

Some Consideration in Microstretch Thermoelastic Diffusive Medium with Mass Diffusion-A Review

Devinder Singh⁺, Rajneesh Kumar, and Arbind Kumar

Deptt. of (Mathematics), Guru Nanak Dev Engg.College Ludhiana (Punjab) INDIA (141006)
Department of Mathematics, K.U.Kurukshetra (Haryana) INDIA
Punjab Technical university Jalandhar (Punjab) INDIA

Abstract. The general solution of the equations for a homogeneous isotropic microstretch thermo elastic medium with mass diffusion for two dimensional problems is obtained due to normal and tangential forces. The Integral transform technique is used to obtain the components of displacements, microrotation, stress and mass concentration, temperature change and mass concentration. A particular case of interest is deduced from the present investigation. The transformed displacements, stresses and pore pressures are functions of the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f(s, \xi, z)$. To obtain the solution of the problem in the physical domain.

Keywords: Normal and tangential force, microstretch, thermoelastic, The Integral transform technique.

1. Introduction

Eringen [1] developed a theory of thermomicrostretch elastic solids in which he included microstructural expansions and contractions. The material points of microstretch solids can stretch and contract independently of their translations and rotations. Microstretch continuum is a model for Bravais Lattice with a basis on the atomic level and a two phase dipolar solid with a care on the macroscopic level. The asymptotic behavior of solutions and an existence result were presented by Bofill and Quintanilla [2]. Cicco [3], discussed the stress concentration effects in microstretch elastic bodies. Eringen [4,5] established a uniqueness theorem for the mixed initial boundary valued problem. This theory was illustrated with the solution of one-dimensional wave and compared with lattice dynamical results. The theory of microstretch elastic solid differs from the theory of micropolar elasticity in the sense that there is an additional degree of freedom called stretch and there is an additional kind of stress called microstretch vector. Diffusion is defined as the spontaneous movement of the particles from a high concentration region to the low concentration region, and it occurs in response to a concentration gradient expressed as the change in the concentration due to change in position. Kumar and Partap [6] discussed the dispersion of axisymmetric waves in thermo microstretch elastic plate. Kumar and Partap [7] analyzed free vibrations for Rayleigh-Lamb waves in a microstretch thermoelastic plate with two relaxation times. Kumar et al [8] investigated the disturbance due to force in normal and tangential direction and porosity effect by using normal mode analysis in fluid saturated porous medium. Othman [9] studied the effect of rotation on plane wave propagation in the context of Green-Naghdi (GN) theory type-II by using the normal mode analysis. Ezzat and Awad adopted the normal mode analysis technique to obtain the temperature gradient, displacement, stresses, couple stress, micro rotation etc. Othman [10] studied the effect of diffusion on 2-dimensional problem of generalized thermoelastic with Green-Naghdi theory and obtained the expressions for displacement components, stresses, temperature fields, concentration and chemical potential by using normal mode analysis. Othman [11] used normal mode analysis technique to obtain the expressions for displacement

⁺ Corresponding author. Tel.: 9878634245; fax: 016125050,
E-mail: despathania@yahoo.com

components, force, stresses, temperature, couple stress and microstress distribution in a thermomicrostretch elastic medium with temperature dependent properties for different theories. A spherical inclusion in an infinite isotropic microstretch medium was discussed by Liu and Hu [12]. Quintanilla [13] studied the spatial decay for the dynamical problem of thermo-microstretch elastic solids. Singh and Tomar [14] discussed Rayleigh-Lamb waves in a microstretch elastic plate cladded with liquid layers. Tomar and Garg [15] discussed the reflection and refraction of plane waves in microstretch elastic medium. Kumar et al [16] investigated the effect of viscosity on plane wave propagation in heat conducting transversely isotropic micropolar viscoelastic half space. In the present paper general model of the equations of microstretch thermoelastic with mass diffusion for a homogeneous isotropic elastic solid is developed. Some special cases have been deduced from the present investigation.

