2012 4th International Conference on Signal Processing Systems (ICSPS 2012)
IPCSIT vol. 58 (2012) © (2012) IACSIT Press, Singapore
DOI: 10.7763/IPCSIT.2012.V58.29

Optimal Design and Performance Analysis for Chaotic Spreading Sequences with Arbitrary Period

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Abstract. Chaos is a deterministic random process in nonlinear dynamic systems. In this article, I propose a new optimal design method of chaotic spreading sequences with arbitrary period, which optimizes the binary-quantization thresholds with different fractal parameters and initial values. This article simulates and analyzes the design results. The results show that the method can generate a huge amount of high-performance chaotic spreading sequences with arbitrary period. The article supports the application of the chaotic sequences in spread spectrum communications.

Keywords: chaotic spreading sequences, logistic mapping, optimal design.

1. Introduction

Spread spectrum communication is a widely used communication mode, of which Direct Sequence Spread Spectrum is the most widely used. In the Direct Sequence Spread Spectrum, the nature of spreading sequences is very important, which need good autocorrelation and cross-correlation properties [1]. The traditional spreading sequences, such as m sequences, Gold sequences and so on, have the limited set of available codes and low linear complexity, which greatly affect the capacity and security performance of the system [2]. In addition, the period of m and Gold sequence is generally 2^n-1, and the choice of period is limited. People have always been looking for the spreading sequences which have better performance, larger number.

Chaos is the deterministic random process in the nonlinear dynamic systems, which is non-periodic, non-convergence, but boundary and extremely sensitive dependent on the initial value [3]. In the recent years, with the further research of the chaos, the chaotic spreading sequences are paid more and more attention [4] [5]. The chaotic systems have the white noise properties of the class random, so the chaotic spreading sequences have the good correlation properties; the sensitive dependence on the initial value can provide many available sequences with arbitrary period, which is the advantage relative to the traditional m sequences and Gold sequences. In addition, the chaotic systems have good randomness, high complexity and good security, whose generating methods could not easily be detected.

The chaotic sequences need optimal design when they are used as the spreading sequences. Taking the Logistic chaotic mapping for example, there are two kinds of common optimal methods. One method is mentioned in the literature [2], under the situation of the Logistic full mapping, selecting the sequence generated by the initial value of which the performance meets the set threshold as the preferred chaotic sequences. While the disadvantages of the method are the limited number of the generated chaotic sequences and the generating methods are oversimplified so that it's easy to be detected. The either method is mentioned in the literature [6], getting a very complicated generating function expression by optimally designing the mapping function of the Logistic chaotic mapping. The disadvantages of the method are that the generating

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function expressions are too complicated, which is not good for the implementation and application of engineering, and the performances are not improved significantly.

This article focuses on optimal design of the chaotic spreading sequences with arbitrary period. We select different binary-quantization threshold ε for different parameters r and initial value x_0, and optimal design the parameters by analyzing the balance and the correlation of the sequences when the different parameters of $\{r\ \varepsilon\ x_0\}$ compose. The method can generate a huge amount of high-performance chaotic spreading sequences with arbitrary period. The article supports the application of the chaotic sequences in spread spectrum communications.

2. Characteristics of the Original Logistic Chaotic Sequence

Logistic map is a dynamic system which has been extensively studied. It shows chaotic behavior. Its expression is:

$$x_{k+1} = rx_k(1-x_k) \tag{1}$$

Studies have shown that the dynamic behavior of Logistic mapping is closely related to the external parameters r (also known as fractal parameters). Logistic mapping shows distinct periodicity or chaotic state with different r. More specifically, Logistic mapping shows periodicity when $0\le r\le 3.5699456\ldots$.When $3.5699456\ldots\le r\le 4$, Logistic mapping shows chaotic state. In this condition, the sequence $\{x_k$: $k=0,1,2\ldots\}$ generated by the Logistic mapping is non-periodic, non-convergence and very sensitive to the initial value.

The steps of generating chaotic sequence by using chaotic mapping are as follows: Give an initial value x_0 to chaotic mapping. Then, iterate to generate the original chaotic sequence $\{x_k$: $k=0,1,2\ldots\}$. The amplitude of this sequence is continuous and analog, which is called the original chaotic sequence. Binary quantify the original chaotic sequences to obtain chaotic spreading sequences $\{1,-1\}$.

