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Angle Estimation Using Capon with Non-Orthogonal Signals in MIMO Radar

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Abstract. Multiple-Input Multiple-Output (MIMO) radar has been shown to provide enhanced performance in theory and practice. This paper focuses on MIMO Radar angle estimation. We show that the spatially orthogonal signal transmission is equivalent to additional virtual sensors which extend the array aperture with virtual spatial tapering. These virtual sensors can be used to form narrower beams with lower sidelobes, therefore, provide higher performance in angular estimation accuracy and increase the upper limit on the number of targets. In this paper, by linear transformation of independent signals and by using the information in the transmit and the receive modes, then using Capon to search for target location spectrum peak. It is shown that the method proposed in this paper is achieved for an arbitrary signal coherence matrix. Target localization performances are evaluated via simulation results.

Keywords: MIMO radar, angle estimation, no-orthogonal signal, capon.

1. Introduction

Recently, Multiple-Input Multiple-Output (MIMO) radar has drawn considerable attention [1-9]. MIMO radar utilizes multiple antennas at both the transmitter and receiver, and can be distinguished into two main classses: MIMO radars with widely separated antennas and MIMO radars with colocated antennas [4-5]. The first class capitalizes on the rich scattering properties of a target [4]. The second class allows to model a target as a point-source in the far-fild, which means that the target is assumed to be sufficiently far such that it is essentially at the same distance with respect to all [5]. It has been shown that waveform diversity enables the MIMO radar superiority in several fundamental aspects, including: 1) significantly improved parameter identifiability 2) direct applicability of many adaptive techniques to achieve high resolution and excellent interference rejection capability. In this paper, we discuss the second class, and assume the transmitted probing signals be narrowband signals, whose amplitudes do not change appreciably across the target.

We address one of the most basic issues of MIMO radar-its parameter identifiability, as we known; angle estimation is an important aspect for MIMO radar and a variety of methods are developed. In practice, orthogonal waveforms with low integrated sidelobe levels (ISLs) can be difficult to attain. We consider herein applying the Capon algorithm to estimate the target locations using MIMO radar with non-orthogonal signals.

2. MIMO Radar Signal Model

Consider a MIMO radar system with M_t transmit antennas and M_r receiving antennas. Suppose that the antenna array is a uniform linear array (ULA) with half-wavelength inter-element spacing. In the case of

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L targets in the given range-Doppler bin and in the presence of targets at direction θ_l in a multipath-free environment, then under the simplifying assumption of point targets, the received signal is given by

$$\mathbf{Y} = \sum_{l=1}^{L} \alpha_l(\theta_l)\mathbf{a}_r^*(\theta_l)\mathbf{a}_t^H(\theta_l)\mathbf{S} + \mathbf{V} \tag{1}$$

Where,

$$\mathbf{S} = \left[s_1(n), s_2(n), \cdots, s_{M_t}(n)\right]^T \tag{2}$$

$$\mathbf{a}_t(\theta) = \left[e^{j2\pi f_0 \tau_1(\theta)}, e^{j2\pi f_0 \tau_2(\theta)}, \cdots, e^{j2\pi f_0 \tau_{M_t}(\theta)}\right]^T \tag{3}$$

$$\mathbf{a}_r(\theta) = \left[e^{j2\pi f_0 \tilde{\tau}_1(\theta)}, e^{j2\pi f_0 \tilde{\tau}_2(\theta)}, \cdots, e^{j2\pi f_0 \tilde{\tau}_{M_r}(\theta)}\right]^T \tag{4}$$

$\mathbf{S} \in C^{M_t \times N}$ denotes the samples of baseband equivalent signals, $[\cdot]^T$ denotes the transpose of amatrix or a vector, N denotes the number of samples of each transmitted signal pulse. $\mathbf{a}_t(\theta) \in C^{M_t \times 1}$ is transmit steering vector, $\tau_m(\theta)$ is the time needed by the signal emitted via m_{th} transmit antenna to arrive at the target. $\mathbf{a}_r(\theta) \in C^{M_r \times 1}$ is receive steering vector, $\tilde{\tau}_n(\theta)$ is the time needed by the signal reflected by the target located at θ to arrive at the n_{th} receive antenna.