2. Basic Equations

The basic equations for homogeneous, isotropic microstretch generalized thermoelastic diffusive solids in the absence of body force, body couple, stretch force and heat source are given by:

$$(\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) + (\mu + K)\nabla^2 \mathbf{u} + K\nabla \times \boldsymbol{\phi} + \lambda_0 \nabla \phi^* - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla T - \beta_2 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \nabla C = \rho \mathbf{u} \quad (1)$$

$$(\gamma \nabla^2 - 2K)\boldsymbol{\phi} + (\alpha + \beta)\nabla(\nabla \cdot \boldsymbol{\phi}) + K\nabla \times \mathbf{u} = \rho j \ddot{\boldsymbol{\phi}} \quad (2)$$

$$(\alpha_0 \nabla^2 - \lambda_1)\phi^* - \lambda_0 \nabla \cdot \mathbf{u} + \nu_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T + \nu_2 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = \frac{\rho j_0}{2} \ddot{\phi}^* \quad (3)$$

$$K^* \nabla^2 T = \rho c^* \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + \beta_1 T_0 \left(\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2}\right) \nabla \cdot \mathbf{u} + \nu_1 T_0 \left(\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2}\right) \phi^* + a T_0 \left(\frac{\partial}{\partial t} + \gamma_1 \frac{\partial^2}{\partial t^2}\right) C \quad (4)$$

$$D\beta_2 \nabla^2 (\nabla \cdot \mathbf{u}) + Da \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla^2 T + \left(\frac{\partial}{\partial t} + \varepsilon \tau^0 \frac{\partial^2}{\partial t^2}\right) C - Db \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \nabla^2 C = 0 \quad (5)$$

$$t_{ij}(\lambda_0 \phi^* + \lambda u_{r,r})\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \epsilon_{ijk} \phi_k) - \beta_1 \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \delta_{ij} T - \beta_2 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \delta_{ij} C = 0 \quad (6)$$

$$m_{ij} = \alpha \phi_{r,r} \delta_{ij} + \beta \phi_{i,j} + \gamma \phi_{j,i} + b_0 \epsilon_{mji} \phi_{,m}^* \quad (7)$$

3. Formulation of the problem

We consider a rectangular Cartesian coordinate system $OX_1X_2X_3$ with x_3 -axis pointing vertically outward the medium. We consider a normal or tangential force to be acting at the free surface of microstretch thermoelastic medium with mass diffusion half space. For two dimensional problems the displacement and microrotation vectors are of the form:

$$\mathbf{u} = (u_1, 0, u_3), \boldsymbol{\phi} = (0, \phi_2, 0), \quad (8)$$

We have introduced the dimensionless quantities in (1)-(5) and obtained

$$u'_r = \frac{\rho \omega^* c_1}{\beta_1 T_0} u_r, \quad x'_r = \frac{\omega^*}{c_1} x_r, \quad t' = \omega^* t, \quad \phi^{*'} = \frac{\rho c_1^2}{\beta_1 T_0} \phi^*, \quad T' = \frac{T}{T_0}, \quad \tau'_1 = \omega^* \tau_1 \quad (9)$$

With the aid of equations (9) the equations (1)-(5) reduce to:

$$a_1 \frac{\partial e}{\partial x_1} + a_2 \nabla^2 u_1 - a_3 \frac{\partial \phi_2}{\partial x_3} + a_4 \frac{\partial \phi^*}{\partial x_1} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial x_1} - a_5 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \frac{\partial C}{\partial x_1} = \ddot{u}_1, \quad (10)$$

$$a_1 \frac{\partial e}{\partial x_3} + a_2 \nabla^2 u_3 + a_3 \frac{\partial \phi_2}{\partial x_1} + a_4 \frac{\partial \phi^*}{\partial x_3} - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \frac{\partial T}{\partial x_3} - a_5 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \frac{\partial C}{\partial x_3} = \dot{u}_3, \quad (11)$$

$$\phi_2 - 2a_6 \phi_2 + a_6 \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right) = a_7 \ddot{\phi}_2, \quad (12)$$

$$\nabla^2 \phi^* - a_8 \phi^* - a_9 e + a_{10} \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T + a_{11} \left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = a_{12} \ddot{\phi}^*, \quad (13)$$

$$\nabla^2 T = a_{13} \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) T + a_{14} \left(\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2}\right) e + a_{15} \left(\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2}\right) \phi^* + a_{16} \left(\frac{\partial}{\partial t} + \gamma_1 \frac{\partial^2}{\partial t^2}\right) C, \quad (14)$$

$$\nabla^2 e + a_{17} \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla^2 T + a_{18} \left(\frac{\partial}{\partial t} + \varepsilon \tau^0 \frac{\partial^2}{\partial t^2}\right) C - a_{19} \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \nabla^2 C = 0, \quad (15)$$