First, we analyze the statistic properties of the performance of the original chaotic sequences. The invariant measure of chaotic sequences $\{x_k$: $k=0,1,2\ldots\}$ [3]:

$$\rho(x) = \begin{cases} \dfrac{1}{\pi\sqrt{x(1-x)}} & 0<x<1 \\ 0 & other \end{cases} \tag{2}$$

2.1 . Using knowledge of probability theory, the mean can be obtained as follows:

$$\begin{aligned} \bar{x} &= \lim_{x\to\infty}\sum_{i=1}^{N} x_i = \int_0^1 x\rho(x)\,dx = \int_0^1 \frac{1}{\pi}\sqrt{\frac{x}{1-x}}dx \\ &= \frac{1}{\pi}\Big[-\sqrt{x(1-x)} - \arcsin\sqrt{1-x}\Big]_0^1 \\ &= 0.5 \end{aligned} \tag{3}$$

Theoretical mean is 0.5. This characterizes in the statistics that the number of elements greater than 0.5 in the original chaotic sequence is almost the same as the number less than 0.5, indicating that the sequence has good balance.

2.2 . Calculate the correlation of original chaotic sequence according to the known invariant measure.

Autocorrelation function is $R_{AC}(m)$, when m=0:

$$\begin{aligned} R_{AC}(0) &= \lim_{x\to\infty}\frac{1}{N}\sum_{k=0}^{N-1} x_k^2 - \bar{x}^2 \\ &= \int_0^1 x^2\rho(x)\,dx \\ &= \frac{1}{\pi}\left[\int_0^1\sqrt{\frac{x}{1-x}}dx - \int_0^1\sqrt{x(1-x)}dx\right] - 0.25 \\ &= 0.125 \end{aligned} \tag{4}$$

When $m \neq 0$:

$$\begin{aligned} R_{AC}(m) &= \lim_{x\to\infty} \frac{1}{N-|m|} \sum_{k=0}^{N-|m|-1} x_k x_{k+m} - \bar{x}^2 \\ &= \int_0^1 x f^m(x)\rho(x)\,dx - 0.25 \\ &= 0 \end{aligned} \tag{5}$$

Cross-correlation function:

$$\begin{aligned} R_{CC}(m) &= \lim_{x\to\infty} \frac{1}{N-|m|} \sum_{k=0}^{N-|m|-1} (x_{1k}-\bar{x}_1)(x_{2k+m}-\bar{x}_2) \\ &= \int_0^1\int_0^1 x_1 x_2 f^m(x_2)\rho(x_1)\rho(x_2)\,dx_1 dx_2 - \bar{x}^2 \\ &= 0 \end{aligned} \tag{6}$$

From the above results, we can obtain the following conclusions: the mean of original chaotic sequence is constant, the autocorrelation function is the δ function, and cross-correlation function is zero. Its features of probability and statistics are consistent with white noise and it can be used as spreading sequences in spread spectrum communications. The influencing factor and performance evaluation of quantified chaotic spreading sequences need further analysis.

3. Binary-Quantified Logistic Chaotic Sequence

The following expression (7) is of spreading sequence after binary-quantization:

$$c_k = \begin{cases} 1 & x_k > \varepsilon \\ -1 & x_k < \varepsilon \end{cases} \tag{7}$$

In expression (7), x_k is the original chaotic sequence and ε is the threshold of binary-quantization.

We get $c_k \in \{-1, 1\}$, k=0, 1, 2… after quantization. Factors affecting its properties are fractal parameters r and the initial value x_0 of logistic mapping function and the threshold of binary-quantization ε. In other words, we can determine the properties of chaotic sequence after we defined $\{r, \varepsilon, x_0\}$.

In this article, we need to analyze the balance and correlation of spreading sequence. The balance of sequence uses the following method to measure. Suppose P is the number of "1" and Q is the number of "-1". The standard of the sequence balance E is given by:

$$E = (P-Q)/(P+Q) \tag{8}$$

In practical applications, the sequence length is not infinitely long. The elements number of the upper and lower than 0.5 is not entirely same as shown in Fig. 1, so we need to select binary-quantization threshold according to requirements.

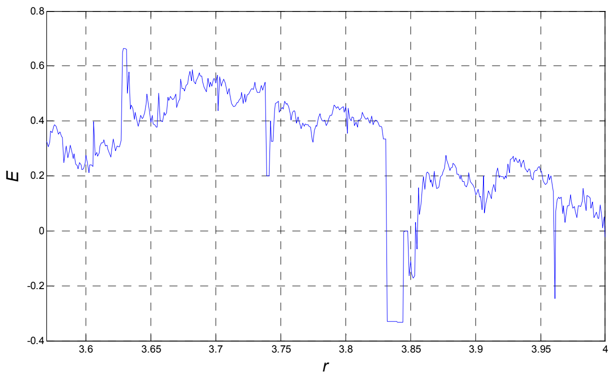

Fig. 1: Performance of Balance with different fractal parameters and x_0=0.24.

