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Nonlinear Adaptive Bilinear Filters for Multichannel Active Noise Control Systems

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Abstract. A bilinear filtered-x least mean square (BFXLMS) algorithm for nonlinear adaptive filters is proposed to suppress nonlinear distortions which occur in multichannel nonlinear active noise control (ANC) systems in this paper. The performance of the bilinear adaptive filter is evaluated in terms of convergence characteristics. Simulation results demonstrate that the BFXLMS algorithm is more effective in reducing nonlinear distortions in multichannel ANC systems than the linear filtered-x least mean square (FXLMS) and second-order Voltterra FXLMS (VFXLMS) algorithms.

Keywords: active noise control, adaptive volterra filter, adaptive bilinear filter, filtered-x LMS algorithm.

1. Introduction

In many actual systems, the acoustic noise generated from dynamic system is nonlinear and predictable (chaotic), colored, and non-Gaussian. Researches show that the linear adaptive techniques used to control the noise exhibit degradation in performance. In addition, when the secondary path transfer function between the speaker and the error microphone has nonminimum phase, and the primary path may exhibit nonlinearity, a linear controller for ANC system yields poor performance. Furthermore, in each of these cases, researches have reliably shown that nonlinear ANC based on nonlinear adaptive filter outperforms ANC-based linear adaptive filter, and shows good performance for attenuating low frequency noises [1-5]. However, up to the present, there is no unique theory for modeling and characterizing the nonlinear ANC systems. Adaptive nonlinear controllers for ANC systems can be mainly divided into two categories: neural networks (NNs) [1-4] and adaptive polynomial filters [5]. Nonlinear ANC controllers, based on neural networks using a training algorithm developed from a back propagation scheme, were reported in applications where the actuators exhibit nonlinear characteristics [1-4]. However, the major disadvantages of nonlinear ANC system-based neural networks are complicated structure, heavy computational burden and slow convergence.

The adaptive Volterra filter, one of adaptive polynomial filter, has been studied in the literature as an alternative to the nonlinear filter, offering the advantage of its global convergence because its output is linear with respect to various higher order kernels or impulse responses [6]. For nonlinear ANC systems with a linear secondary path (LSP), Tan and Jiang proposed an adaptive Volterra filter using the FXLMS (VFXLMS) algorithm [5], and outperformed the linear adaptive filter with the FXLMS algorithm. However, to accurately model the nonlinear systems, the computational complexity of adaptive Volterra filters increases exponentially as the order (or the memory size) increases. Recently, to reduce the computational burden, Zhou and Debrunner developed computationally efficient filtered-error LMS (FELMS) algorithm based on a functional expansion nonlinear filter, which includes the SOV, TOV and the functional link

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artificial neural network (FLANN) models [7]. To improve the performance of the adaptive Volterra filter for nonlinear ANC systems, the adaptive output-error bilinear filter using the corresponding FXLMS algorithm with less computational complexity was presented by Kuo and Wu [8], and it provides the faster convergence and more effective in reducing saturation effects in ANC systems than adaptive second-order and third-order Volterra filters with the FXLMS algorithm.

In this paper, a computationally efficient multichannel nonlinear ANC system with the bilinear FXLMS (BFXLMS) algorithm is presented which is developed in the single-channel bilinear filtered-x LMS algorithm based nonlinear ANC as reported in [8].

2. Multichannel BFXLMS Algorithm

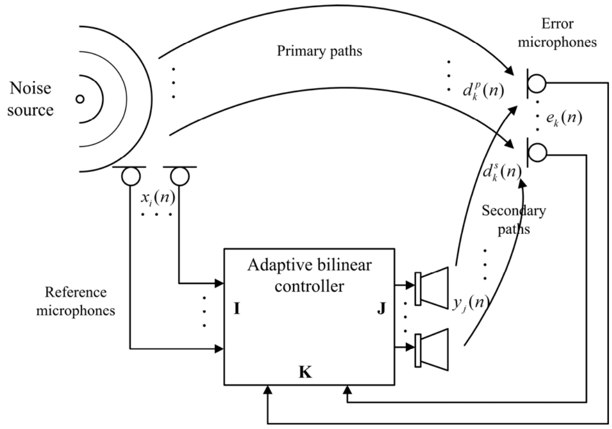

Fig.1. Multichannel ANC system with adaptive bilinear filter.

