

Complex Algorithms for Lattice Adaptive IIR Notch Filter

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Abstract. The problem of detecting the complex narrowband signal using complex adaptive IIR notch filter is investigated. In this paper, three complex coefficient adaptive IIR notch filters using gradient-based algorithms are proposed. These three methods are compared among themselves in terms of detecting performance and estimation accuracy. Simulation results show that these proposed algorithms can detect and estimate the frequency of sinusoid, but an “improved” simplified lattice complex algorithm outperforms the other two in the convergence speed, tracking speed and steady-state mean square error at low SNR.

Keywords: narrowband signal, adaptive IIR notch filter, algorithms.

1. Introduction

Adaptive IIR notch filter can eliminate narrowband or sinusoidal interference while leaving broadband signals unchanged, in other hand, it can enhance narrowband or sinusoidal signal embedded in broadband noise.

In recent years, real coefficient adaptive IIR notch filter has been widely used in the areas such as biomedical engineering, communications, control, and radar system. But, when the signal consists of in-phase and quadrature-phase components, its sample values are complex numbers, such as radar and communications, the complex coefficient adaptive filter must be developed.

The adaptive notch filter (ANF) and adaptive line enhancer (ALE) are generally realized by using FIR filters. For FIR filter, corresponding to polynomial transfer functions, typically require large filter orders to obtain satisfactorily sharp cutoff characteristics. On the other hand, an IIR notch filter is one whose magnitude response vanishes at a particular value (the notch frequency w_0) on the unit circle, and whose magnitude response is nearly constant at other points on the unit circle. Excellent approximations may be obtained using second-order filter sections. Generally speaking, the IIR filter is more computationally efficient and has better statistical performance than the FIR filter. Although so many complex ANF and ALE algorithms have been studied, only a few adaptive IIR filters have been considered [1] [2] [3]. The purpose of this paper is to present class of complex IIR filters with lattice formulations to construct the ANF, for the lattice formulation allows independent tuning of the notch frequency and attenuation bandwidth. Simulation results are provided to show the detection and frequency estimation performance of three novel gradient-based adaptive IIR complex algorithms notch filter.

The rest of this paper is organized as follows. Section 2 introduces the structure of second order IIR lattice notch filter. Section 3 presents three proposed complex algorithms. In Section 4 describes the numerical results of three algorithms which demonstrate the frequency estimation and sinusoid signal detection performance of the three proposed complex algorithms at lower SNR. Section 5 is a conclusion of the paper.

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2. Second Order IIR Lattice Notch Filter

Consider a noisy sinusoidal with unknown frequency ω_0 , amplitude A, and phase ϕ that is uniformly distributed between 0 and 2π , that is:

$$y(n) = Ae^{j(\omega_0 k + \phi)} + w(n) \quad (1)$$

where $w(n)$ is a zero-mean complex Gaussian noise.

Fig.1 depicts the basic block diagram of a second order IIR notch filter with lattice form structure. $y(n)$ is the output signal of notch filter.

The transfer function of an adaptive IIR notch filter with lattice form structure [4] from input to notch output is expressed as $H(z)$:

$$H(z) = \frac{1 + \sin q_2}{2} \frac{1 + 2 \sin q_1 z^{-1} + z^{-2}}{1 + \sin q_1 (1 + \sin q_2) z^{-1} + \sin q_2 z^{-2}} \quad (2)$$

with ω_0 denoting the notch frequency and B the 3dB attenuation bandwidth, one can show that

$$q_1 = \omega_0 - p/2 \quad \omega_0 \in [0, \pi] \quad (3)$$

$$\sin q_2 = \frac{1 - \tan(B/2)}{1 + \tan(B/2)} \quad 0 < q_2 < p/2 \quad (4)$$

So, the independent tuning of the notch frequency and attenuation bandwidth can be realized. The proposed algorithm to tune the notch frequency parameter θ_1 under held the bandwidth parameter θ_2 constant will be introduced in the following section.

