2012 4th International Conference on Signal Processing Systems (ICSPS 2012)
IPCSIT vol. 58 (2012) © (2012) IACSIT Press, Singapore
DOI: 10.7763/IPCSIT.2012.V58.18

Impacts of Signals Cross-Correlation Characteristics on Matched Filtering Performance for MIMO Radar

Qing Wang+, Huai-tie Xiao and Hao-wen Chen

ATR Key Lab, National University of Defense Technology
Changsha, 410073, China

Abstract. Considering that completely orthogonal signals cannot be found in reality, the impacts of correlation characteristic of MIMO radar signals on the performance of matched filtering output at the receiver are discussed in this paper. Firstly, the outputs of matched filters are precisely modeled by auto-correlation functions and cross-correlation functions of multiple signals. And an impact factor is also defined to evaluate the degree that the correlations can affect the filter outputs under different conditions. The impact factor function contains the influence of sensor spacing, the reflection coefficient of different transmit signals and target azimuth angle simultaneously. Then, we qualitatively analyze the relationship between the correlation effect on matched filtering and those three parameters, and obtain some useful results. Finally, some numerical examples are presented to demonstrate the conclusions. Theoretical analysis and experiments both show that the outputs of matched filters may be obviously influenced by the correlation of signals and the effect varies under different conditions, particularly enormous in some situation. Consequently, MIMO radar system with multiple signals requires more highly on the orthogonality of transmit waveforms.

Keywords: MIMO radar, matched filtering, orthogonal signals.

1. Introduction

MIMO radar [1] is a new born concept that has been proposed for a few years, of which both system architecture and signal design are very flexible and its performances in target detection, parameter estimation and high resolution are relatively extraordinary compared to conventional radars. Therefore, it is paid much attention to by many researchers and institutions. Since its naissance in 2004, a large number of letters have come forth, advancing it into realization by theoretical supports. For example, in [2], J. Li and the other authors summarized some new conclusions in recent years, deeply interpreted the idea of MIMO radar and proclaimed its potentials. The authors in [3] and [4] studied on the detection performance and parameter estimation capability of the new radar, respectively. Many other papers alike the previous also demonstrated the great superiority of MIMO radar from different perspectives. However, as a fresh new idea, the studies on MIMO radar are far from in-depth. Its concept needs to be understood more precisely and numerous problems in theory and implementation demand settlements.

Essentially, the advantages of MIMO radar are obtained from diversity techniques such as spatial diversity, waveform diversity and so on. In other words, MIMO radar can improve target information acquisition capabilities by optimizing array and waveform design. Usually, the multiple waveforms used in MIMO radar need to be orthogonal to each other, so that matched filters can be employed at the receiver to separate all the transmit signals in order to achieve the effects of waveform diversity. After that, we can carry out the signal processing procedures like beam forming and parameter estimation subsequently, [5-6]. The orthogonality of transmit waveforms is thereby the basis for MIMO radar signal processing. Currently, many

+ Corresponding author.
E-mail address: qwang_87@163.com.

researchers have been making their efforts to find orthogonal waveforms and some of their approaches are designed to achieve good correlation performance, [7] taken as an example. However, in practice, completely orthogonal waveforms do not exist, even if small cross-correlation value is difficult to realize. When transmit waveforms are not completely orthogonal, they may not be able to be separated by passing received signals through matched filters and the follow-up processing will inevitably be affected. Hence, the matched filtering of incomplete orthogonal or quasi-orthogonal signals used in the scene of MIMO radar is studied in this paper, and some important conclusions are drawn.

First, the output of quasi-orthogonal MIMO radar waveforms passed through matched filters is modeled. The model involves in the reflection coefficients of different transmit waveforms, sensor positions and target azimuth that may impact on the output of filters. Then, an impact factor function is proposed to indicate the degree of influence caused by signal coherence in filter output. Finally, we present some numerical results to illustrate this effect.

2. Matched Filtering Model

As shown in Fig. 1, assume that there is a narrow-band monostatic MIMO radar system, of which both transmitter and receiver are linear arrays, with M and N elements, respectively. All of the elements at transmitter and receiver are omnidirectional antennas, located at known locations given by $d_{t,m}$ and $d_{r,n}$. The transmit signals' carrier frequency is f_0 and their complex envelopes are different from each other, represented by $s_m(t)$. The azimuth of the fixed point target is θ and the distances from transmit and receive array elements to the point target is $r_{t,m}$ and $r_{r,n}$, separately.