The multichannel ANC system with adaptive bilinear filter is shown in Fig. 1, with reference to which the following notations have been defined:

I	the number of primary source signals
J	the number of secondary source signals
K	The number of error sensors
$x_i(n)$	ith input signal at nth instant, $1 \le i \le I$
$y_j(n)$	jth secondary source at nth instant, $1 \le j \le J$
$s_{k,j}(n)$	the impulse response of the secondary path transfer function which connects the jth secondary source with kth error microphone;
$d_k^P(n)$	signal at the end of primary path near the kth error microphone;
$d_k^s(n)$	signal at the end of secondary path near the kth error microphone;

In Fig. 1, input signals $x_i(n)$, generated by the noise source, are collected by I microphones and fed to the adaptive bilinear controller. Then, using a truly recursive estimate, any output $y_j(n)$ of the multichannel adaptive bilinear controller can be expressed by

$$y_j(n) = \sum_{i=1}^{I} y_{j,i}(n) \tag{1}$$

and

$$\begin{aligned} y_{j,i}(n) &= \sum_{l=0}^{L} a_{j,i,l}(n) x_i(n-l) + \sum_{q=1}^{L} b_{j,i,q}(n) y_{j,i}(n-q) \\ &+ \sum_{l=0}^{L} \sum_{p=1}^{L} c_{j,i,l,p}(n) x_i(n-l) y_{j,i}(n-p) \end{aligned} \tag{2}$$

where L+1 is the memory length of the bilinear filter, $a_{j,i,l}(n)$, $b_{j,i,q}(n)$ and $c_{j,i,l,p}(n)$ denotes feedforward, feedback and cross coefficients, respectively.

To derive the BFXLMS algorithm for multichannel ANC system, using vector notation, (2) connecting the input i to the output j can be rewritten in the filter bank as

$$y_{j,i}(n) = A_{j,i}(n)X_i(n) + B_{j,i}(n)Y_{j,i}(n-1) + C_{j,i}(n)V_{j,i}(n) \tag{3}$$

where $A_{j,i}(n) = [a_{j,i,0}(n), \cdots, a_{j,i,l}(n), \cdots, a_{j,i,L}(n)]^T$ is the feedforward coefficient vector of length L+1, the corresponding input signal vector $X_i(n)$ is defined by

$$X_i(n) = [x_i(n), x_i(n-1), \cdots, x_i(n-L)]^T \tag{5}$$

The feedback coefficient vector $B_{j,i}(n)$ of length L is given by

$$B_{j,i}(n) = [b_{j,i,1}(n), \cdots, b_{j,i,q}(n), \cdots, b_{j,i,L}(n)]^T \tag{6}$$

And the corresponding output signal vector $Y_{j,i}(n-1)$ with one unit delay is defined by

$$Y_{j,i}(n-1) = [y_{j,i}(n-1), y_{j,i}(n-2), \cdots, y_{j,i}(n-L)]^T \tag{7}$$

The cross coefficient vector $C_{j,i}(n)$ of length $L(L+1)$ is defined as

$$C_{j,i}(n) = [c_{j,i,0,1}(n), \cdots, c_{j,i,l,p}(n), \cdots, c_{j,i,L,L}(n)]^T \tag{8}$$

And the corresponding cross signal vector $V_{j,i}(n)$ is defined as

$$\begin{aligned} V_{j,i}(n-1) = [&x_i(n)y_{j,i}(n-1), \cdots, x_i(n)y_{j,i}(n-L), \\ &x_i(n-1)y_{j,i}(n-1), \cdots, x_i(n-L)y_{j,i}(n-L)]^T \end{aligned} \tag{9}$$