The displacement components u_1 and u_3 are related to potential functions ϕ and ψ as:

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1} \quad (16)$$

Using the relation (16), in the equations (10)-(15), we obtain:

$$(a_1 + a_2) \nabla^2 \phi - \ddot{\phi} + a_4 \phi^* - \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T - a_2 \left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = 0, \quad (17)$$

$$\left(\nabla^2 - a_8 - a_{12} \frac{\partial^2}{\partial t^2}\right) \phi^* - a_9 \nabla^2 \phi + a_{10} \left(1 + \tau_1 \frac{\partial}{\partial t}\right) T + a_{11} \left(1 + \tau^1 \frac{\partial}{\partial t}\right) C = 0, \quad (18)$$

$$a_{13} \left(1 + \tau_0 \frac{\partial}{\partial t}\right) \dot{T} + a_{14} \left(\frac{\partial}{\partial t} + \varepsilon \tau_0 \frac{\partial^2}{\partial t^2}\right) \nabla^2 \phi + a_{15} \left(1 + \varepsilon \tau_0 \frac{\partial}{\partial t}\right) \dot{\phi}^* + a_{16} \left(1 + \gamma_1 \frac{\partial}{\partial t}\right) \dot{C} - \nabla^2 T = 0, \quad (19)$$

$$\nabla^4 \phi + a_{17} \left(1 + \tau_1 \frac{\partial}{\partial t}\right) \nabla^2 T + a_{18} \left(\frac{\partial}{\partial t} + \varepsilon \tau^0 \frac{\partial^2}{\partial t^2}\right) C - a_{19} \left(1 + \tau^1 \frac{\partial}{\partial t}\right) \nabla^2 C = 0, \quad (20)$$

$$a_2 \nabla^2 \psi - \ddot{\psi} + a_3 \phi_2 = 0, \quad (21)$$

$$\nabla^2 \phi_2 - 2a_6 \phi_2 - a_6 \nabla^2 \psi = a_7 \ddot{\phi}_2, \quad (22)$$

Here $\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_3^2}$ is the Laplacian operator. Now, we define the Laplace transform:

$$\bar{f}(x, z, s) = \int_0^\infty f(x, z, t) e^{-st} dt \quad (23)$$

So, the equations (10)-(15) become:

$$(a_1 + a_2) \nabla^2 \bar{\phi} - s^2 \bar{\phi} + a_4 \bar{\phi}^* - (1 + \tau_1 s) \bar{T} - a_2 (1 + \tau^1 s) \bar{C} = 0, \quad (24)$$

$$\left(\nabla^2 - a_8 - a_{12} s^2\right) \bar{\phi}^* - a_9 \nabla^2 \bar{\phi} + a_{10} (1 + \tau_1 s) \bar{T} + a_{11} (1 + \tau^1 s) \bar{C} = 0 \quad (25)$$

$$a_{13} (s + \tau_0 s^2) \bar{T} + a_{14} (s + \varepsilon \tau_0 s^2) \nabla^2 \bar{\phi} + a_{15} (s + \varepsilon \tau_0 s^2) \bar{\phi}^* + a_{16} (s + \gamma_1 s^2) \bar{C} - \nabla^2 \bar{T} = 0 \quad (26)$$

$$\nabla^4 \bar{\phi} + a_{17} (1 + \tau_1 s) \nabla^2 \bar{T} + a_{18} (s + \varepsilon \tau^0 s^2) \bar{C} - a_{19} (1 + \tau^1 s) \nabla^2 \bar{C} = 0, \quad (27)$$

$$a_2 \nabla^2 \bar{\psi} - s^2 \bar{\psi} + a_3 \bar{\phi}_2 = 0, \quad (28)$$

$$\nabla^2 \bar{\phi}_2 - 2a_6 \bar{\phi}_2 - a_6 \nabla^2 \bar{\psi} = s^2 a_7 \bar{\phi}_2, \quad (29)$$