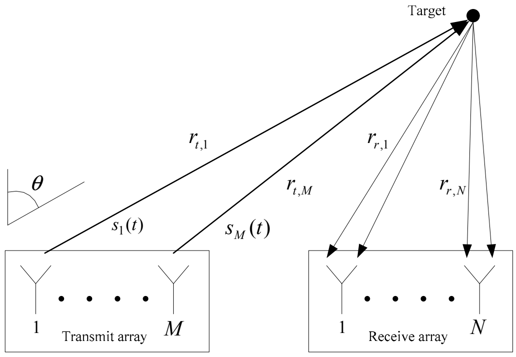

Fig. 1: MIMO radar system model.

Each transmit signal propagates through different path and arrives at the receiver with a delay equal to $r_{t,m}/c + r_{r,n}/c$, so we can express the received signal at the nth receiver as

$$x_n(t) = \sum_{m=1}^{M} \alpha_m s_m\left(t - \frac{r_{t,m}}{c} - \frac{r_{r,n}}{c}\right) e^{-j2\pi f_0\left(t - \frac{r_{t,m}}{c} - \frac{r_{r,n}}{c}\right)} + n_n(t) \quad (1)$$

where α_m is the reflection coefficient of the mth transmit signal, $n_n(t)$ is zero mean complex Gaussian noise and assumed irrelevant to all the transmit signals (here the propagation power loss is omitted so as to simplify the formula).

If the transmit signals are completely orthogonal to each other, which means they can satisfy the following condition:

$$\int_{Tp} s_k(t) s_l^*(t-\tau) dt = \begin{cases} \delta(\tau) & k = l \\ 0 & k \neq l \end{cases} \quad (2)$$

where T_p is pulse width, τ is time delay and $-T_p \le \tau \le T_p$, $\delta(\tau)$ is an impulse function. Then the matched filter can be used to isolate the echo of different transmit signals from the received signals. The matched filter structure of MIMO is usually the form shown in Fig. 2.

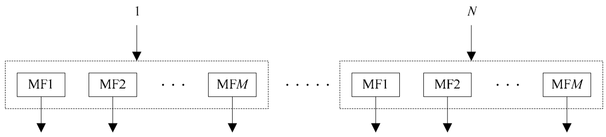

Fig. 2.Matched filter structure of MIMO radar.

The act of a matched filter is the same as a cross correlator, which suggests that we only need to correlate the received signal with transmit signals respectively to achieve the purpose of transmit signal separation. However, (2) actually can not be met in practice, that is, when the kth transmit signal pass through the lth correlator, the output will be as below:

$$\int_{Tp} s_k(t)s_l^*(t-\tau)dt = \begin{cases} c_{kk}(\tau) & k=l \\ c_{kl}(\tau) & k \neq l \end{cases} \tag{3}$$

where τ is the time delay of the matched filter and satisfies $-T_p \leq \tau \leq T_p$, $c_{kk}(\tau)$ is the autocorrelation function of the kth transmit signal, $c_{kl}(\tau)$ is the cross-correlation of the kth and the lth signal.

Through waveform optimization algorithms, such as the law in [7], we can design some waveforms that have relatively small cross-correlation values. But we should note that, the received signal at the receiver is some kind of superposition of multiple transmit signals. Thereby, the overall effect cannot be just neglected. Furthermore, matched filtering is a sliding correlation process. While filtering, different signals' cross-correlation output will cause sidelobes which may produce false target or mask off the real target [8].

From here on, we will start to achieve a precise model of the matched filter output. In order to make the model more concise, some parameter substitutions are presented ahead. First, let

$$t' = t - \frac{r_{t,i}}{c} - \frac{r_{r,j}}{c} \tag{4}$$

This step is actually to move the origin of time axis to $r_{t,i}/c + r_{r,j}/c$, which is the delay of the signal from the ith transmit element to the jth receive element. By substituting (4) into (2), we can get the output of the jth receiver as bellow

$$\begin{aligned} x_j(t') &= \sum_{m=1}^{M} \alpha_m s_m\left(t' + \frac{r_{t,i}}{c} + \frac{r_{r,j}}{c} - \frac{r_{t,m}}{c} - \frac{r_{r,j}}{c}\right) \\ &\quad \bullet e^{-j2\pi f_0\left(t' + \frac{r_{t,i}}{c} + \frac{r_{r,j}}{c} - \frac{r_{t,m}}{c} - \frac{r_{r,j}}{c}\right)} + n_j(t') \\ &= \sum_{m=1}^{M} \alpha_m s_m\left(t' + \frac{r_{t,i}}{c} - \frac{r_{t,m}}{c}\right) e^{-j2\pi f_0\left(t' + \frac{r_{t,i}}{c} - \frac{r_{t,m}}{c}\right)} + n_j(t') \end{aligned} \tag{5}$$

