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Analysis on Range Resolution of Distributed MIMO Radar with Coherent Processing

Gang Chen +, Hong Gu and Weimin Su
Department of Electronic Engineering, Nanjing University of Science and Technology
Nanjing, Jiangsu, P.R. China

Abstract. It is studied that the effect of the signal effective bandwidth and the angular spread that sensors placed over on the range resolution of distributed MIMO radar with coherent processing. The relation expression is developed. It reveals that the high resolution beyond support by bandwidth can be attained, when the angular spread is larger than a threshold. Finally, Simulations verify the results.

Keywords: MIMO radar; range resolution, bandwidth, coherent processing.

1. Introduction

Recently, MIMO (Multiple Input Multiple Output) Radar is a technology that has drawn considerable attention. There are two kinds of MIMO radar architectures, one is collocated scheme [1-3], and the other is distributed scheme [4-6]. In MIMO radar with collocated antennas, the work focused on waveform diversity, whereas in MIMO radar with distributed antennas, the emphasis has been on the use of spatial diversity. Previous works on localization of targets in MIMO radar systems has shown that MIMO radar with coherent processing over widely dispersed sensor elements that partly surround the target may lead to resolutions higher than supported by the radar bandwidth [7]. Localization systems that exploit the phase information are referred to as coherent, in contrast to noncoherent systems, which exploit envelope measurements [8].

MIMO radar systems with widely distributed elements are able to resolve scatterers with a high range resolution, but are subject to the requirement of placing sensors over a larger area. When sensors positioned over sufficiently wide angular spread with respect to target position, the range resolution obtained with coherent processing is higher than supported by bandwidth. Below a threshold angular spread however, coherent MIMO radar resolves targets through exploiting only envelop information which mainly dependent on its signal effective bandwidth. The motivation of this work is the availability of a relation expression for the effect of the signal bandwidth and angular spread on range resolution that enables performance analysis without lengthy simulations, and develops ways to predict the resources required to achieve desired performance by distributed MIMO radar.

This paper is organized as follows: Section II introduces the system signal model. In Section III, relation between the effect of angular spread and signal bandwidth on range resolution is examined. Simulations are performed to verify the relation expression in Section IV. Finally, Section V concludes the paper.

2. System Model

System layout of MIMO radar with distributed (widely separated) antennas is illustrated in Fig.1. There are M transmitters $T_1, T_2, \cdots, T_M$ and N receivers $R_1, R_2, \cdots, R_N$ in the system. All the sensors and target are located in a two-dimensional plane, and time and phase synchronized. It is convenient to introduce a coordinate system with the origin at the centre of the monitored area and estimated the target location

+ Corresponding author.
E-mail address: gang.chen@foxmail.com.

$P=(x,y)$. The *i*-th transmitter and *j*-th receiver are located at angles $\theta_{t,i}$ and $\theta_{r,j}$ with respect to the Target as illustrated in Fig.1. The target model developed here generalizes the model in [9] to signal point scatterer and distributed sensors. In Skolnik's Model [9], the returns of signal point scatterer have fixed amplitude and phase, and are independent of angle. In the model developed below, path loss effects are neglected, i.e., the model accounts for the effect of the sensors/targets locations only through time delays (or phase shifts) of the signals.

The signal transmitted from the *i*-th transmit element is

$$g_i(t)=2\,\mathrm{Re}\{\hat{s}_i(t)\}\ \ (i=1,2,\cdots,M) \tag{1}$$

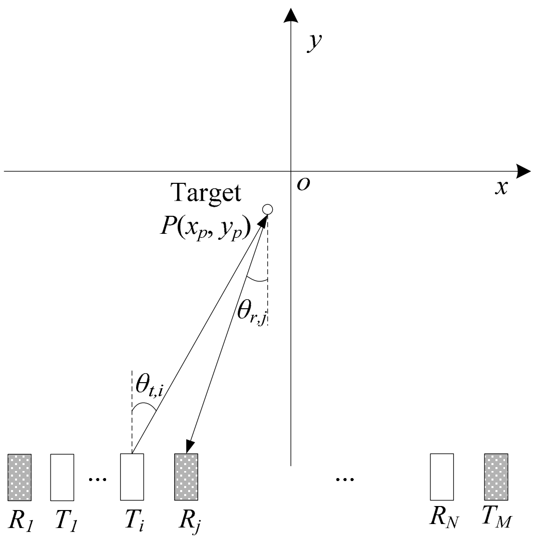

Fig.1 Distributed MIMO radar system layout

The effective bandwidth *B* and time duration *T* are considered to be constant for all waveforms $\hat{s}_i(t)$. All transmitters operate at the same centre frequency f_c. Moreover, let all transmit waveforms are narrowband signals. Hence $\hat{s}_i(t)$ can be given as

$$\hat{s}_i(t)=s_i(t)e^{j2\pi f_c t} \tag{2}$$

where $s_i(t)$ is the complex envelope of the *i*-th waveform, that is, baseband waveform. It is assumed that the baseband waveforms maintain approximate orthogonality even for different mutual delays. The condition is

$$\int s_l(t)s_k^*(t-\tau)\mathrm{d}t\approx 0\ \ \forall\tau,\ l\neq k \tag{3}$$