For the simplicity, we can combine the coefficient vectors (4), (6) and (8) as an overall vector $W_{j,i}(n)$ of length $L+1+L+(L+1)L = L^2+3L+1$ expressed as

$$W_{j,i}(n) = [A^T_{j,i}(n), B^T_{j,i}(n), C^T_{j,i}(n)]^T \tag{10}$$

Similarly, a generalized signal vector $U_{j,i}(n)$ by combining the signal vectors (5), (7) and (9) is written by

$$U_{j,i}(n) = [X^T_i(n), Y^T_{j,i}(n-1), V^T_{j,i}(n)]^T \tag{11}$$

Then, $y_{j,i}(n)$ given in (2) can be simplified to

$$\begin{aligned} y_{j,i}(n) &= A^T_{j,i}(n)X_i(n) + B^T_{j,i}(n)Y_{j,i}(n-1) + C^T_{j,i}(n)V_{j,i}(n) \\ &= W^T_{j,i}(n)U_{j,i}(n) \end{aligned} \tag{12}$$

Therefore, the output of the bilinear filter for a multichannel ANC system can be given as

$$y_j(n) = \sum_{i=1}^{I} W^T_{j,i}(n)U_{j,i}(n) \tag{13}$$

Similar to the adaptive FIR, Volterra and FLANN filters for the multichannel ANC cases, the objective of the multichannle controller based on the adaptive bilinear filter is to minimize the sum of the instantaneous squared errors $e^2_k(n)$ using the FXLMS algorithm expressed as

$$\xi(n) = \sum_{k=1}^{K} e^2_k(n) \tag{14}$$

and

$$e_k(n) = d_k^P(n) + d_k^S(n) \tag{15}$$

where $d_k^P(n)$ represents the sound generated by the noise source, $d_k^S(n)$ denotes the sound generated through the secondary paths by the kth actuators as

$$d_k^S(n) = \sum_{j=1}^{J} s_{k,j}(n) * \sum_{i=1}^{I} W_{j,i}^T(n) U_{j,i}(n) \tag{16}$$

where * denotes convolution operation.

In accordance with the steepest descent algorithm rule, the weight update equation based on the LMS algorithm is derived as follows.

$$W_{j,i}(n+1) = W_{j,i}(n) - \frac{u}{2} \nabla_W \xi(n) \tag{17}$$

where $\nabla_W \xi(n)$ the gradient estimator is defined by

$$\nabla_W \xi(n) = 2\sum_{k=1}^{K} e_k(n) \nabla_W e_k(n) \tag{18}$$

Based on the above mentioned (12), (13), (15) and (16) equations, $\nabla_W e_k(n)$ can further given as

$$\begin{aligned}
\nabla_W e_k(n) &= s_{k,j}(n) * \frac{\partial y_{j,i}(n)}{\partial W_{j,i}(n)} \\
&= \hat{s}_{k,j}(n) * \left[\frac{\partial y_{j,i}(n)}{\partial a_{j,i,0}(n)}, \cdots, \frac{\partial y_{j,i}(n)}{\partial a_{j,i,L}(n)}, \right. \\
&\qquad \frac{\partial y_{j,i}(n)}{\partial b_{j,i,1}(n)}, \cdots, \frac{\partial y_{j,i}(n)}{\partial b_{j,i,L}(n)}, \\
&\qquad \left. \frac{\partial y_{j,i}(n)}{\partial c_{j,i,0,1}(n)}, \cdots, \frac{\partial y_{j,i}(n)}{\partial c_{j,i,L,L}(n)} \right]^T
\end{aligned} \tag{19}$$

where $s_{k,j}(n)$ is replaced by its estimate $\hat{s}_{k,j}(n)$ for actual applications.

Define

$$\begin{aligned}
y_{a_{j,i,l}}(n) &= \frac{\partial y_{j,i}(n)}{\partial a_{j,i,l}(n)} \\
&= x_i(n-l) + \sum_{q=1}^{L} b_{j,i,q}(n) \frac{\partial y_{j,i}(n-q)}{\partial a_{j,i,l}(n)} \\
&\quad + \sum_{l=0}^{L} \sum_{p=1}^{L} c_{j,i,l,p}(n) x_i(n-l) \frac{\partial y_{j,i}(n-p)}{\partial a_{j,i,l}(n)}
\end{aligned} \tag{20a}$$

$$\begin{aligned}
y_{b_{j,i,q}}(n) &= \frac{\partial y_{j,i}(n)}{\partial b_{j,i,q}(n)} \\
&= y_{j,i}(n-q) + \sum_{q=1}^{L} b_{j,i,q}(n) \frac{\partial y_{j,i}(n-q)}{\partial b_{j,i,q}(n)} \\
&\quad + \sum_{l=0}^{L} \sum_{p=1}^{L} c_{j,i,l,p}(n) x_i(n-l) \frac{\partial y_{j,i}(n-p)}{\partial b_{j,i,q}(n)}
\end{aligned} \tag{20b}$$