When transmit signals are not completely orthogonal, the output of the ith matched filter can be given by, [8]

$$\begin{aligned} y_{ji}(\tau) &= \int_{Tp} x_j(t') s_i^*(t'-\tau) e^{j2\pi f_0 t'} dt' \\ &= \alpha_i c_{ii}(\tau) + \sum_{m \neq i}^{M} \alpha_m c_{mi}\left(\tau + \frac{r_{t,i}}{c} - \frac{r_{t,m}}{c}\right) e^{-j2\pi f_0\left(\frac{r_{t,i}}{c} - \frac{r_{t,m}}{c}\right)} \end{aligned} \tag{6}$$

For a far field point target

$$\frac{r_{t,i}}{c} - \frac{r_{t,m}}{c} = \frac{(d_{t,m} - d_{t,i})\sin\theta}{c} = \frac{d_{mi}\sin\theta}{c} \tag{7}$$

Then we have

$$y_{ji}(\tau)=\alpha_i c_{ii}(\tau)+\sum_{m\neq i}^{M}\alpha_m c_{mi}(\tau+\frac{d_{mi}\sin\theta}{c})e^{-j2\pi f_0\frac{d_{mi}\sin\theta}{c}} \quad (8)$$

Now we have obtained the output by passing the jth receive signal through the ith matched filter. And the output formula is simply composed of shifted correlation functions weighted by reflection coefficients and exponential factors.

As can be seen from the above equation, the effect of signal correlation function between signals is mainly reflected in the second term which is also affected by array element spacings, target location and reflection coefficients. Therefore, in order to evaluate the effect on the performance of matched filtering when the parameters are different, we define an impact factor:

$$F(\boldsymbol{\alpha},\mathbf{d},\theta)=\max_i\frac{|\sum_{m\neq i}^{M}\alpha_m c_{mi}(\tau+\frac{d_{mi}\sin\theta}{c})e^{-j2\pi f_0\frac{d_{mi}\sin\theta}{c}}|}{|\alpha_i c_{ii}(0)|} \quad (9)$$

where $\boldsymbol{\alpha}$ is the target reflection coefficient vector which is made up of all the reflection coefficients of different signals, $\mathbf{d}$ is transmit array element position vector and θ is target azimuth. When the number and form of the transmitted signals are all known, the signals' correlation functions are determinate and $F(\boldsymbol{\alpha},\mathbf{d},\theta)$ is only related to the three parameters, $\boldsymbol{\alpha}$, $\mathbf{d}$ and θ. The impact factor is actually the maximum magnitude of the cross-correlation part normalized by autocorrelation peak in all the channels. It represents the greatest impact of signal correlation characteristic on the performance of matched filtering. Smaller impact factor suggests that cross-correlation has less effect on matched filtering, while the larger factor may indicate that false peak would be caused and even the real target signal would probably be offset.

3. Impact with Different Parameters

It can be seen from (8) that when the transmit signals are not completely orthogonal, the output of matched filter is the superposition of signal autocorrelation and cross-correlation functions and affected by transmit array element position, target reflection coefficients and target azimuth. Separately, we qualitatively discuss as follows:

3.1. Impact of transmit array

The distances between transmit array elements will induce time delay of complex envelope of transmit signals, resulting in cross-correlation function shift. Under normal circumstances, the magnitude of cross-correlation function generally decrease as time delay grows, which is shown in Fig. 3 as an example. Therefore, when the target location is fixed, with the array element spacing increases, the correlation effect on the matched filter output will gradually decreases. In particular, when $|d_{mi}\sin\theta/c|\geq 2T_p$, the transmit signals have a time interval of two times of T_p, and the two signals' outputs through the matched filter do not overlap. So at this time the impact of correlation is minimized.