The waveform transmitted by the *i*-th transmitter, reflected by a point scatterer at P, leads to a signal component at the *j*-th receiver of the following form

$$\tilde{r}_{i,j}=\tilde{\alpha}_{i,j}g_i(t-\tau'_{i,j}(\vec{P}))+\tilde{n}_{i,j}(t) \tag{4}$$

here $\tilde{\alpha}_{i,j}$ is complex coefficient. According to Skolnik model, all the complex coefficients are identical, $\tilde{\alpha}_{i,j}=\tilde{\alpha}$. $\tilde{n}_{i,j}(t)$ represents background noise for the (*i,j*) transmit-receive channel. After complex demodulation, the received signal is

$$r_{i,j}=\alpha s_i(t-\tau'_{i,j}(\vec{P}))e^{j2\pi f_c\tau'_{i,j}(\vec{P})}+n_{i,j}(t) \tag{5}$$

where $n_{i,j}(t)$ is white Gaussian. $\tau'_{i,j}(\vec{P})$ denote the propagation delay from transmitter T_i, to scatterer *P*, to receiver R_j, which associated with the locations $T_i=(x_{ti},y_{ti})$, $R_j=(x_{rj},y_{rj})$ and $P=(x_p,y_p)$ through the relation

$$\tau'_{i,j}(x_p, y_p) = \frac{\sqrt{(x_{ti}-x_p)^2+(y_{ti}-y_p)^2}+\sqrt{(x_{rj}-x_p)^2+(y_{rj}-y_p)^2}}{c} \quad (6)$$

where c represents the speed of light. Assuming that the phases and the time references at the transmit and receive elements are calibrated to a hypothetical scatter location at the origin of axes, and using a Taylor series expansion, we can express the propagation time in the following simplified form

$$\tau_{i,j} = -\frac{1}{c}[(\sin\theta_{t,i}+\sin\theta_{r,j})x+(\cos\theta_{t,i}+\cos\theta_{r,j})y] \quad (7)$$

3. Analysis on Range Resolution

Ambiguity function is an important tool to characterize the resolution of MIMO radar. Normalized two-dimensional ambiguity function of MIMO radar with coherent processing is reduced as [7]

$$\begin{aligned} A(x,y) &= \frac{1}{M^2N^2}\left|\sum_{i=1}^{M}\sum_{j=1}^{N}\int \tilde{r}_i^*(t)g_i(t-\tau_{i,j})dt\right|^2 \\ &= \frac{1}{M^2N^2}\left|\sum_{i=1}^{M}\sum_{j=1}^{N}e^{-j2\pi f_c\tau_{i,j}}\int s_i^*(t)s_i(t-\tau_{i,j})dt\right|^2 \end{aligned} \quad (8)$$

In [10], [11], it revealed that the bandwidth has a very limited impact on resolution, and that high resolution requires a large angular spread. As such, when the angular spread that sensors placed over is narrow, the system cannot enjoy the spatial diversity gain. In this case, MIMO radar resolves scatterers through employing only envelop information, while the range resolution is mainly supported by radar's signal bandwidth. The range resolution is approximately given by

$$\Delta r_c \approx \frac{c}{2}\frac{1}{B} \quad (9)$$

Base on the relation $c = f_c\lambda$, (9) can now be expressed

$$\Delta r_c \approx \frac{f_c}{2B}\lambda \quad (10)$$

where λ is the carrier wavelength. It is apparent that, the range resolution inversely proportional to the ratio of the effective bandwidth to the signal carrier frequency, and proportional to the carrier wavelength.

To assess the effect of the phase information on the range resolution of distributed MIMO radar with coherent processing, we assume that the range resolution of distributed MIMO radar is independent of signal bandwidth, regardless of angular spread. As such, for estimating the mainlobe width, the integrals in the sums of the last line of (8) can be ignored. Then the ambiguity function simplifies to

$$A_p(x,y) = \frac{1}{M^2N^2}\left|\sum_{i=1}^{M}\sum_{j=1}^{N}e^{-j2\pi f_c\tau_{i,j}}\right|^2 \quad (11)$$