$$
\begin{aligned}
y_{c_{j,i,l,p}}(n) &= \frac{\partial y_{j,i}(n)}{\partial c_{j,i,l,p}(n)} \\
&= x_i(n-l)y_{j,i}(n-p) + \sum_{q=1}^{L} b_{j,i,q}(n)\frac{\partial y_{j,i}(n-q)}{\partial c_{j,i,l,p}(n)} \\
&+ \sum_{l=0}^{L}\sum_{p=1}^{L} c_{j,i,l,p}(n)x_i(n-l)\frac{\partial y_{j,i}(n-p)}{\partial c_{j,i,l,p}(n)}
\end{aligned} \tag{20c}
$$

and assume that the step size u is small for slow convergence for multichannel ANC system, we have

$$
\frac{\partial y_{j,i}(n-p)}{\partial a_{j,i,l}(n)} \approx \frac{\partial y_{j,i}(n-p)}{\partial a_{j,i,l}(n-p)} = y_{a_{j,i,l}}(n-p) \tag{21a}
$$

$$
\frac{\partial y_{j,i}(n-q)}{\partial b_{j,i,q}(n)} \approx \frac{\partial y_{j,i}(n-q)}{\partial b_{j,i,q}(n-q)} = y_{b_{j,i,q}}(n-q) \tag{21b}
$$

$$
\frac{\partial y_{j,i}(n-p)}{\partial c_{j,i,l,p}(n)} \approx \frac{\partial y_{j,i}(n-p)}{\partial c_{j,i,l,p}(n-p)} = y_{c_{j,i,l,p}}(n-p) \tag{21c}
$$

Thus, equations (20a-c) can be modified as

$$
\begin{aligned}
y_{a_{j,i,l}}(n) &\approx x_i(n-l) + \sum_{q=1}^{L} b_{j,i,q}(n)y_{a_{j,i,l}}(n-q) \\
&+ \sum_{l=0}^{L}\sum_{p=1}^{L} c_{j,i,l,p}(n)x_i(n-l)y_{a_{j,i,l}}(n-p)
\end{aligned} \tag{22a}
$$

$$
\begin{aligned}
y_{b_{j,i,q}}(n) &\approx y_{j,i}(n-p) + \sum_{q=1}^{L} b_{j,i,q}(n)y_{b_{j,i,q}}(n-q) \\
&+ \sum_{l=0}^{L}\sum_{p=1}^{L} c_{j,i,l,p}(n)x_i(n-l)y_{b_{j,i,q}}(n-p)
\end{aligned} \tag{22b}
$$

$$
\begin{aligned}
y_{c_{j,i,l,p}}(n) &\approx x_i(n-l)y_{j,i}(n-p) + \sum_{q=1}^{L} b_{j,i,q}(n)y_{c_{j,i,l,p}}(n-q) \\
&+ \sum_{l=0}^{L}\sum_{p=1}^{L} c_{j,i,l,p}(n)x_i(n-l)y_{c_{j,i,l,p}}(n-p)
\end{aligned} \tag{22c}
$$

When L is large, the equations (22a-c) are too complicated to implement due to the recursive nature of $y_{a_{j,i,l}}(n)$, $y_{b_{j,i,q}}(n)$ and $y_{c_{j,i,l,p}}(n)$. Assume that the recursion based on the past output gradients is negligible [8]. That is

$$
y_{a_{j,i,l}}(n-p) = y_{b_{j,i,q}}(n-p) = y_{c_{j,i,l,p}}(n-p) = 0 \tag{23}
$$

for all i,j,l and p. Then,

$$
\begin{aligned}
\nabla_W e_k(n) &= \hat{s}_{k,j}(n) * \big[x_i(n), \cdots, x_i(n-L), \\
&\qquad y_{j,i}(n-1), \cdots, y_{j,i}(n-L), \\
&\qquad x_i(n)y_{j,i}(n-1), \cdots, x_i(n-L)y_{j,i}(n-L)\big]^T \\
&= \hat{s}_{k,j}(n) * U_{j,i}(n)
\end{aligned} \tag{24}
$$

Therefore, the coefficient update equation of the BFXLMS algorithm is given as

$$
W_{j,i}(n+1) = W_{j,i}(n) - u\sum_{k=1}^{K} e_k(n)U'_{k,j,i}(n) \tag{25}
$$

where $U'_{k,j,i}(n) = s_{k,j}(n) * U_{j,i}(n)$.