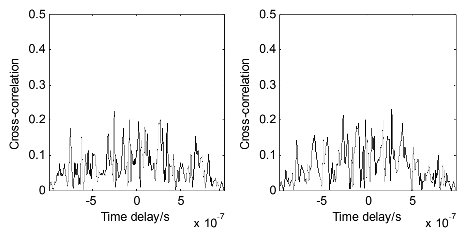

Fig. 3. Cross-correlation of transmit signals.

3.2. Impact of reflection coefficients

The size of reflection coefficients affect the power of each impinged signal at the receiver, so will it change the magnitude of autocorrelation and cross-correlation function in the output of each matched filter. Especially, weak target that has small reflection coefficient is more vulnerable to be influenced by cross-correlation part (the second term in (8)), so that the real signal is very likely to be masked off.

3.3. Impact of target azimuth

Similar to the impact of array element spacing, when the array element spacing is large, the changes in azimuth will affect the delay of signal envelope and also the phase of the correlation function weight $e^{-j2\pi f_0 \frac{d_{mi}\sin\theta}{c}}$. Especially, when array element spacing is large, a small azimuth change will result in significant phase variety.

In addition, while transmit array aperture is small (can be comparable with the carrier wavelength), the propagation delay has little effect on signal envelope, so the matched filter output can be approximated by:

$$y_{ji}(\tau) = \alpha_i c_{ii}(\tau) + \sum_{m\neq i}^{M} \alpha_m c_{mi}(\tau) e^{-j2\pi f_0 \frac{d_{mi}\sin\theta}{c}} \tag{10}$$

Then the array element spacing and target position primarily impact on the phase of correlation functions.

4. Numerical Results

The transmit waveforms used in our experiments are poly-phase orthogonal code waveforms given by [7]. The number of the code sequences is 4, length 40 and phase number 4. The pulse width is 1μs and carrier wavelength is 0.1 m. Meanwhile, in order to more accurately characterize the correlation between signals, complex correlation function is employed here [9].

4.1. Array element spacings

In considering the influence of array element spacings on the output of matched filter, all reflection coefficients are set to 1 so as to eliminate their impacts. And the azimuth of the fixed target is 20^0. Then we force the distribution of array elements to be uniform and observe the impact factor varying with the array aperture increases. The result is illustrated in Fig. 4.

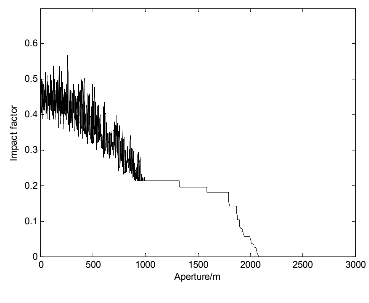

Fig. 4. Impact of element spacing on matched filtering.

As expected, while the array aperture increases, the spacing between array elements grows accordingly, so the impact factor gradually decreases. Besides, when all array element spacings meet $|d_{mi}\sin\theta/c| \geq 2T_p$, that is when the array aperture is greater than 2078 m, the impact factor changes to 0.

4.2. Reflection coefficients

Assuming that the reflection coefficients of the four transmit signals are 0.7, 1, 1 and 0.6 respectively, transmit array position vector is d = [0 10 20 30], in units of m, target azimuth is $\theta = 20^0$. The outputs of the four matched filters are shown in Fig. 5.

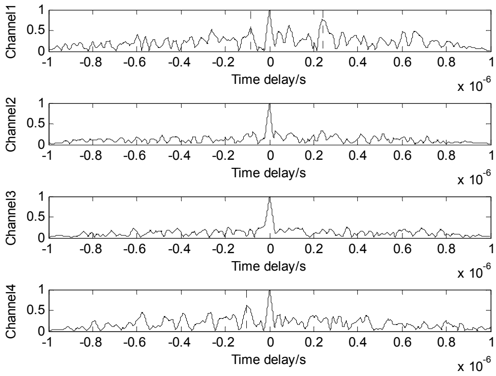

Fig. 5. Impact of Reflection coefficients.

It can be seen that, in channels 1 and 4, in addition to the peaks at τ=0 the positions identified by the dotted lines also have large peaks, resulting in false targets. Therefore, if the target reflection coefficient of the signal is smaller, the real signal would be concealed in the sidelobes.

4.3. Target azimuth

The impact of azimuth change is bound by array element spacings, so we firstly assume a uniform linear array case with an aperture of 300m. The result is plotted in Fig. 6, which illustrates the change of impact factor in azimuth from $-\pi/2$ to $\pi/2$.