α stands for the complex amplitudes proportional to the radar-cross-sections (RCSs) of those targets, f_0 is the carrier frequency, $[\cdot]^*$ denotes the conjugate of a matrix or a vector, and $[\cdot]^H$ denotes the conjugate transpose of a matrix or a vector. The noise vectors $\mathbf{V} \in \mathbf{C}^{M_r \times N}$ are assumed to be independent, zero-mean complex Gaussian with known covariance matrix $\boldsymbol{\sigma}_v^2$, and assumed the noise is uncorrelated with transmittied signal $\mathbf{S}$.

Assume that transmitter and receiver phase centers are constrained to occupy the same locations. Let

$$\mathbf{F}(\theta) = \mathbf{a}_r(\theta)\mathbf{a}_t^T(\theta) \tag{5}$$

Note that $\mathbf{F}$ is the array response matrix, the elements of $\mathbf{F}$ through all possible combinations of delays in transmit and receive modes.

Then we can rewrite (1) as

$$\mathbf{Y} = \sum_{l=1}^{L} \alpha_l \mathbf{F}^*(\theta_l)\mathbf{S} + \mathbf{V} \tag{6}$$

3. Spectrum Estimation with Non-Orthogonal Signals

By matched filtering using $\mathbf{S}^H$, the sufficient statistic matrix can be defined as

$$\mathbf{X} = \frac{1}{\sqrt{N}}\mathbf{Y}\mathbf{S}^H \tag{7}$$

In the case of orthogonal transmitted signals and the coherence matrix of transmitted signals is an identity matrix. The sufficient statistics are statistically independent. However, it can be shown that for non-orthogonal signals, the sufficient statistics are statistically dependent. We first assume that the coherence matrix of non-orthogonal signals, $\mathbf{R}_{SS}$, is nonsingular, and then we will refer to the general case, the matrix $\mathbf{R}_{SS}$ can be decomposed using singular value decomposition(SVD)as

$$\mathbf{R}_{SS} = \frac{1}{N}\sum_{n=1}^{N} s(n)s^H(n) = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H \tag{8}$$

Where $\mathbf{U}$ and $\mathbf{\Lambda}$ are the matrices of eigenvectors and eigenvalues of $\mathbf{R}_{SS}$, respectively.

Accordingly, $\overline{\mathbf{S}}$ is a linear transformation of a vector of independent signals defined as

$$\overline{\mathbf{S}} = \mathbf{\Lambda}^{-\frac{1}{2}}\mathbf{U}^H\mathbf{S} \tag{9}$$

The independent sufficient statistic vector can be obtained as [8]

$$\begin{aligned}\overline{\mathbf{X}} &= \frac{1}{\sqrt{N}}\mathbf{Y}\overline{\mathbf{S}}^H = EU\Lambda^{-\frac{1}{2}} \\ &= \sqrt{N}\sum_{l=1}^{L}\alpha_l \mathbf{F}(\theta_l)\mathbf{U}\mathbf{\Lambda}^{\frac{1}{2}} + \overline{\mathbf{V}}\end{aligned} \tag{10}$$

Actually, the sufficient statistic can be obtained by a matched filter: temporal matching the measurement vectors to different signal subspace components $\overline{\mathbf{S}}$.

Insertion of (3) into (7) yields the sufficient statistic can be written in the form

$$\overline{\mathbf{\eta}} = vec(\overline{\mathbf{X}}) = \sum_{l=1}^{L}\mathbf{a}(\theta_l)\alpha_l + \tilde{\mathbf{V}} \tag{11}$$

Where, $vec(\cdot)$ denotes the vectorization operator. $\mathbf{a}(\theta) = vec(\sqrt{N}\mathbf{F}(\theta_l)\mathbf{U}\mathbf{\Lambda}^{\frac{1}{2}})$, a_l is the equivalent array response at the direction θ_l. By linear transformation of a vector of independent signals, therefore the sufficient statistics can result in M_tM_r virtual array. Consequently, the array aperture is virtually extended. Then we can obtain narrower beams, improve the angular resolution by using the information in the transmit and the receive modes and increase the upper limit on the number of targets which can be detected and localized by the array (this is attributed to the virtual sensors). $\tilde{\mathbf{V}} = vec(\overline{\mathbf{V}})$ which is zero-mean complex Gaussian with $\mathbf{\sigma}_v^2\mathbf{I}_{M_tM_r}$.