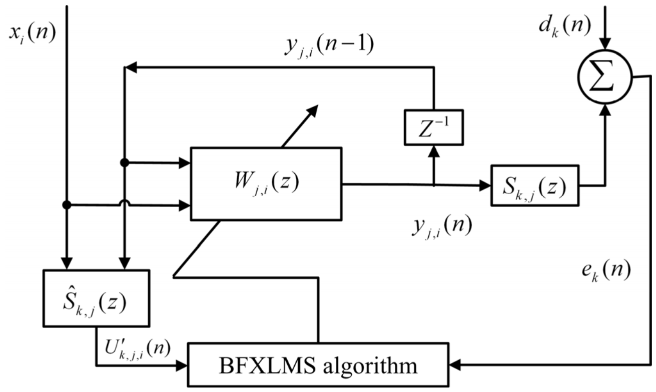

Fig. 2. The implementation of the BFXLMS algorithm.

The block diagram of this adaptive bilinear filter with the FXLMS algorithm for multichannel nonlinear ANC systems is illustrated in Fig. 2.

3. Simulation

To test the performance of the BFXLMS algorithm over the FXLMS and second-order VFXLMS algorithms, computer simulations are conducted in multichannel nonlinear ANC system in this section. In all the experiments, the value of the memory size L is set to be 10, and the number of reference microphone, loudspeakers and error microphone are chosen to be 1, 2 and 4, respectively. Moreover, the ensemble average of square error (EASE) defined by

$$EASE = 10\log_{10}\left[\frac{\sum_{k=1}^{K} e_k^2(n)}{K}\right] \tag{26}$$

is obtained after averaging over 50 independent runs each consisting of 4000 iterations, where $E(\cdot)$ is the expectation operator.

A. Experiment-I

In the first experiment, four linear primary paths and eight nonminimum phase secondary path transfer functions are used as follows:

$B_{1,1}(z) = z^{-5} - 0.3z^{-6} + 0.2z^{-7}$, $B_{1,2}(z) = z^{-5} - 0.2z^{-6} + 0.1z^{-7}$,

$B_{1,3}(z) = z^{-5} - 0.3z^{-6} + 0.1z^{-7}$, $B_{1,4}(z) = z^{-5} - 0.2z^{-6} + 0.2z^{-7}$,

$S_{1,1}(z) = z^{-2} + 1.5z^{-3} - z^{-4}$, $S_{2,1}(z) = z^{-2} + 1.7z^{-3} - z^{-4}$,

$S_{3,1}(z) = z^{-2} + 1.8z^{-3} - z^{-4}$, $S_{4,1}(z) = z^{-2} + 1.9z^{-3} - z^{-4}$,

$S_{1,2}(z) = z^{-2} + 1.5z^{-3} - z^{-4}$, $S_{2,2}(z) = z^{-2} + 1.2z^{-3} - z^{-4}$,

$$S_{3,2}(z) = z^{-2} + 1.1z^{-3} - z^{-4} \text{ , } S_{4,2}(z) = z^{-2} + z^{-3} - z^{-4} \text{ .} \tag{27}$$

A logistic chaotic noise, a second-order white and predictable nonlinear process, is chosen as the reference noise source in simulations, which is generated using the recursive equation [3]

$$x(n+1) = \lambda x(n)[1 - x(n)] \tag{28}$$

where $\lambda = 4$ and x(0)=0.9. This nonlinear noise process is then normalized to have unity signal power. The step sizes for all the four algorithms are chosen as: a) FXLMS $u = 0.001$; b) VFXLMS $u_1 = 0.0008, u_2 = 0.0001$; and c) BFXLMS $u_1, u_2 = 0.001$. Comparison of convergence performance with chaotic input signal and nonminimum phase secondary path transfer function is depicted in Fig. 3. As can be seen from the figure, the convergence performance of BFXLMS and algorithm is superior to the linear FXLMS, and second-order VFXLMS algorithms.

