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Research on Modeling and Compensating Method of Random Drift of MEMS Gyroscope

Yuan Li+, Fengyang Duan and Zanping Li

Department of Control Engineering, Aviation University, Changchun, China

Abstract. Aiming at the performance of Micro Electro Mecha-nical Systems (MEMS) gyroscope influenced by random drift largely, its error compensating method is analyzed. Based on the time series analysis method, the raw measurement from the MEMS gyroscope is processed and modeled by the Auto-regressive (AR) model. With the AR model, a time-varying factor adaptive Kalman filter (AKF) is designed. The results show that, after filtering in the case of constant and changing angular rate, the standard deviations are 16% and 70% of that before filtering, and the drift error can be reduced by the filter effectively.

Keywords: MEMS, random drift, AR model, AKF.

1. Introduction

MEMS gyroscopes, with the advantages of small size, light weight and low power consumption, are more and more widely applicated in low cost inertial system. As the precision level of MEMS gyroscopes are not high till now, even in low precision measurement system, the random drift estimation and compensation of gyroscopes are quite necessary. And the main ways of improving the system precision are the model identification and filtering of the MEMS gyroscopes random drift [1].

The common methods of modeling of gyroscopes random drift always get high model order and large amount of computation, such as neural network [2] and wavelet analysis [3], which applicated in low cost system can hardly ensure real-time property. Based on the time series analysis theory, an AR model of gyroscope random drift is established. Then we process the gyroscope random drift with a time-varying factor adaptive Kalman filter, and the method, which is helpful in improving real-time property of the low cost system, is efficient with low computation.

2. Error Model

Modeling based on the time series analysis theory requires collecting long-term measurements, and ensure that the signal is zero means value, stationary and normal [4].

2.1. Data acquisition and processing

In this paper, we take STIM202 gyroscope [5] as research object. Under the condition of constant temperature 25°C， we recorded 12 hours' measurements of gyroscope at uniform intervals of two seconds while it is static and processed the output of Z-Axis.

The raw data contains constant and random components.

So after the mean of it has been removed, the rest is the signal of gyro random drift. During the long-term of sampling, the change of electrical parameters and outside environment could cause the signal to generate trend, so it is necessary to take a processing to ensure stationary.

+ Corresponding author.
E-mail address: 56423664@qq.com.

The linear trend in the signal can be removed by the first or second order differential treatment [6], [7]. Fig. 1 shows the signal after it has been processed.

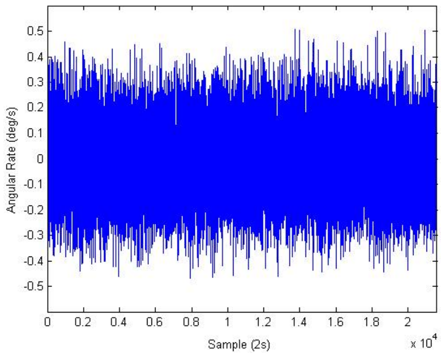

Fig.1. Processed data of gyro random drift.

2.2. Model Establishment

After processed, the static data from the MEMS gyroscope turns into random error signal.

The result of signal testing through reverse order testing method and its third-order and forth-order cumulants show that the signal has satisfied modeling of time series analysis theory requirements.

Auto-Regressive and Moving Average (ARMA) Model [8] is a basic and widely used time series model, and it is defined as

$$\begin{aligned} & X_t - \varphi_1 X_{t-1} - \varphi_2 X_{t-2} - \cdots - \varphi_m X_{t-m} \\ & = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \cdots - \theta_n a_{t-n} \end{aligned} \tag{1}$$

Where X_t is the time series $\{X_t\}$ at time t, m and n are the orders of AR model and Moving Average (MA) model, $\varphi_i (i = 1, 2, \cdots, m)$ is the AR parameter, $\theta_j (j = 1, 2, \cdots, n)$ is the MA parameter and $\{a_t\}$ is the standard deviation of the sensor white noise.

When $\theta_j = 0 (j = 1, 2, \cdots, n)$, ARMA model reduces to AR model as

$$X_t = a_t + \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + \cdots + \varphi_n X_{t-n} \tag{2}$$

ARMA model and AR model have different characteri-stics in auto-correlation function (ACF) and partial auto-correlation function (PACF). In AR model, the ACF is 'tailed' and the PACF is 'truncated', while in ARMA model both of ACF and PACF are 'tailed'. Therefore, the model of signal can be identified by the characteristics of its ACF and PACF. The ACF and PACF of gyro random drift signal are presented as figure 2, which shows that the PACF cuts off whereas the ACF has a jagged exponential decay. Therefore the AR model is adopted to model the gyro random drift signal.

After the parameters of each low order model have been estimated by least-square method, the order determination test of the AR model can be conducted via Akaike Informa-tion Criterion (AIC) and Bayesian Information Criterion (BIC), which can be normally written as

$$AIC(p) = N \ln \sigma_a^2 + 2p \tag{3}$$

$$BIC(p) = N \ln \sigma_a^2 + p \ln N \tag{4}$$

Where p denotes the order of model, N denotes the number of Sampling points and σ_a^2 denotes is the variance of residual a_t.

