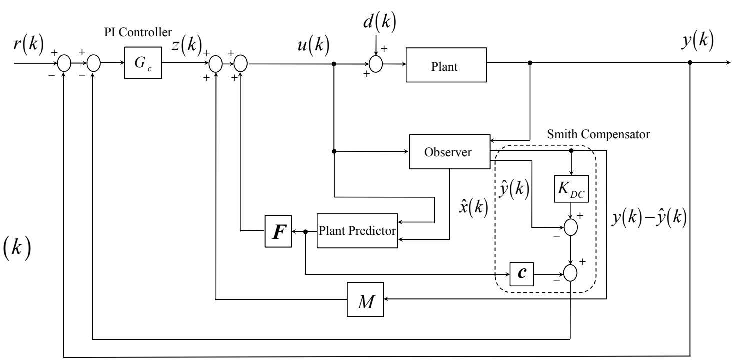

Fig.2 Block diagram of the proposed method

The predicted-state feedback system with disturbance compensator is written by (1), (6), and (8). Note that (1), (6), and (8) are changed (9) (10), and (11) by z- transformation.

$$(z\boldsymbol{I}-\boldsymbol{A})\boldsymbol{X}(z)-\boldsymbol{b}z^{-d}U(z)=0 \tag{9}$$

$$(z\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{Lc})\hat{\boldsymbol{X}}(z)-\boldsymbol{b}z^{-d}U(z)-\boldsymbol{LcX}(z)=0 \tag{10}$$

$$z^{d}\hat{\boldsymbol{X}}(z)=\boldsymbol{A}^{d}\hat{\boldsymbol{X}}(z)+\left(I-\boldsymbol{A}^{d}z^{-d}\right)(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{b}U(z) \tag{11}$$

A manipulated valuable $U(z)$ is written (12)

$$U(z)=\boldsymbol{F}z^{d}\hat{\boldsymbol{X}}(z)+M\left\{Y(z)-\hat{Y}(z)\right\} \tag{12}$$

From (9), (10), and (12), we express in matrix form.

$$\begin{bmatrix} z\boldsymbol{I}-\boldsymbol{A} & \boldsymbol{0} & -\boldsymbol{b}z^{-d} \\ -\boldsymbol{Lc} & z\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{Lc} & -\boldsymbol{b}z^{-d} \\ -M\boldsymbol{c} & -\boldsymbol{FA}^{d}+M\boldsymbol{c} & 1-\boldsymbol{F}\left(\boldsymbol{I}-\boldsymbol{A}^{d}z^{-d}\right)(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{b} \end{bmatrix}\begin{bmatrix} \boldsymbol{X}(z) \\ \hat{\boldsymbol{X}}(z) \\ U(z) \end{bmatrix}=0 \tag{13}$$

The characteristic equation is given by

$$\begin{aligned}\phi(z)&=\det\begin{bmatrix} z\boldsymbol{I}-\boldsymbol{A} & \boldsymbol{0} & -\boldsymbol{b}z^{-d} \\ -\boldsymbol{Lc} & z\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{Lc} & -\boldsymbol{b}z^{-d} \\ -M\boldsymbol{c} & -\boldsymbol{FA}^{d}+M\boldsymbol{c} & 1-\boldsymbol{F}\left(\boldsymbol{I}-\boldsymbol{A}^{d}z^{-d}\right)(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{b} \end{bmatrix}\\ &=\det(z\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{Lc})\det(z\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{bF})\end{aligned} \tag{14}$$

If the characteristic equation does not equal 0 by an observer gain *L* and a Feedback gain *F*, the system is stable.

When we input a reference input *R* (*z*), a manipulated valuable *U* (*z*) is written (15).

$$U(z)=\boldsymbol{F}z^{d}\hat{\boldsymbol{X}}(z)+M\left\{Y(z)-\hat{Y}(z)\right\}+R(z) \tag{15}$$

From (9), (10), and (15), the target value response *Y* (*z*)/*R* (*z*) is

$$\frac{Y(z)}{R(z)}=\begin{bmatrix}\boldsymbol{c} & \boldsymbol{0} & 0\end{bmatrix}\begin{bmatrix} z\boldsymbol{I}-\boldsymbol{A} & \boldsymbol{0} & -\boldsymbol{b}z^{-d} \\ -\boldsymbol{Lc} & z\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{Lc} & -\boldsymbol{b}z^{-d} \\ -M\boldsymbol{c} & -\boldsymbol{FA}^{d}+M\boldsymbol{c} & 1-\boldsymbol{F}\left(\boldsymbol{I}-\boldsymbol{A}^{d}z^{-d}\right)(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{b} \end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{0} \\ \boldsymbol{0} \\ 1\end{bmatrix} \tag{16}$$

(16) is change into a small matrix and we solve the equation. And we get the transfer function with a feedback gain.

