# A Segmentation of Non-rigid Shape with Heat Diffuse

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**Abstract.** In this article, we propose a novel approach of 3D shape segmentation. The heat diffuse on discrete geometry is used in this approach. Using this segmentation method, we can obtain some sematic shape parts, and also a consistent segmentation result can be gotten for shapes under different postures. The whole process includes two stages. Firstly, the heat kernel signature is calculated for human shape. And then, the points are clustered by the combination of heat kernel signature and location.

Keywords: Heat diffuse, heat kernel signature, shape segmentation, non-rigid shape

# 1. Introduction

3D Shape has been applied extensively in various industries because of increasingly improved data acquisition techniques. As one important branch of shape analysis, shape segmentation arouses huge interest among graphic field.

Some surveys of 3D shape segmentation algorithms have been provided by Chen et al<sup>[1,2,3]</sup>. In 1999, Mangan<sup>[4]</sup> used watershed segmentation to do 3D surface partition. And then Lavou' e<sup>[5]</sup> proposed a new CAD mesh segmentation method through region growing. K-mean algorithm is a classic iterative clustering method, which is used excessively in shape segmentation. In 2004, Spectral cluster is applied to 3D mesh segmentation by Liu<sup>[6]</sup>, whose one disadvantage is to be sensitive to shape deformation and perturbation. In the last several years, diffuse theory is used, for example GPS<sup>[7]</sup>, HKS<sup>[8]</sup> and WKS<sup>[9]</sup>. The calculated heat kernel function has some advantages: multi-scale, invariant to isometric deformation, robust under perturbation<sup>[10]</sup>. Heat Walk<sup>[8]</sup> and Persistence Diagram<sup>[11]</sup> combined with HKS have been applied for shape segmentation. Take human body shape for example, Heat Walk is unable to identify the arms and head separately. In PD Method, there are always some mixed parts. For instance, some areas of torso are recognized as parts of arms.

Our algorithm will do human shape segmentation with heat kernel signature. This article includes two parts to introduce the definition of heat kernel signature and shape segmentation. In shape segmentation, a primary cut, that points are clustered by heat kernel signature, is applied firstly. And then the shape is segmented by point location. The main structure of paper is as following: In Section 3, heat kernel signature will be introduced. In Section 4, a primary cut by heat kernel and an agglomerative clustering by points' coordination will be introduced. And finally, the experiments results and limits are exhibited in Section 5.

# 2. Heat Kernel Signature

In this section, we will introduce the heat diffuse on manifold and discrete it on shape's geometry process firstly. M presents a Riemannian manifold with compact boundary. The heat diffuse can be calculated by the following equation:

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$$\Delta_M u(x,t) = -\frac{\partial u(x,t)}{\partial t}$$

 $\Delta_M$  is a Laplace-Beltrami operator. u Satisfies the Dirichlet Boundary Condition, and u(x,t) = 0 is available for any point x at time t. Given point x's initial heat distribution  $f: M \to \mathbb{R}$ , let  $H_t(f)$  denote the total heat distribution at time t. It is easy to verify  $H_t = e^{-t\Delta_M}$ . Meanwhile, we know that a function  $k_t(x, y)$  satisfies the below equations<sup>[9]</sup>:

$$H_t(f) = \int_M k_t(x, y) f(y) dy$$
$$k_t(x, y) = \sum_{i=0}^{\infty} e^{-\lambda_i t} \phi_i(x) \phi_i(y)$$

 $\lambda_i$  is the eigenvalue and  $\phi_i$  is the eigenfunction of  $\Delta_M$ . So the heat kernel function can be extended to discrete diffuse geometry. For each point on shapes, a heat kernel signature (HKS) is defined as.

$$HKS(x): \mathbb{R}^+ \to \mathbb{R}, HKS(x,t) = k_t(x,x)$$

In [10], it has been proved that when t is small, the value of HKS is proportional to heat diffuse distance of point x. The advantage is that heat diffusion distance is intrinsic and thus deformation-invariant, which makes it available in deformable shape analysis.

#### **3.** Shape Segmentation

The goal of this section is to segment the shape into meaningful parts. Before illustrating the algorithm in detail, we firstly give the summary of segmentation process as Fig. 1 showed. Using the definition in Section2, each point's HKS value of the tested shape is calculated in step (a). Then a primary cut will be applied and the medial result is gotten as step (b). Finally, the model is cut by agglomerative clustering method and we can obtain the meaningful result as (c).



Fig: 1. a) The calculated HKS value in the first step. b) The primary cut. c) The result after agglomerative clustering.