Now, we define the Fourier's transform:

$$\hat{f}(z, \xi, s) = \int_{-\infty}^\infty \bar{f}(x, z, s) e^{i\xi x} dx \quad (30)$$

Applying Fourier's transform to equations (24)-(29), we obtain the equations:

$$(a_1 + a_2) \left(\frac{d^2}{dx_3^2} - \xi^2 \right) \widehat{\phi} - s^2 \widehat{\phi} + a_4 \widehat{\phi}^* - (1 + \tau_1 s) \widehat{T} - a_2 (1 + \tau^1 s) \widehat{C} = 0, \quad (31)$$

$$\left(\frac{d^2}{dx_3^2} - \xi^2 - a_8 - a_{12} s^2 \right) \widehat{\phi}^* - a_9 \left(\frac{d^2}{dx_3^2} - \xi^2 \right) \widehat{\phi} + a_{10} (1 + \tau_1 s) \widehat{T} + a_{11} (1 + \tau^1 s) \widehat{C} = 0, \quad (32)$$

$$a_{13} (s + \tau_0 s^2) \widehat{T} + a_{14} (s + \varepsilon \tau_0 s^2) \left(\frac{d^2}{dx_3^2} - \xi^2 \right) \widehat{\phi} + a_{15} (s + \varepsilon \tau_0 s^2) \widehat{\phi}^* + a_{16} (s + \gamma_1 s^2) \widehat{C} - \left(\frac{d^2}{dx_3^2} - \xi^2 \right) \widehat{T} = 0 \quad (33)$$

$$\left(\frac{d^2}{dx_3^2} - \xi^2 \right)^2 \widehat{\phi} + a_{17} (1 + \tau_1 s) \left(\frac{d^2}{dx_3^2} - \xi^2 \right) \widehat{T} + a_{18} (s + \varepsilon \tau^0 s^2) \widehat{C} - a_{19} (1 + \tau^1 s) \left(\frac{d^2}{dx_3^2} - \xi^2 \right) \widehat{C} = 0, \quad (34)$$

$$a_2 \left(\frac{d^2}{dx_3^2} - \xi^2 \right) \widehat{\psi} - s^2 \widehat{\psi} + a_3 \widehat{\phi}_2 = 0, \quad (35)$$

$$\left(\frac{d^2}{dx_3^2} - \xi^2 \right) \widehat{\phi}_2 - 2a_6 \widehat{\phi}_2 - a_6 \left(\frac{d^2}{dx_3^2} - \xi^2 \right) \widehat{\psi} = s^2 a_7 \widehat{\phi}_2, \quad (36)$$

On solving equations (24)-(27), we obtain:

$$\left[A_0 \frac{d^8}{dx_3^8} + A_1 \frac{d^6}{dx_3^6} + A_2 \frac{d^4}{dx_3^4} + A_3 \frac{d^2}{dx_3^2} + A_5 \right] (\widehat{\phi}, \widehat{\phi}^*, \widehat{T}, \widehat{C}) = 0,$$

And on solving equations (31)-(34), we obtain:

$$\left[\frac{d^4}{dx_3^4} + A_6 \frac{d^2}{dx_3^2} + A_7 \right] (\widehat{\phi}_2, \widehat{\psi}) = 0,$$

The solution of the above system satisfying the radiation conditions that $(\widehat{\phi}, \widehat{\phi}^*, \widehat{T}, \widehat{C}, \widehat{\phi}_2, \widehat{\psi}) \rightarrow 0$ as $x_3 \rightarrow \infty$ are given as following:

$$(\widehat{\phi}, \widehat{\phi}^*, \widehat{T}, \widehat{C}) = \sum_{i=1}^4 (1, \alpha_{1i}, \alpha_{2i}, \alpha_{3i}) M_i e^{-m_i x_3}, \quad (\widehat{\phi}_2, \widehat{\psi}) = \sum_{i=5}^6 (1, \beta_{1i}) M_i' e^{-m_i x_3},$$