$$\begin{aligned}\frac{Y(z)}{R(z)}&=\boldsymbol{c}(z\boldsymbol{I}-\boldsymbol{A}-\boldsymbol{bF})^{-1}\boldsymbol{b}z^{-d}\\ &=G_{f}z^{-d}\end{aligned} \tag{17}$$

When we input a disturbance D (z), a plant is written (18). And (18) is changed (19) by z- transformation.

$$\boldsymbol{x}(k+1)=\boldsymbol{A}\boldsymbol{x}(k)+\boldsymbol{b}\{u(k-d)+D(k-d)\} \tag{18}$$

$$(z\boldsymbol{I}-\boldsymbol{A})\boldsymbol{X}(z)-\boldsymbol{b}z^{-d}U(z)=\boldsymbol{b}z^{-d}D(z) \tag{19}$$

From (10), (12), and (19), the target value response $Y(z)/R(z)$ is

$$\frac{Y(z)}{D(z)}=[\boldsymbol{c}\quad \boldsymbol{0}\quad 0]\begin{bmatrix} z\boldsymbol{I}-\boldsymbol{A} & \boldsymbol{0} & -\boldsymbol{b}z^{-d} \\ -\boldsymbol{L}\boldsymbol{c} & z\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{L}\boldsymbol{c} & -\boldsymbol{b}z^{-d} \\ -M\boldsymbol{c} & -\boldsymbol{F}\boldsymbol{A}^{d}+M\boldsymbol{c} & 1-\boldsymbol{F}(\boldsymbol{I}-\boldsymbol{A}^{d}z^{-d})(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{b} \end{bmatrix}^{-1}\begin{bmatrix} \boldsymbol{b}z^{-d} \\ \boldsymbol{0} \\ 0 \end{bmatrix} \tag{20}$$

We solve (20) by same operation in such way (17)

$$\frac{Y(z)}{D(z)}=G_f\left[\begin{array}{l} 1-\boldsymbol{F}(\boldsymbol{I}-\boldsymbol{A}^{d}z^{-d})(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{b} \\ \qquad +(M\boldsymbol{c}-\boldsymbol{F}\boldsymbol{A}^{d})(z\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{L}\boldsymbol{c})^{-1}\boldsymbol{b}z^{-d} \end{array}\right] \tag{21}$$

In case of an unstable plant, (21) is written by

$$\frac{Y(z)}{D(z)}=G_f\left[1-\boldsymbol{F}\sum_{i=1}^{d}\boldsymbol{A}^{d-i}z^{-d+i-1}\boldsymbol{b}+(M\boldsymbol{c}-\boldsymbol{F}\boldsymbol{A}^{d})(z\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{L}\boldsymbol{c})^{-1}\boldsymbol{b}z^{-d}\right] \tag{22}$$

To eliminate the influence of disturbance, we solve the following the equation. And we get the disturbance compensator M in (24).

$$\lim_{z\to 1}\left[1-\boldsymbol{F}\sum_{i=1}^{d}\boldsymbol{A}^{d-i}z^{-d+i-1}\boldsymbol{b}+(M\boldsymbol{c}-\boldsymbol{F}\boldsymbol{A}^{d})(z\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{L}\boldsymbol{c})^{-1}\boldsymbol{b}z^{-d}\right]=0 \tag{23}$$

$$M=\frac{-1+\boldsymbol{F}\boldsymbol{A}^{d}(\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{L}\boldsymbol{c})^{-1}\boldsymbol{b}+\boldsymbol{F}\sum_{i=1}^{d}\boldsymbol{A}^{d-i}\boldsymbol{b}}{\boldsymbol{c}(\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{L}\boldsymbol{c})^{-1}\boldsymbol{b}} \tag{24}$$

In this method, the disturbance is eliminated by the disturbance compensator M.

Furthermore, we introduce the predicted-state feedback system with disturbance compensator to the modified smith predictor proposed by authors [11]. Fig.2 is a block diagram of the proposed method.

4. Simulation Study

In this section, we show the simulation result of proposed method. Fig.3 is shown a response of nominal system and Fig.4 is shown a response of robust system. A unit step input is introduced at time t=0[sec]. We discrete the system by the zero-order hold method at sampling time T_s=10[msec].

$$G(s)e^{-Ls}=\frac{2}{s^2-2.2s+1}e^{-Ls} \tag{25}$$

The weight of the cost function are Q/R=0.002. The observer gain is L = [0.0482 0.0220] ^{T} and the feedback gain is F= [4.3539 -0.0179]. The disturbance compensator is M=-14.9247. The compensate gain is K_{DC} =1.0862. The PI controller parameters are K_P = 1.0, and K_I = 0.288. An input-side disturbance is introduced $D(s)$ = -0.01/s at time t=50[sec]. We add +20% error in time delay to show the robustness of the proposed method.

