

# IIR Digital Filter Design Research and Simulation on MATLAB

ZHANG Chengliang<sup>1 +</sup> and WANG Aihong<sup>2</sup>

<sup>1</sup> Department of Physical and electronic science, Binzhou University  
Binzhou, China

<sup>2</sup> College of Information Science and Engineering, Northeastern University  
Shenyang, China

**Abstract.** In modern communication systems, filtering is the most common and extremely important signal processing technology, it is an effective method of interference suppression, and the design of filter has become the core issues of the signal processing. Generally speaking, filter can be divided into analog filter and digital filter. Today, the development of analog filter has been more mature. However, digital filter has many advantages, such as higher stability, higher precision. With the development of digital technology, using digital technology to realize filter function is widely used. Then this paper analyzes the design principle of digital filter, and introduced the impulse invariance method and window function method. This paper presents a new design of a effective IIR filter, which realized via two methods.

MATLAB is a popular numerical analysis software, providing a wealth of design tool. A MATLAB-based digital filter design procedure designs the filter, and mainly analyzes the 5-order IIR filter, which is designed using Simulink. Through the MATLAB visualization procedures given the frequency characteristic curve, comparing the simulation results. It has the high practical value.

**Keywords:** digital filters, infinite-impulseresponse (IIR) filter, impulse invariance method.

## 1. Introduction

Filter, is a circuit that allows a particular frequency band through, and attenuation of signal which outside the band. Filter is used extensively, involving tele-communications, military, medical, electricity, etc. In the modern communication system, filter is one kind of signal processing technology which is used very commonly, and digital filters is better than analog filters in many ways, So, with digital technology to achieve filter function been widely used.

In digital signal processing, filtering occupies an extremely important role. The input sequences of digital filter are through certain operations transform into output sequence. Digital filter core thought is prominent and effective wave, restrain the interference wave. Compared with simulation filtering, digital filtering has many outstanding qualities, such as: it can satisfy the strict requirement on amplitude and phase characteristic, overcome voltage drift, temperature drift and noise etc.

## 2. Digital Filter

Digital filter is the important foundation of digital signal processing; it is the most widely used a linear system. The object of digital filter processing is analog signal conversion for digital signals. Compared with analog filter, digital filters have prominent advantages such as high stability, high precision and flexible. With the development of digital technology, using digital technology to achieve filter function more and more attention by people, and been widely used.

### 2.1 . The realization of digital filter

There are two realization ways to realize Digital filter: one is the frequency domain method, using FFT fast algorithm of the input signal to the discrete Fourier transform, and analyzes its spectrum, and then based on the expected frequency characteristics of filter, finally reuse Fourier inverse transform time signal recovers.

---

<sup>+</sup> Corresponding author.  
E-mail address: zclbysj@163.com.

Another method is the time domain method; this method is based on discrete sampling data for difference numerical computation to achieve filtering purpose.

## 2.2 The design of digital filter

According to temporal characteristics of impulse response function, Digital filter can be divided into two kinds of, an infinite impulse response (IIR) filter and finite impulse response (FIR) filter. By digital signal processing, IIR filter's characteristic is infinite duration of impulse response, while FIR filter is limited, lasting only a certain amount of time. The design method of digital filter is varied, like impulse invariance method and bilinear transform method, window function method and frequency sampling method.

General design procedure is as follows:

- a) According to actual needs, to determine the filter performance requirements;
- b) Using a causal, stable discrete linear system to approached this constant index;
- c) Through the operation to realize the filtering system designed;
- d) Through the simulation system meets the given technical requirements.

IIR filter is the Impulse Response with unlimited duration, this kind of Filter generally need to realize using a recursive structure, and called a recursive filter. Impulse response reform is a method for converting simulation filter to digital filter.

## 3. IIR Digital Filter

IIR digital filter is unlimited duration of impulse response, this kind of filter generally need to realize using a recursive structure, and called a recursive filter. IIR filters filter expression can be defined as a difference equation:

$$y(n) = \sum_{k=0}^M a_k x(n-k) - \sum_{k=1}^N b_k y(n-k) \quad (1)$$

Where  $x(n)$  and  $y(n)$  are input and output signal sequences; and  $a_k$ ,  $b_k$  are filter coefficients.

The system transfer function can be expressed by the following equation:

$$H(z) = \frac{\sum_{k=0}^M a_k z^{-k}}{1 + \sum_{k=1}^N b_k z^{-k}} \quad (2)$$

$N$  is IIR filter's order number (or filter system transfer function's poles count),  $M$  for filter system transfer function's zero count, where  $a_k$  and  $b_k$  are weight function coefficients.