#### 3.1. primary cut

After calculating the HKS value for the shape, our tasks are to analysis the obtained data and find the common properties and differences between points, which are the basis of segmentation.

Due to the relationship between heat diffuse distance and HKS value, which has been described in section 2, we can get the HKS value distribution as Fig. 1(a). A conclusion can be obtained from that when t is a small value, the HKS value from center to boundary is increasing. So in this paper, we firstly cluster the torso as one part by a given threshold value  $v_{threshold}$ , which splits the arms and head from the torso.

$$hks_t(x, x) \le v_{threshold}, v_{threshold} = \frac{avg(HKS)}{W}$$

avg(HKS) is the average of all points' HKS value, w is an input parameter by manual. As Fig.1 (b) showed, the points with HKS value below  $v_{threshold}$  are classified as the black part, and the left ones belong to the blue part. In experiment, the value of  $v_{threshold}$  depends on the experience. It is worthwhile noting that the

threshold value gives rise to segmentation results. So it is significant to find an appropriate value automatically. This problem will be researched in our future work.

#### **3.2.** Agglomerative Clustering

In the second cut phase, the work is to recognize the head and limbs. From Fig. 1(b), we find that every part is easy to be distinguished from each other according to its point location. So the point location of left parts will be considered during segmentation. This intention can be denoted as the below formula.

$$I = AggCluster(L,k)$$

k is the clustering number and set by manual. We can get the clustering result from vector I. In this formula, L is a  $n \times 3$  matrix which includes the points' coordinates in Euclidean space.

$$L = [Shape.X, Shape.Y, Shape.Z]$$

In the following clustering process, a nearest neighbour method will be used for L. It is illustrated as:

$$D_{pq} = \min(d_{ij}), l_i \in G_p, l_j \in G_q$$

 $G_p, G_q$  present two clustering sets.  $d_{ij}$  is the Chebychev distance. It can maximum coordinate difference of points and the adjacent points will be clustered into the same classification. Chebychev distance is denoted as:

$$d_{ij} = \max_{1 \le k \le 3} \left| l_{ik} - l_{jk} \right|, \quad l_i, l_j \in L$$

Given the above approach, one clustering set includes one point at the beginning. In the clustering iterations, the points with long distance are separated, and the adjacent points are concentrated. In section 4, we will display the experiment results of some shapes from shrec2011 and sherc2010.

## 4. Experiment Results

We have applied our algorithm for different kinds of shapes. And the expected results are achieved as below.

**Suitability:** The results of Fig. 2 show that this algorithm is available for many kinds of shapes including human, animal and so on. Taking human body shape for example, the head and torso can be separated clearly by our approach.

**Consistency:** The invariance of the heat kernel signature under isometric deformations is a direct consequence of the invariance of the Laplace-Beltrami operator, which implies that the heat equation only involves intrinsic properties of the manifold <sup>[5]</sup>. When this approach is applied to one non-rigid shape under various postures in Fig.3, the consistent decomposition results can be obtained with the same w = 3 value.



Fig. 2: Segmentation of shapes.



Fig. 3: Consistent segmentation.

**Robustness:** In [10], the robustness against noise of HKS has been proved and verified. So when we used some models with noise and holes to do segmentation, as expected, the segmentation results of various models are insensitive to the perturbation, as Fig. 4 showed. This is a significant property for many shape analysis actions, such as shape retrieval, shape feature detection, etc. It can improve the currency of shape recognition.



Fig. 4: Noise shapes.

Fig. 5: Negative shape.

**Limits:** Because of relationship with location, this approach is unavailable for the human shape with overlapped parts. As Fig. 5 showed, the overlapped arms cannot be identified correctly. Also, the threshold value of HKS is given by manual, which is a bottleneck for automatically anthropometric work. Those will be the key points of our future working.

#### 5. Conclusion and Future Work

In our research, the contribution is to discover a meaningful and consistent segmentation for shapes under different postures or with noise. With the consistency and robustness, we can get an ideal segmentation for some uncompleted or deformed shapes. And also this approach can be applied to various kinds of models. It is notable that the process of segmentation is consistent with heat diffusion, so it is very easy to be understood and the segmentation result accords with human's visual perception. In future, our workings are to get the threshold value automatically and then use it in our anthropometric project.

## 6. Acknowledgements

This article would not have been possible without the help of several people and organizations who provided the data, code, and funding. We especially thank AIM@SHAPE for providing the meshes from SHREC 2011 and 2012. We also would like to thank Sun Jian who provided source code of calculating points' HKS value. Finally, we appreciate the help of Teng Ma in Tencent.Inc.

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