4. Boundary Conditions

Consider normal and tangential force acting on the surface $x_3 = 0$ along with vanishing of couple stress, microstress, mass diffusion and temperature gradient at the boundary and considering insulated and infinite boundary at $x_3 = 0$. Mathematically this can be written as:

$t_{33} = -F_1 \delta(x) \delta(t), t_{31} = -F_2 \delta(x) \delta(t), m_{32} = 0, \lambda_3 = 0, \frac{\partial T}{\partial x_3} = F_3 \delta(x) \delta(t), \frac{\partial C}{\partial x_3} = F_4 \delta(x) \delta(t)$, Here F_1 and F_2 are the magnitude of the applied force. On applying the Laplace transform and then Fourier transform the above conditions reduces to:

$$\widehat{t}_{33} = -\widehat{t}_{31} = -F_2 \widehat{m}_{32} = 0, \widehat{\lambda}_3 = 0, \frac{\partial \widehat{T}}{\partial x_3} = F_3, \quad \frac{\partial \widehat{C}}{\partial x_3} = F_4 \quad (37)$$

Using these boundary conditions and solving the linear equations formed, we obtain:

$$\widehat{t}_{33}, \widehat{t}_{31}, \widehat{m}_{32}, \widehat{\lambda}_3, \widehat{u}_1, \widehat{u}_3, \widehat{T}, \widehat{C} \quad (38)$$

Case I- Normal Stress: To obtain the expressions due to normal stress we must set $F_2 = 0$ in the boundary conditions (37).

Case II- Tangential Stress: To obtain the expressions due to tangential stress we must set $F_1 = 0$ in the boundary conditions (37).

Particular cases:

- (i) If we take $\tau_l = \tau^l = 0, \varepsilon = I, \gamma_l = \tau_0$, in Eqs. (38) we obtain the corresponding expressions of stresses, displacements and temperature distribution for L-S theory.
- (ii) If we take $\varepsilon = 0, \gamma_l = \tau^0$ in Eqs. (38) the corresponding expressions of stresses, displacements and temperature distribution are obtained for G-L theory.
- (iii) Taking $\tau^0 = \tau^l = \tau_0 = \tau_l = \gamma_l = 0$ in Eqs. (38) yield the corresponding expressions of stresses, displacements and temperature distribution for Coupled theory of thermoelasticity.

Special cases:

- (a) Microstretch Thermoelastic Solid: If we neglect the diffusion effect in Eqs. (38) We obtain the corresponding expressions of stresses, displacements and temperature for microstretch thermoelastic solid.
- (a) Micropolar Thermoelastic Diffusive Solid: If we neglect the microstretch effect in Eqs. (38) We obtain the corresponding expressions of stresses, displacements and temperature for micropolar thermoelastic diffusive solid.

5. Conclusion

The general solution of the equations for a homogeneous isotropic microstretch thermo elastic medium with mass diffusion for two dimensional problems is obtained due to normal and tangential forces. The Integral transform technique is used to obtain the components of displacements, microrotation, stress and mass concentration, temperature change and mass concentration. A particular case of interest is deduced from the present investigation. The transformed displacements, stresses and pore pressures are functions of the parameters of Laplace and Fourier transforms s and ξ respectively and hence are of the form $f(s, \xi, z)$. To obtain the solution of the problem in the physical domain.

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Professor (Dr.) Devinder Singh is in Guru Nanak Dev Engineering College Ludhiana Punjab(INDIA) teaching Mathematics since 2007 to B.Tech/M.Tech/MBA/MCA students. Singh is also technical members of International conference in Singapore. Singh is also member of Punjab Science Academy, International association of Engineers (IAENG), International J.of Mathematical Sciences and Engineering Applications, Member, International Association of Computer Science and Information technology. Arbind Kumar is a research scholar at PTU Jalandhar. Rajneesh Kumar(Dr.)is Professor at Kurukshetra university ,Kurukshetra.