$A_p(x,y)$ isolates the effect of the signal bandwidth on the MIMO radar range resolution gain performance. In the system layout of this paper, y-direction is the range direction for MIMO radar (see Fig.1). Hence, the range resolution is characterized by the mainlobe width of the y cut of the ambiguity function. $A_p(x,y)$ in y-direction only can be evaluated according to (11) as

$$\begin{aligned} A_p(0,y) &= \frac{1}{M^2N^2}\left|\sum_{i=1}^{M}e^{j\frac{2\pi y}{\lambda}\cos\theta_{t,i}}\right|^2\cdot\left|\sum_{j=1}^{N}e^{j\frac{2\pi y}{\lambda}\cos\theta_{r,j}}\right|^2 \\ &= \frac{1}{M^2N^2}[M+2\sum_{l=2}^{M}\sum_{k=1}^{l-1}\cos(\frac{2\pi y}{\lambda}(\cos\theta_{t,l}-\cos\theta_{t,k}))] \\ &\quad \cdot[N+2\sum_{l=2}^{N}\sum_{k=1}^{l-1}\cos(\frac{2\pi y}{\lambda}(\cos\theta_{r,l}-\cos\theta_{r,k}))] \end{aligned} \quad (12)$$

Since the ambiguity function is normalized, $A_p(0,y)$ has unity gain at the peak. We fit a parabola $P(x)=a-bx^2$ to the peak, and calculate the 3dB width of the approximating parabola. This calculation yields the parabolic width PW [12] which is given by the expression:

$$PW = 2\sqrt{\frac{A_p([0,y]^T)}{-\dfrac{d^2A_p([0,y]^T)}{dy^2}}}\Bigg|_{\text{peak}} \tag{13}$$

where the subscript 'peak' denotes evaluation of the $A'(0,y)$ and its second derivative at the peak of the mainlobe, which occurs at $y=0$. According to (12), we obtain follows

$$\begin{aligned} &A_p(0,y)\big|_{\text{peak}} \\ &= \frac{1}{M^2N^2}[M+2\cdot\frac{M(M-1)}{2}][N+2\cdot\frac{N(N-1)}{2}]=1 \end{aligned} \tag{14}$$

$$\begin{aligned} &\frac{d^2A_p(0,y)}{dy^2}\bigg|_{\text{peak}} \\ &= \frac{-2}{M^2N^2}(\frac{2\pi}{\lambda})^2[N^2\sum_{l=2}^{M}\sum_{k=1}^{l-1}(\cos\theta_{t,l}-\cos\theta_{t,k})^2 \\ &\qquad +M^2\sum_{l=2}^{M}\sum_{k=1}^{l-1}(\cos\theta_{r,l}-\cos\theta_{r,k})^2] \end{aligned} \tag{15}$$

Using (14) and (15) in (13), yields

$$\begin{aligned} PW &= \frac{MN\lambda}{\sqrt{2}\pi} \\ &\bullet\frac{1}{\sqrt{[N^2\sum_{l=2}^{M}\sum_{k=1}^{l-1}(\cos\theta_{t,l}-\cos\theta_{t,k})^2+M^2\sum_{l=2}^{M}\sum_{k=1}^{l-1}(\cos\theta_{r,l}-\cos\theta_{r,k})^2]}} \end{aligned} \tag{16}$$

It is apparent from (16) that the parabolic width dependent on the geometric layout of the MIMO radar system, which is means that wider angular spread of the radars results in better resolution. Coherent processing leading to a range resolution beyond supported by the radar bandwidth means

$$PW \le \Delta r_c \tag{17}$$

Applying (10) and (16) to (17), we get the following expression:

$$\frac{\pi\sqrt{[N^2\sum_{l=2}^{M}\sum_{k=1}^{l-1}(\cos\theta_{t,l}-\cos\theta_{t,k})^2+M^2\sum_{l=2}^{M}\sum_{k=1}^{l-1}(\cos\theta_{r,l}-\cos\theta_{r,k})^2]}}{\sqrt{2}MN} \ge \frac{B}{f_c} \tag{18}$$

For MIMO radar with coherent processing, both angular spread that sensors distributed over and effective bandwidth can improve the range resolution. The threshold of angular spread which sensors located over that can lead to higher resolution than supported by radar bandwidth is expressed in (18).

For a better insight, some special schemes are evaluated. In Fig.2 a 5×4 MIMO radar system (*M*=5, *N*=4) is illustrated, with different antenna locations, accounting for different resolution characteristics. The sensors are placed evenly on part of the circle centred at the origin as shown in Case 1. In case 2, transmit and receive elements are uniformly spaced in a line. In Case 3, all sensors are located in part of circle whose centre is separated from the origin by

array. For each case, The sensors are distributed over $|\theta|<\theta_{\max}$, and the thresholds are evaluated for different ratios of effective bandwidth to carrier frequency. The threshold of spread that can lead to resolution higher than supported by radar bandwidth for each case is list in the Table 1.