Usually, the design of IIR digital filter is via analog filter prototype, then convert analog filter to digital filter. The design of simulated filter is mature, therefore, make full use of the existing resources will brings many advantages for the design of digital filter.

### 3.1 Impulse invariance method

Impulse invariance method is a way to converting analog filter to digital filter. It gets digital filter unit impulse response  $h[k]$  by sampling interval of analog filter unit impulse response  $h(t)$ , that is:

$$h[k] = h(kT) \quad (3)$$

Set a causal analog filter's system function only have first-order poles, launched by partial fraction:

$$H(s) = \sum_{l=1}^M \frac{A_l}{s + p_l} \quad (4)$$

By Laplace inverse transform, this system's unit impulse response is as follows:

$$h(t) = \sum_{l=1}^M A_l e^{-p_l t} u(t) \quad (5)$$

From formula (3), digital filter unit impulse response is:

$$h[k] = h(kT) = \sum_{l=1}^M A_l e^{-p_l kT} u[k] \quad (6)$$

The z-transform is given by:

$$H(z) = \sum_{l=1}^M \frac{A_l}{1 - e^{-p_l T} z^{-1}} \quad (7)$$

So, impulse invariance method can be finished by formula (8):

$$\frac{1}{s + p_l} \rightarrow \frac{1}{1 - e^{-p_l T} z^{-1}} \quad (8)$$

Owing to  $\Omega = T\omega$ , it is linear relationship between Simulation frequency and digital frequency.

### 3.2 Characteristics of impulse invariance method

The characteristics of impulse invariance method can be conclude and given by

- Using impulse invariance method to realize analog to digital filter, frequency conversion meet linear phase;
- Existing aliasing phenomenon, so can't design high-pass and band-stop filters.

### 3.3 Design of IIR digital filter

Using impulse invariance method to design a digital low-pass filter, design requirement:  $f_p=2000\text{rad/s}$ ,  $A_p<3\text{dB}$ ,  $f_s=3000\text{rad/s}$ ,  $A_s>15\text{dB}$ , Sampling frequency:  $F_s=10000\text{H}$ 。 Assume a signal  $x(t)=\sin(2*\text{Pi}*f_1*t)+0.5\cos(2*\text{Pi}*f_2*t)$ , and  $f_1=1000\text{Hz}$ ,  $f_2=5000\text{Hz}$ 。

Run MATLAB program can get order number:  $N = 5.0000$ , and filter frequency response curve, as shown in Fig. 1:

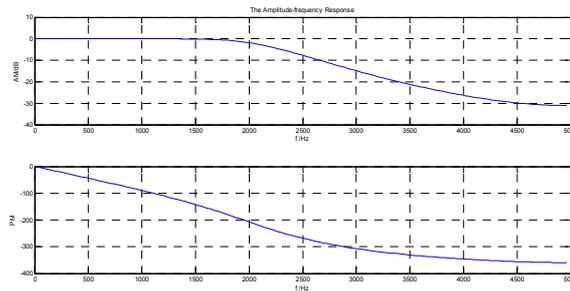


Fig. 10: The amplitude-frequency response

From the amplitude-frequency characteristic curve, in 2000Hz place attenuation is less than 3dB and in more than 3000Hz place attenuation is more than 15dB, so it can satisfy the attenuation of the filter design indexes.

Input function  $x(t)$  through this filter, the rendering is as shown in Fig. 2:

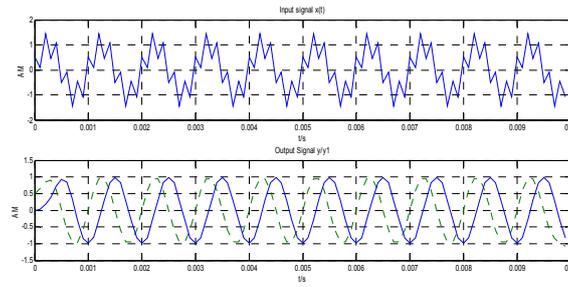


Fig.2: Filter rendering

It can be seen from Fig. 2, using the function `filtfilt` (zero phase filter) for the input signal  $x(t)$ , after filtering the output signal  $y(t)$  (solid line) is consistent in phase with the input signal 1000Hz signal, that is to say, it does not change with the phase, which is the advantage of using the function; while using the function `filter`, the output  $y_1(t)$  (dotted line), has a certain delay compared with the original signal.