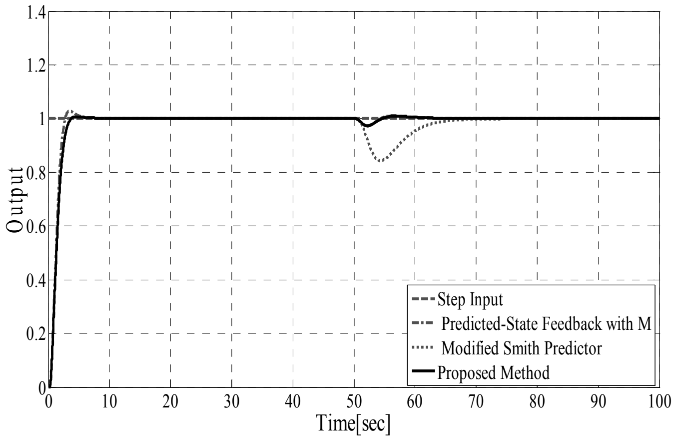

Fig.3 Nominal System

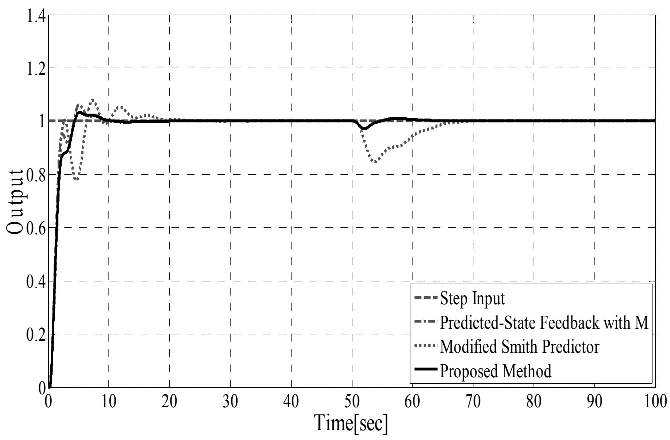

Fig.4 Robust System (+20% error in time delay)

5. Conclusion

In this paper, we have proposed smith compensator using modified IMC for an unstable plant with time delay. The first of all, we have proposed the predicted-state feedback system with the disturbance compensator to eliminate the large influence for the input-side disturbance and it is introduced to the modified smith predictor. In this simulation study, our proposed method has high robustness against an input-side disturbance.

6. References

[1] O.J. Smith, "A Controller to Overcome Dead Time," *ISA Journal*, vol.6, pp. 28-33, Feb., 1959

[2] M. Morari and E. Zafiriou, Robust *Process Control*, Prentice Hall, New Jersey, 1989

[3] K.J. Astrom, C.C. Hang and B.C. Lim, "A Smith predictor for controlling a process with an integrator and long dead –time,"*IEEE Trans. On* Automatic *Control*, vol.39, pp. 343-345, Feb., 1994.

[4] I. Yoichi, M. Yuichi and I. Yoshihisa, "Discrete Modified Internal Model Control for plants with an integrator and time delay," *IEICE Trans. On* Fundamentals *of Electronics, Communication and Computer Sciences*, vol.J90-A, pp. 663-665, Aug., 2007. (in Japanese)

[5] Annraoi M.De Paor, "A Modified Smith Predictor and Controller for Unstable Processes with Time Delay, " *Int. J.Control*,Vol. 41,No.4,1985,pp.1025-1036.

[6] Annraoi M.De Paor and Ruth P. K. Egan, "Extension and Partial Optimization of a Modified Smith Predictor and Controller for Unstable Process with Time Delay," *Int. J.Control*,Vol. 50(4),pp1315-1326,1989.

[7] K. Watanabe and M. Ito, "A Process-model Control for Linear Systems with Delay," *IEEE Trans. Automatic Control*, vol. AC-26, no. 6, Dec., 1981.

[8] T.Furukawa and E. Shimemura, "Predictive Control for Systems with Time Delay,"*Int.J.control*,Vol. 37,No.2,1983,pp.399-412.

[9] K. K. tan, T.H.Lee, and F. M. Leu, "Optimal Smith-predictor Design Based on a GPC Approch," Ind .Eng .Chem .Res. 41, pp. 1242-1248, 2002.

[10] B. del-Muro-Cuellar, M. Velasco-Villa, O. Jiménes-Ramírez, G. Fernández-Anaya, and J. Álvarez-Ramírez, "Observer-Based Smith Prediction Scheme for Unstable Plus Time Delay Processes," *Ind. Eng. Chem. Res.*, 46(14) , pp.4906-4913 , 2007.

[11] O.Manato, S. Hiroki, M. Kazusa, S. Kazuhiro and I. Yoshihisa, "Discrete Modified Smith Predictor Based on Optimal Control Method for a Plant with an Integrator," *2010 IEEE International Conference on Systems, Man and Cybernetics* (SMC 2010) ,pp.630-636,Octorber,2010