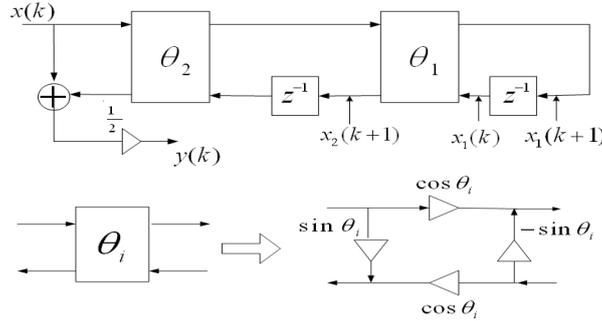


Fig. 1. Second order lattice IIR notch filter.

3. Proposed Three New Complex Algorithms

The gradient descent real algorithm [4] for lattice form adaptive notch filter to tune the notch frequency parameter θ_1 is introduced in table I. Where $\varphi(n) = \partial y(n) / \partial \theta_1$, $y(n)$ is the output of notch filter, $\mu(n)$ is a variable step-size parameter, $\mu_0 > 0$ is a initial step-size, λ is a forgetting factor.

Table 1: The lattice gradient real algorithm

$$\begin{aligned} \begin{bmatrix} g_1 \\ \omega_1 \end{bmatrix} &= \begin{bmatrix} \cos \theta_2 & -\sin \theta_2 \\ \sin \theta_2 & \cos \theta_2 \end{bmatrix} \begin{bmatrix} x(n) \\ x_2(n) \end{bmatrix} \\ y(n) &= \frac{1}{2} [x(n) + \omega_1] \\ \begin{bmatrix} g_2 \\ \omega_2 \end{bmatrix} &= \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 \\ \sin \theta_1 & \cos \theta_1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ \xi_2(n) \end{bmatrix} \\ \theta_1(n+1) &= \theta_1(n) - \mu(n)y(n)\varphi(n) \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} &= \begin{bmatrix} \cos \theta_1(n+1) & -\sin \theta_1(n+1) \\ \sin \theta_1(n+1) & \cos \theta_1(n+1) \end{bmatrix} \begin{bmatrix} g_1 \\ x_1(n) \end{bmatrix} \\ \begin{bmatrix} \xi_1(n+1) \\ \xi_2(n+1) \end{bmatrix} &= \begin{bmatrix} \cos \theta_1(n+1) & -\sin \theta_1(n+1) \\ \sin \theta_1(n+1) & \cos \theta_1(n+1) \end{bmatrix} \begin{bmatrix} g_1 \\ \xi_1(n) \end{bmatrix} \\ \mu(n) &= \frac{\mu_0}{\sum_{k=0}^n \lambda^{n-k} [\varphi(k)]^2} \end{aligned}$$

Based on the lattice gradient real algorithm (LGRA), the lattice gradient complex algorithm (LGCA) is proposed. For the adaptive notch filter, θ_1 is chosen to minimize the notch filter output power, $y^{(k)}y^{*(k)}$, where “*” denotes a complex conjugate,

$$\frac{\partial [y(n)y^*(n)]}{\partial \theta_1(n)} = 2 \operatorname{Re}[y(n)\varphi^*(n)]. \text{ For LGCA, the updating formula becomes:}$$

$$\theta_1(n+1) = \theta_1(n) - \mu \operatorname{Re}[y(n)\varphi^*(n)] \quad (5)$$

$$\mu(n) = \frac{\mu_0}{\sum_{k=0}^n \lambda^{n-k} |\varphi(k)|^2} \quad 0 \leq \lambda < 1 \quad (6)$$

The simplified lattice real algorithm(SLRA) [4] for adaptive notch filter to tune the notch frequency parameter θ_1 and μ are as follows:

$$\theta_1(n+1) = \theta_1(n) - \mu(n)[y(n)x_1(n)] \quad (7)$$

$$\mu(n) = \frac{\mu_0}{\sum_{k=0}^n \lambda^{n-k} [x_1(k)]^2} \quad 0 \leq \lambda < 1 \quad (8)$$