And the test result is presented in Table 1. As shown, by increasing the order of the AR model, AIC and BIC values don't decrease remarkably. Although those values are the lower the better, the amount of computation raises with the cubic of the order of AR model in the system. To further improve the system real-time property, the AR (1) model is used for the random drift error model.

After the Model plan has been finalized, it is necessary to perform the applicability test, testing whether $\{a_t\}$ is white noise. By calculating, the one-step auto-correlation coeffic-ient of a_t is $\rho_{a,1} = 0.078$ and the two-step cross-correlation coefficient between a_t and X_t is $\rho_{aX,2} = -0.0095$. Both of the coefficients approach zero, so $\{a_t\}$ is considered to be white noise and the model is applicable.

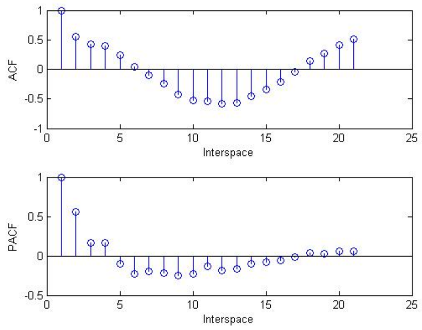

Fig.2. Value of ACF and PACF.

Table 1: Value of AIC and BIC of Models （×105）

	Model			
Criterion	***AR(1)***	***AR(2)***	***AR(3)***	***AR(4)***
AIC	-4.3560	-4.3562	-4.3564	-4.3564
BIC	-4.3559	-4.3560	-4.3561	-4.3561

The model is expressed in the form of

$$X_t = \varphi X_{t-1} + a_t \quad a_t \sim NID(0, \sigma_a{}^2) \tag{5}$$

3. Filter Design

After the drift error model has been determined, we choose Kalman filer (KF) [8] to compensate the drift error. The linear state equation and measurement equation are given as follows

$$X_k = \phi_{k,k-1} X_{k-1} + \Gamma_{k-1} W_{k-1} \tag{6}$$

$$Z_k = H_k X_k + V_k \tag{7}$$

Where X_k is the system state vector at time t_k, $\phi_{k,k-1}$ is the state transition matrix between time point t_{k-1} and t_k, Γ_{k-1} is the system noise distribution matrix, H_k is measurement observation matrix, W_{k-1} and V_k are the random error of system and measurement and they are per the following error statistics

$$E[W_k] = 0, Cov[W_k, W_j] = Q_k \delta_{kj} \tag{8}$$

$$E[V_k] = 0, Cov[V_k, V_j] = R_k \delta_{kj} \tag{9}$$

$$Cov[W_k, V_j] = 0 \tag{10}$$

Where Q_k and R_k are variances of W_k and V_k.

The regular KF established on this basis can effectively reduce the random drift error of gyro in the static or constant angular rate state, but in the state of changing angular rate the KF doesn't work well.

Through the analysis we learned that the filter model changed while the angular rate was changing. The more dramatic changes in angular velocity, the applicability of the filter model is poorer, causes the greater system error. But the direct reason for this is that the old measurement value influences the estimate of the current too much. Increasing the gain of current measurement value in the state estimation equation is an effective mean to solve this problem.

Based on the drift model before, AR(1) model takes the place of KF state equation and variance of residual Q_k takes the place of σ_a^2. H_k is set by 1. The AKF multiple update steps may be performed

One-step predicted state estimate

$$\hat{X}_{k/k-1} = \varphi \hat{X}_{k-1} \tag{11}$$

Updated state estimate

$$\hat{X}_k = \hat{X}_{k/k-1} + K_k (Z_k - \hat{X}_{k/k-1}) \tag{12}$$

Filter gain

$$K_k = P_{k/k-1} (P_{k/k-1} + R_k)^{-1} \tag{13}$$

One-step predicted estimate covariance

$$P_{k/k-1} = s_k \varphi^2 P_{k-1} + \sigma_a^2 \tag{14}$$

Updated estimate covariance

$$P_k = (I - K_k) P_{k/k-1} \tag{15}$$

A factor s_k is added into recursive equation of regular KF. The value selection of s_k is not only to be able to reduce the error under the condition of changing angular rate, but also to ensure the accuracy in the constant angular rate state, and expressed as follows

$$s_k = \frac{\omega_0}{\omega_0 - |\Delta\omega_k|} \tag{16}$$

Where $\Delta\omega_k$ is the first order differential of gyroscope measurements at the current moment, ω_0 is the reference value obtained by experiments.

In the condition of constant angular rate, $\Delta\omega_k$ approaches zero and s_k approaches 1, the AKF doesn't change too much from KF. When the angular rate is changing, s_k will adjust the filter.

4. Simulation and Experiment

4.1. Constant angular rate experiment

In order to verify the effectiveness of the model and filter, use both regular KF and AKF methods to process the static data at first. We set $\varphi = 0.1374$ and $\hat{X}_0 = 0$, P_0 [10] and R_k [11] are set by 10 times of standard deviation of raw data and one tenth of variance. Filtering effect is shown in figure 3 and figure 4, and the error reduces obviously.