Let $\varphi \triangleq [\alpha_1, \cdots, \alpha_L]$, then the covariance matrix of n_{th} virtual sub-array can be given as

$$R_{\eta_n} = \mathbf{a}(\theta)E[\varphi_n\varphi_n^H]\mathbf{a}^H(\theta) + E[v_n v_n^H] \tag{12}$$

According to the signal model developed in Eq. (11), the Capon spatial spectrum can be formulated as

$$P_{capon} = \frac{1}{a^H R_{\eta_n}^{-1} a} \tag{13}$$

4. Simulation Results

In this section, several numerical examples are presented to demonstrate the effectiveness of the presented method. Consider a MIMO radar system with $M_t = M_r = 10$ antennas and 0.5-wavelength spacing between adjacent antennas is used both for transmitting and for receiving. Consider a scenario in which 8 targets are located at $-60^\circ, -55^\circ, -50^\circ, -45^\circ, -40^\circ, -30^\circ, -20^\circ, -10^\circ$, the number of snapshots is 256, signal-to-noise ratio (SNR) = 10. The received signal is corrupted by a spatially and temporally white circularly symmetric complex Gaussian noise with mean zero and by jammer located at 0° with an unknown waveform (uncorrelated with the waveforms transmitted by the radar) with interference-to-noise ratio (INR) = 60db. The simulation results are shown in Fig. 1 and Fig. 2. Note that all 8 target locations can be approximately determined from the peak locations of the capon spatial spectrum, seen from the spectrum, the angle estimation performance degrades as the jammer as incurred.

Next reconsider a scenario in which 4 targets are located at $-40^\circ, -30^\circ, -20^\circ, -10^\circ$, the received signal is corrupted by a spatially and temporally white circularly symmetric complex Gaussian noise with mean zero SNR=10 , but with no jammer is shown in Fig. 3.

In Fig. 4, solid line denotes the linear transformation of a vector of independent signals, the marked dash line denotes the non-orthogonal transmitted signals. From the figure 4, it can be shown that by linear transformation of a vector of independent signals and by using the information in the transmit and the receive modes, the spectrum can form narrower beams with lower sidelobes, targets can be approximately determined with the linear transformation of the transmitted signals, therefore, provide higher performance in angular estimation accuracy.

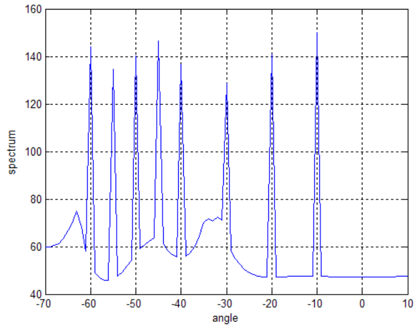

Fig.1. Targets corrupted by complex Gaussian noise.

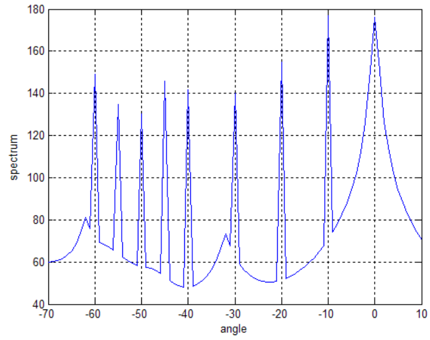

Fig. 2. Targets corrupted by noise and jammer.

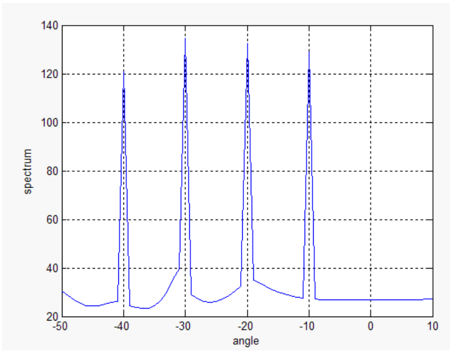

Fig.3. 4 targets corrupted by complex Gaussian noise.

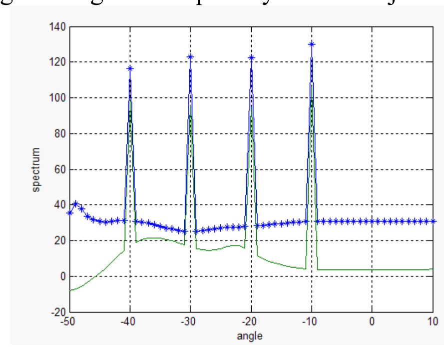

Fig. 4. Transmitted signals with linear transformation or not.

5. Using the Template

The method proposed in this paper is achieved for an arbitrary signal coherence matrix, by linear transformation of a vector of independent signals and by using the information in the transmit and the receive modes, array aperture can be extended. By using the virtual array, we can provide higher performance in angular estimation accuracy.

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