So, the simplified lattice complex algorithm(SLCA) based on SLRA in this paper is given by

$$\theta_1(n+1) = \theta_1(n) - \mu \operatorname{Re}[y(n)x_1^*(n)] \quad (9)$$

$$\mu(n) = \frac{\mu_0}{\sum_{k=0}^n \lambda^{n-k} |x_1(k)|^2} \quad 0 \leq \lambda < 1 \quad (10)$$

where $x_1(k)$ is regressor signal obtained from a transfer function $H'_b(z) = \frac{\cos \theta_1 \cos \theta_2 z^{-1}}{1 + \sin \theta_1 (1 + \sin \theta_2) z^{-1} + \sin \theta_2 z^{-2}}$.

In order to control the steady of algorithm at lower SNR, Reference [5] proposed a steady variable step-size algorithm, called improved simplified lattice real algorithm (ISLRA):

$$\mu(n) = \frac{\mu_0}{c + \sum_{k=0}^n \lambda^{n-k} [x_1(k)]^2} \quad 0 \ll \lambda < 1 \quad (11)$$

where $1 < c \ll \sum_{k=0}^n \lambda^{n-k} [x_1(k)]^2$ is a factor controls the steady of algorithm at lower SNR.

According to ISLRA, the improved simplified lattice complex algorithm (ISLCA) is also proposed:

$$\theta_1(n+1) = \theta_1(n) - \mu \operatorname{Re}[y(n)x_1^*(n)] \quad (12)$$

$$\mu(n) = \frac{\mu_0}{c + \sum_{k=0}^n \lambda^{n-k} |x_1(k)|^2} \quad 0 \leq \lambda < 1 \quad (13)$$

4. The simulation results

For some active sonar waveforms which are bandpass with high frequency, the algorithm can not be realize at real-time in microcomputer if it sample at Nyquist rate. However, it is shown that all the information content of a bandpass signal is contained in the complex envelope. This results in significant computational and storage savings. Thus a typical means for obtaining digital samples of a sonar waveform is to demodulate the waveform to dc, low-pass filter the waveform and sample. The samples are obtained by quadrature sampling or delay sampling.

In the simulations, the center frequency of input signal is 30kHz, $f_s = 200\text{kHz}$ is a sampling rate, 300ms and 100ms are observation time and duration of signal respectively, zero mean Gaussian noise. Demodulate frequency is 25kHz, $f_0 = 30.05\text{ kHz}$ is an envelope sampling rate. We can obtain the complex date using quadrature sampling:

$$x_r(i) = [x(8i) \cos(16\pi f_0 i / f_s)]_{LP} \quad (i = 1, 2, 3 \cdots N)$$

$$x_i(i) = [x(8i) \sin(16\pi f_0 i / f_s)]_{LP} \quad (i = 1, 2, 3 \cdots N)$$

where $[*]_{LP}$ is a low-pass operator, $x_r(i)$ and $x_i(i)$ are the real and imaginary parts of the desired complex signal respectively.

In order to study the performance of detection and estimation a single sinusoid using these three complex algorithms, the results of the output envelope (Fig.2(c),(d),(e))and estimated frequency after demodulated (Fig.2(f)) (correct demodulated frequency is 50Hz) using 100 Monte Carlo simulations at SNR=-20dB are illustrated in Fig.2. It is shown that the proposed ISLCA is clearly outperforms the other two algorithms in detection and frequencies estimation accuracy at lower SNR.

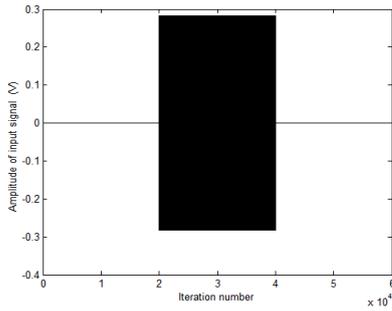


Fig. 2(a). The original signal.