The standard deviation of error before and after filtering is shown in table 2. The result shows that the filter and model established in the static condition are reasonable. Compared with the regular KF, the accuracy of AKF doesn't reduce.

Under the condition constant angular rate, we get the same result as it in the static condition. After processed by the two kinds of filter, the standard deviation of error is reduced to 16% of the raw data's. Because of the limited space, the detail description of the experiment is omitted.

4.2. Changing angular rate experiment

In the same environment condition, we take out a part of raw changing angular rate data (figure 5) from the MEMS gyroscope for experiment.

Filtering effect is shown in figure 6 and figure 7. The result shows that filtering changing angular rate data with the regular KF get poor effect and the error value is changing with the angular rate changing, while the error of AKF isn't influenced by angular rate changing almost. The standard deviation of error before and after filtering in the table 3 shows that AKF can reduce the random drift error under the condition of changing angular rate effectively.

From above, it shows that based on the AR model before, regular KF just can reduce the random error in the case of constant angular rate, while AKF can not only ensure the accuracy in the constant angular rate condition, but also improve the filter effect when angular rate is changing. Thus it can be seen that this algorithm worked well.

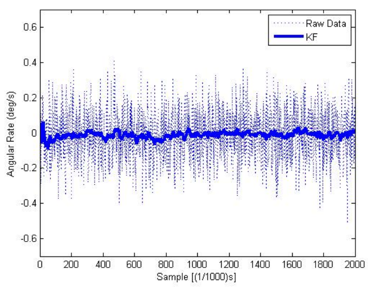

Fig.3. Kalman filter result of static experiment.

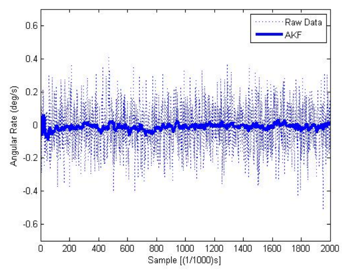

Fig.4. Adaptive Kalman filter result of static experiment.

Table 2: Standard Deviation of Error In Static Experiment

	Raw Data	***Regular KF***	***AKF***
Standard deviation (deg/s)	0.1285	0.0198	0.0198

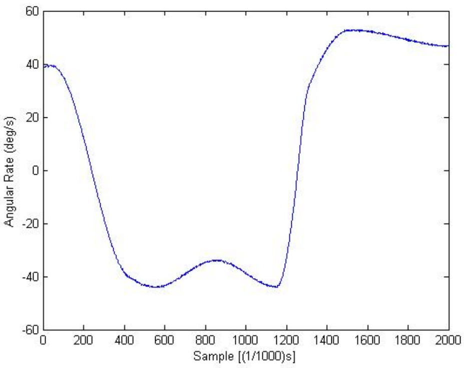

Fig.5. Changing angular rate data of gyro.

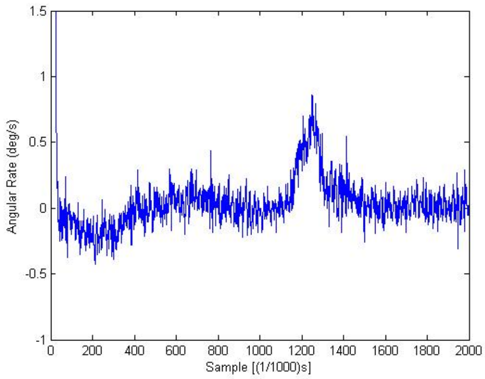

Fig.6. Kalman filter result of changing angular rate experiment.

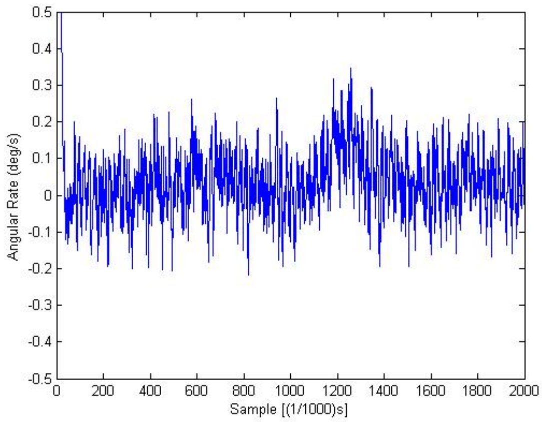

Fig.7. Adaptive Kalman filter result of changing angular rate experiment.

Table 3: Standard Deviation of Error In Changing Angular Rate Experiment

	Raw Data	***Regular KF***	***AKF***
Standard deviation (deg/s)	0.1285	0.2044	0.0892

5. Conclusion

In this paper, we analysis the MEMS gyroscope output raw date, extract the random drift signal based on requirements of time series analysis theory and establish an AR(1) model for filter. The results of experiments show that in the constant angular rate case, both regular KF and AKF have good performances, but when the angular rate is changing regular KF loses efficacy, while AKF still performs well and the standard deviation of drift error reduces by 30%. The AKF algorithm based on AR model, which is simple, effective and high real-time property, has some practical value in MEMS gyroscope-based low cost integrated navigation system.

6. References

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