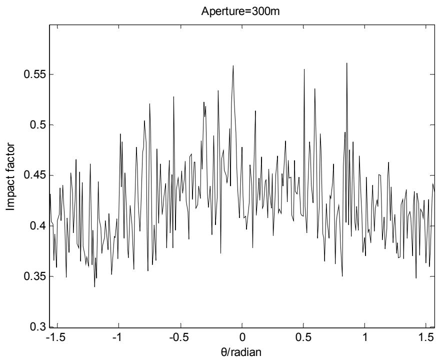

Fig. 6. Azimuth impact when aperture is 300 m.

It can be seen that as the target azimuth increases, the impact factor function shows dramatic changes in the ups and downs. And on some locations there are very high peaks, indicating that the matched filter output will be even worse at these angles.

Then, suppose that the array aperture is 3m, which is 30 times the carrier wavelength and the process same to the former is repeated. We obtain a curve drawn in Fig. 7. Clearly, when the array element spacings are small, the impact factor curve fluctuates more smoothly, but there are still some high peaks emerge at certain angles.

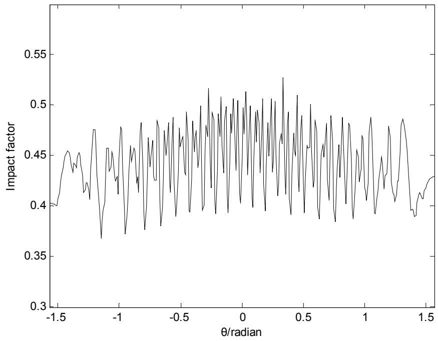

Fig. 7. Azimuth impact when aperture is 3m.

5. Conclusions

In this paper, we studied the impacts of transmit signal cross-correlation on matched filter output when using quasi-orthogonal multiple signals in MIMO radar. By more accurately modeling the process of matched filtering, the performance of matched filter output under different conditions is analyzed. Theoretical analysis and experiments show that in the incomplete orthogonal transmit signal case, the influence on matched filter output performance is subject to different degrees. These impacts are not only related to autocorrelation and cross-correlation performance of the signals themselves but also the distribution of transmit array, target reflection coefficient of each signal and the target azimuth angle. Therefore, the multiple signal system of MIMO radar requires more perfect orthogonal transmit signals.

Although we still have large room in designing signals, it is an indisputable fact that the signals can not be completely orthogonal. More rational as it is, we may be able to utilize the cross-correlation characteristics of the transmit signals so as to enhance the performances of some other aspects of MIMO radar. Therefore, it is also the significance of this paper to stimulate more realistic and novel ideas and concepts to promote the implementation of MIMO radar system.

6. References

[1] E. Fishler, A. Haimovich, R. Blum, L. Cimini, D. Chizhik, and R. Valenzuela, "MIMO radar: An idea whose time has come," in *Proc.of the IEEE Int. Conf. on Radar*. Philadelphia, PA, April 2004.

[2] J. Li and P. Stoica, "MIMO Radar Signal Processing," New York: Wiley & Sons, 2008.

[3] E. Fishler, A. Haimovich, R. S. Blum, L. J. Cimini, D. Chizhik, and R.A. Valenzuela, "Spatial diversity in radars-models and detection performance," *IEEE Trans. Signal Process*., vol 54, pp. 823–838, 2006.

[4] L.Z. Xu, J. Li, and P. Stoica, "Target detection and parameter estimation for MIMO radar systems," *IEEE Trans, Aerospace and Electronic Systems*, vol. 44, no. 3, July 2008

[5] C.Y. Chen and P.P. Vaidyanathan, "Minimum redundancy MIMO radars," in Proc. ISCAS, 2008, pp. 45–48.

[6] M. Jin, G.S. Liao, and J. Li, "Joint DOD and DOA estimation for bistatic MIMO radar," Signal Process., vol. 89, pp. 244–251, 2009.

[7] B. Liu, Z. S. He, J. K. Zeng, and B. Y. Liu, "Polyphase Orthogonal Code Design for MIMO Radar Systems," proceeding of IEEE, 2006.

[8] G. Richard Curry, "Radar System Performance Modeling," 2nd ed. London: Artech House, 2005.

[9] S.C. Joseph, R.S. Yahya, M.A. Jensen, and G.J. Pottie, "Evaluation of Personal Communications Dual-Antenna Handset Diversity Performance," IEEE Trans. Vehicular Technology, 47(3): 737-746, August 1998.