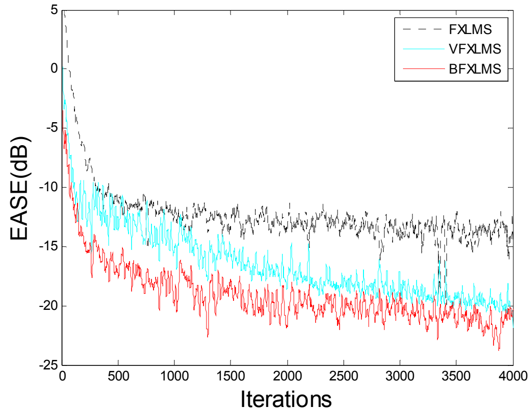

Fig. 3. Comparison of convergence performance with chaotic input signal and nonminimum phase secondary path transfer function.

B. Experiment-II

Assuming that the primary path exhibits the nonlinear behavior, the performance of different ANC controllers is further evaluated in the second experiment. The primary noise at the canceling point is generated based on the following third-order polynomial model [3]:

$$d(n) = t(n-2) + 0.08t^2(n-2) - 0.04t^3(n-2) \tag{29}$$

where $t(n)$ is obtained from the following linear convolution:

$$t(n) = x(n) * f_{i,k}(n) \tag{30}$$

Note that the four different transfer functions with impulse response $f_{i,k}(n)$ are $F_{1,1}(z) = z^{-3} - 0.3z^{-4} + 0.2z^{-5}$, $F_{1,2}(z) = z^{-3} - 0.2z^{-4} + 0.1z^{-5}$ $F_{1,3}(z) = z^{-3} - 0.3z^{-4} + 0.1z^{-5}$, and $F_{1,4}(z) = z^{-3} - 0.2z^{-4} + 0.2z^{-5}$. And the reference noise $x(n)$ is a sinusoidal wave of 500 Hz sampled at the rate of 8000 samples/s, that is

$$x(n) = \sqrt{2}\sin\left(\frac{2\pi \times 500 \times n}{8000}\right) + v(n) \tag{31}$$

and v(n) is a white noise process with the Gaussian distribution. The signal power-to-noise power ratio (SNR) is chosen to be 40 dB. The nonminimum phase secondary path transfer functions are considered to be same as in Experiment-I. The step sizes are used in the following: a) FXLMS $u = 0.00005$; b) VFXLMS $u_1 = 0.00001, u_2 = 0.00005$; and c) BFXLMS $u_1, u_2 = 0.0001$. Fig. 4 illustrates the comparison for the nonlinearity case. Various conclusions drawn from these figures show that the BFXLMS algorithm exhibit better convergence characteristics as compared to FXLMS and VFXLMS algorithms.

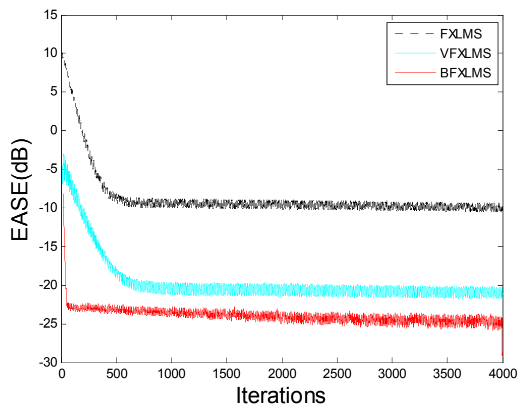

Fig. 4. Comparison of convergence characteristics with periodic input signal, nonlinear primary paths and nonminimum phase secondary path transfer function.

4. Conclusion and Discussion

An adaptive bilinear filter using the FXLMS algorithm is presented to overcome nonlinear distortions that occur in multichannel nonlinear ANC systems in this paper. This nonlinear adaptive filter, which is a nonlinear extension of the IIR filter, can model nonlinear systems accurately with less computational complexity. Computer simulations illustrate that the BFXLMS algorithm exhibits better than FXLMS, and second-order VFXLMS algorithms for multichannel nonlinear ANC systems.

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