### 3.4 Simulink

According to the design of filter index, the Fdatool interface parameters can be set as follows.

Table 1: Fdatool interface parameters settings

| Parameters    | Setting                 |
|---------------|-------------------------|
| Filter Type   | Lowpass                 |
| Design Method | IIR Butterworth         |
| Fs            | Fs=10000Hz              |
| Ap,As         | Ap=3dB, As=15dB         |
| Fp,Fstop      | Fp=2000Hz, Fstop=3000Hz |
| Filter Order  | Minimum order           |

By calling the Simulink modules, a digital filter simulation diagram is formed.

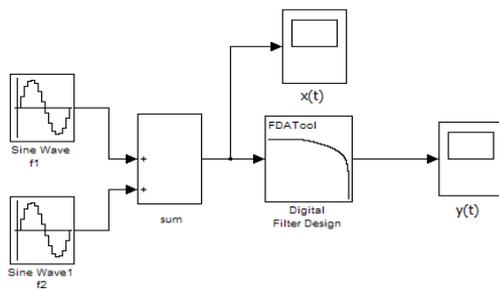


Fig. 3: Simulink figure

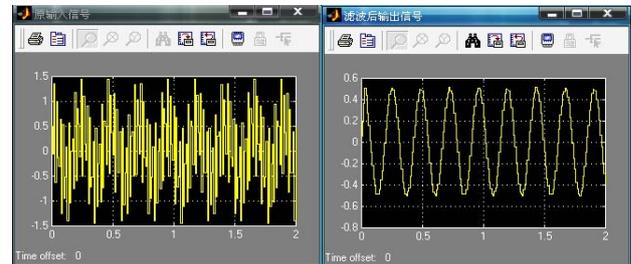


Fig. 4: Filtering rendering

It can be seen from the signal filtering rendering in Fig. 4, the 5000Hz signal is filtered out from the input signal  $x(t)$ . From Fig. 2 and Fig. 4, the amplitude-frequency phase frequency characteristics and filtering effect are consistent by program design and Simulink, the effect is ideal, and meets the design requirements.

## 4. Conclusion

It can be designed quickly and effectively by MATLAB, the characteristics of filters can be compared to achieve the optimum design. The higher the filter order number, the better the filtering effect, but the more computation time is occupied, so in a situation that meets the index requirements, it should try to reduce the filter order number  $N$ . IIR digital filters can use a lower order number to meet the same design requirements compared with FIR digital filters. Designing filters on MATLAB is used in digital signal processing, and has wide application and development prospects.

## 5. Acknowledgment

The completion of this paper should thank all the university teachers. W.A.H. got much knowledge from them who gave many cares and helps, so that W.A.H. accumulated enough knowledge to finish this paper;

more benefit from guiding teacher Zhang Chengliang and Li Shiping, and their elaborate guidance and assistance. All the achievement cannot leave their support and help.

## 6. Reference

- [1] Fesquet, L. IIR Digital Filtering of Non-uniformly Sampled Signals via State Representation. *Signal Processing*; Oct2010, Vol. 90 Issue 10, p2811-2821, 11p.
- [2] HUANG Meng, TANG Lin, ZHEN Yu & ZHANG Jie. (2010). Optimized FIR Filter Design Based on Self-adaptive Genetic Algorithm. *Modern Electronics Technique*, 02.
- [3] Jong-Sik Lim. Design of Low-Pass Filters Using Defected Ground Structure. *IEEE Transactions on Microwave Theory & Techniques*; Aug2005.
- [4] WANG Yang & ZENG Yi-cheng. (2011). A complexity reduction approach for FIR notch filter design. *Microcomputer Information*, 2.
- [5] M. D. Lutovac, D. V. Tasic, and B. L. Evans, *Filter Design for Signal Processing*, :Prentice-Hall R. Nicole, "Title of paper with only first word capitalized," J. Name Stand. Abbrev., in press.
- [6] A. Krukowski and I. Kale, *DSP System Design: Complexity Reduced IIR Filter Implementation for Practical Applications*, Kluwer, Norwell, MA, 2003.
- [7] B. Farhang-Boroujeny, "A square-root Nyquist (M) filter design for digital communication systems," *IEEE Trans. Signal Process.*, vol. 56, no. 5, pp. 2127-2132, May 2008.
- [8] N. Wong and C. U. Lei, "IIR approximation of FIR filters via discrete-time vector fitting," *IEEE Trans. Signal Process.*, vol. 56, no. 3, pp. 1296-1302, Mar. 2008.