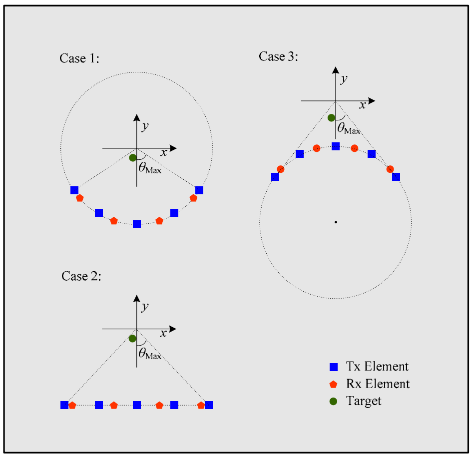

Fig.2: System layout for cases 1 to 3

Table 1. The threshold of angular spread

B/f_c	0.05	0.1	0.15	0.2
Case 1	14.8	20.9	25.9	29.6
Case 2	14.8	21.1	26	29.9
Case 3	14.9	21.3	26.1	30.2

4. Simulations

To verify the relation expression of (18), a comparison between the resolutions of normal coherent processing and coherent processing exploiting only phase information is given. The mainlobe width of two-dimensional ambiguity function (AF), based on expression (8), characterize the resolution of coherent processing, whereas the simplified ambiguity function (AF-PI), based on expression (11), characterize the resolution of coherent processing with exploiting only phase information.

For convenience, it is assumed that transmit elements and receive elements are distributed similarly, and that baseband signals have a rectangular frequency response. System parameters are set as follows: B/f_c=0.1 and 0.2, $f_c = 5\text{GHz}$, sensors positioned evenly over $|\theta| \le \theta_{\max}$, distance is 5000m, design the $\theta_{\max}$ to be in the range 10-90 degree.

Fig.3 and Fig.4 contain mainlobe widths for different values of $\theta_{\max}$ and a B/f_c of 0.1 and 0.2, respectively. The mainlobe widths for y cuts of AF based on expressions (8) and AF-PI based on expressions (11) are plotted for comparison. Fig.3 is based on a ratio of effective bandwidth to carrier frequency of $B/f_c = 0.1$, whereas Fig.4 is based on a ratio of effective bandwidth to carrier frequency. Note, that the ordinate is given in multiples of λ.

From two figures it can be seen, that both mainlobe widths of y-cuts of AF and AF-PI are in excellent agreement

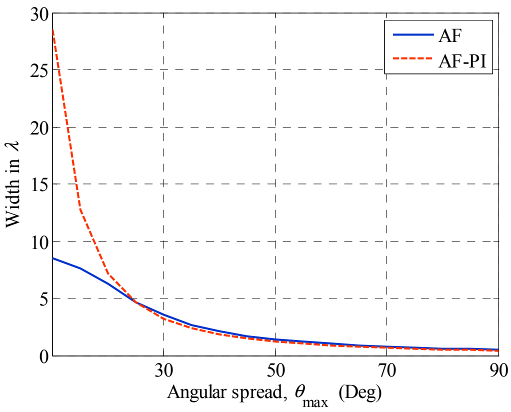

Fig.3. Mainlobe width of y-cuts for $B/f_c = 0.1$

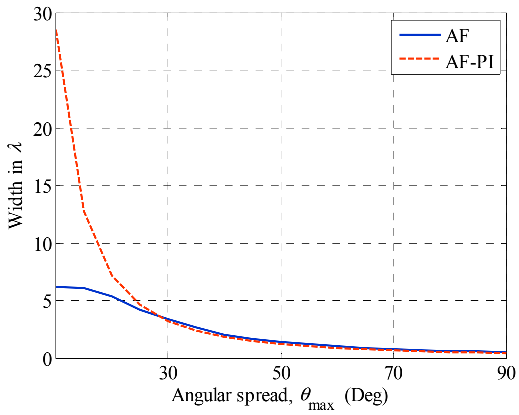

Fig.4. Mainlobe width of y-cuts for $B/f_c = 0.2$

for larger θ_{max} values. In other words, when the spread angular that distributed MIMO radar sensors over is larger than a threshold, the range resolution is independence of signal bandwidth, which is obtained mainly by phase information. As expect, the thresholds are approximately 20 degree and 29 degree, respectively.

5. Conclusions

Relation expression is derived for the effect of signal bandwidth and angular spread on range resolution of MIMO radar with coherent processing. For different ratio of bandwidth to carrier frequency, it needs different angular spread to gain resolution beyond supported by signal bandwidth for distributed MIMO radar. When the angular spread of MIMO radar with respect to target is large enough, the high range resolution could be attained through utilizing just phase information, included in the received signals, with coherent processing.

6. Acknowledgment

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