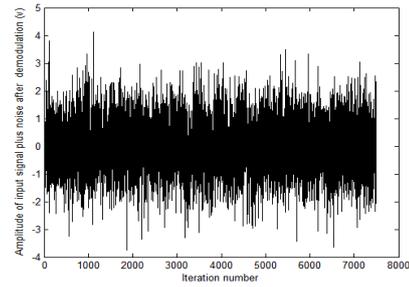


Fig. 2(b). The demodulated real part of input noisy signal.

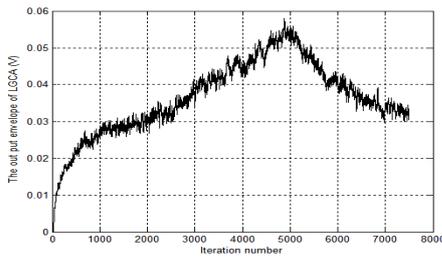


Fig. 2(c). The output envelope of LGCA.

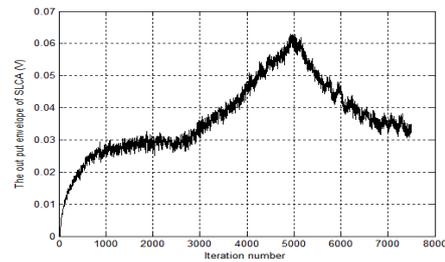


Fig. 2(d). The output envelope of SLCA.

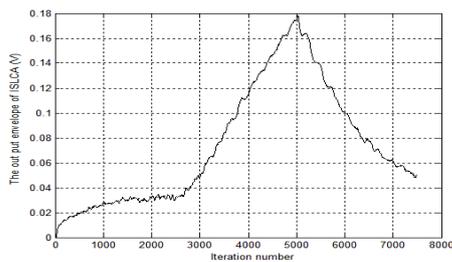


Fig. 2(e). The output envelope of ISLCA.

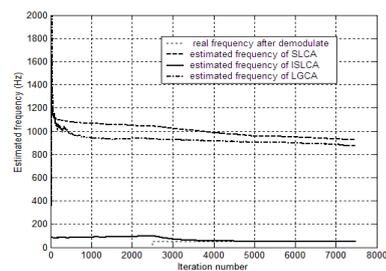


Fig. 2(f). The estimated frequency of three complex algorithm.

5. Conclusions

In this paper, we presented three gradient-based complex algorithms for adaptive IIR notch filter to enhance and estimate a single sinusoid in Gaussian noise. The results of the computer simulation have shown the SLCA has the lowest complexity, and the ISLCA has superiority of detecting performance and estimation accuracy over the other two complex algorithms at lower SNR. We believe that the complex algorithms proposed in this paper will have more applications in other areas.

6. References

- [1] S. Nishimura, H. Y. Jiang, and T. Hinamoto, "Convergence analysis of complex adaptive IIR notch filters with colored noisy signal," *IEEE Pacific Rim Conference on Communications, Computers and Signal Processing*, pp:325-328, 1999.
- [2] S. C. Pei, C. C. Tseng, "Complex adaptive IIR notch filter algorithm and its applications," *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, 41(2):158-163, 1994.
- [3] S. Nishimura, H. Y. Jiang, "Gradient-based complex adaptive IIR notch filters for frequency estimation," *Proceeding of IEEE Asia Pacific Conference on Circuits and Systems '96*, pp: 235-239, 1996.
- [4] P. A. Regalia, "Adaptive IIR filtering in signal processing and control," New York: Marcel Dekker, inc, 1995.
- [5] H. Liang, Y. P. Hong, Z. S. Li. "A steady variable step-size for adaptive IIR notch filter and its using in moving target detection," *Technical Acoustics*, 24(4):197-201, 2005.