Correlation of PN sequence would affect its anti-jamming performance and multi-user performance, etc. We set X and Y are two sequences; whose period is N. Autocorrelation and Cross-correlation are given by the following formula (9):

$$R_{AC}(k)=\frac{1}{N}\sum_{i=0}^{N-1}X_iX_{i+k}\ ,R_{CC}(k)=\frac{1}{N}\sum_{i=0}^{N-1}X_iY_{i+k} \tag{9}$$

4. Optimal Design Method and Performance Simulation

This optimal design method is divided into two steps: preliminary design and detailed design. When we determined length of the sequences, the nature of the sequence is determined by $\{r, \varepsilon, x_0\}$.Studies have shown that the statistical properties of Logistic chaotic sequence are relatively stable with different initial values. In other words, the initial value x_0 in $\{r, \varepsilon, x_0\}$ less impacts the performance of the sequence. Based on the above, we finish the preliminary design of $\{r, \varepsilon\}$ at first, when x_0 takes any value within the range. Then, do the detailed design for different x_0.

4.1. Preliminary design

Preliminary design is the preliminary choice of $\{r, \varepsilon\}$ based on the performance of balance and relevance.

We first select an initial value x_0 and determine ε_m that meet the conditions $E_m<\xi_E$ with different r_m, where ξ_E is the performance evaluation of the balance and positive. Now, we get $\{r, \varepsilon\}_m$ meeting the requirements of balance.

Secondly, based on $\{r, \varepsilon\}_m$ which has been selected, we choose $\{r, \varepsilon\}_n$ according to the given requirements of correlation ξ_{ac} and ξ_{cc}, where $\{r, \varepsilon\}_n \in \{r, \varepsilon\}_m$. Then, we complete the preliminary design.

This paper selected r=3.57,3.58,3.59…3.99,4.00 to optimize. In accordance with above method, we determine $\{r, \varepsilon\}_m$ according to ξ_E.We set x_0=0.38,l=1600, where l is the length of sequence. Generally, the requirement of the balance of signal is 1%. We set ξ_E=0.002. In addition, ξ_E can be changed as needed in practice. Table I shows $\{r, \varepsilon\}_m$ we selected:

Table 1: Quantization thresholds for different fractal parameters

r	3.57	3.58	3.59	3.60	3.61
ε	0.5627	0.5745	0.5871	0.6004	0.6145
r	3.62	3.63	3.64	3.65	3.66
ε	0.6292	0.6437	0.6608	0.6775	0.6948
r	3.67	3.68	3.69	3.70	3.71
ε	0.71258	0.72721	0.72387	0.71836	0.7207
r	3.72	3.73	3.75	3.76	3.77
ε	0.71212	0.71163	0.69777	0.69252	0.69974
r	3.78	3.79	3.80	3.81	3.82
ε	0.68968	0.68951	0.68142	0.65801	0.67539
r	3.86	3.87	3.88	3.89	3.90
ε	0.54941	0.60199	0.65672	0.63966	0.59525
r	3.91	3.92	3.93	3.94	3.95
ε	0.55299	0.61306	0.67542	0.64391	0.61848
r	3.96	3.97	3.98	3.99	4.00
ε	0.54433	0.56581	0.54048	0.54761	0.47981

When r=3.74, 3.83, 3.84, 3.85, the balance of sequence exists singularities. So they are removed from Table 1.

Based on the above work, we use 40 groups of $\{r, \varepsilon\}_m$ to generate chaotic spreading sequences, and select $\{r, \varepsilon\}_n$ which meet the requirements of correlation. We set x_{01}=0.38, x_{02}=0.66. Using the sequence generated by x_{01} to do part-autocorrelation. Then, do part cross-correlation with the sequence generated by x_{02}. The average of normalized cross-correlation of Gold sequence with the period of 2047 is 0.016. However, for l=1600, performance evaluation can be broadened. So we select ξ_{cc}=ξ_{cc}=0.032 to be the performance evaluation of normalized autocorrelation and cross-correlation. Figure 2 and Figure 3 shows the result of each $\{r, \varepsilon\}_m$.

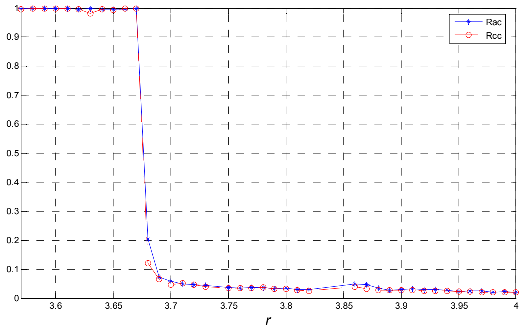

Fig. 2: Curves of Normalized Autocorrelation and Cross-correlation

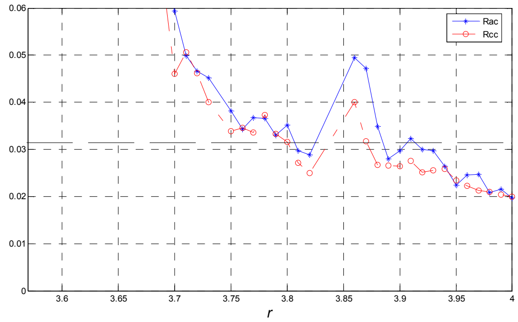

Fig. 3: Enlargement of Figure 2.

According to the given correlation threshold, we get the results of optimal design. There are 13 groups of parameters $\{r, \varepsilon\}_n$, where $r_n \in \{3.81, 3.82, 3.89, 3.90, 3.92, 3.93, 3.94, 3.95, 3.96, 3.97, 3.98, 3.99, 4.00\}$.

In this section, we only choose 44 groups of parameters to illustrate the preliminary design method. In practice, there is an infinite number of parameters r that can be used for screening. It is noteworthy that r within the range (3.92, 4.00) all meet the requirements of the preliminary design. We can infer other parameters within the range (3.92, 4.00), such as r=3.965 and r=3.9882, would also meet the requirements, which have been proven. It is not to undertake in this paper.

4.2. Detailed design

Detailed design is based on $\{r, \varepsilon\}$ selected in preliminary design. We design chaotic spreading sequences $\{C_i\}$ i=1,2,…,N. And N is the number of sequences that the communication system needs.

The specific method is as following: We set the sequences that have been designed $C_1, C_2, \ldots, C_{i-1}$ and are going to design C_i. Firstly, we choose $\{r, x_0\}_i$ which has different x_0 or different r or different x_0 & r with $C_1, C_2, \ldots, C_{i-1}$. Then, based on the ε_i of $\{r, \varepsilon\}_i$ selected in preliminary design, we fine tune ε_i to get ε_{i0} that makes $(E/E_0)^2+(R_{ac}/R_{ac0})^2$ to be minimized Where E_0=0.002, R_{ac0}=0.016. Finally, We test the cross-correlation values between C_i and the designed sequences $C_1, C_2, \ldots, C_{i-1}$. If they meet the requirements, we choose it, otherwise it's abandoned

Using the above method, we design C_1. We choose a fractal parameter r from 13 groups of $\{r, \varepsilon\}$ determined in preliminary design. We may assume that r_1=4.00, x_{01}=0.37. Through simulation, $(E/E_0)^2+(R_{ac}/R_{ac0})^2$ has the minimum when ε_{10}=0.4901. Where E_1=0, R_{ac1}=0.021. C_1 is completed.

The design of C_2 is similar with C_1. We choose r_2=3.95, x_{02}=0.24. When ε_{10}=0.4901, $(E/E_0)^2+(R_{ac}/R_{ac0})^2$ has the minimum. Where E_2=0, R_{ac2}=0.021. And the cross correlation R_{cc12}=0.020<0.32 complies with the requirements. C_2 is completed.

Then, we design $C_3, C_4, \ldots, C_N$ using the same method with C_2 and complete the design of chaotic spreading sequences.

4.3. Compare performance with the gold sequences

We compare the performance of chaotic spreading sequences with truncated Gold code as shown in Table 2. The original period is 2047, and the truncated period is 1600.

Table 2: Comparison of performance

	Balance degree	Autocor relation	Cross-co rrelation	Linear complexity
Chaotic Spreading Sequences	0	0.024	0.020	0.5
Truncated Gold Code	0.0275	0.021	0.020	0.014

Linear complexity in the table is calculated using B-M algorithm [7]. For m, Gold and other linear sequences, the complexity equals the number of shift register required to generate the sequences divided by length of sequences.

As shown in the Table II,the correlation of two sequences is close. But the balance and security of chaotic spreading sequences are significantly better than the truncated Gold sequences. This shows that the chaotic spreading sequences optimally designed in this paper have good performance.

5. Conclusion

The article proposes the method of getting the demanded chaotic spreading sequences by optimally designing the binary quantization threshold for the different fractal parameters and initial values. The article optimizes the logistic chaotic sequences using the mentioned methods, simulates the designing results and compares their performance with the Gold sequences. The simulation results show that the optimal design method is feasible and effective. There are several advantages of the optimal design method as follows. First, the method expands the range of options of the period of spreading sequences. The period is no longer limited to 2^n-1, which greatly improves the flexibility when we design the communication systems. Second, the method expands the range of options of the fractal parameter r in using Logistic mapping, which no longer limits $r = 4$. This advantage greatly increases the number of available sequences. Third, the large number of fractal parameters and initial values that can be chosen makes that the generating function is not so easy to be detected, and the sequences are more complicated than traditional linear sequence, which greatly improves the security of system.

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