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Security Assessment of Bus Voltages in Electric Power Systems

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Abstract. Control room operators often deal with stressed system operating conditions and vulnerable networks. They need tools for continuous monitoring of system conditions to detect any untoward situation as quickly as possible. Security monitoring and assessment ensures that planned or unplanned outages do not lead to costly consequences such as loss of non-interruptible loads, equipment damage, and uncontrolled separation. Proposed method employs probabilistic load flow for bus voltage security assessment. Probability distribution and density function of the cumulants of the bus voltages is plotted. With the help of these distributions a fast and accurate contingency ranking for security assessment is made possible. The method has been applied to the IEEE RTS 7-Bus test system which consists of 7 buses and 10 lines as well as to the larger IEEE 24 bus system which consists of 24 buses and 38 lines.

Keywords: Probalistic load flow, Security, Reliability.

1. Introduction

Electric power system security is a critical part of operational and facility planning of bulk power transmission systems. Security assessment ensures that unplanned outages do not lead to costly consequences such as loss of non-interruptible load tripping of generation, equipment damage, and uncontrolled separation. Generally, the North American Reliability Council (NERC) Planning Standards gives accepted definition of security [1].

“Security is the ability of the electric systems to withstand sudden disturbances such as electric short circuits or unanticipated loss of system elements”.

Electric power systems are always operated with a significant security margin to ensure that the transmission network is capable of withstanding unpredictable events such as unplanned outages. Therefore, the system operator requires operating data, which is necessary for the fast contingency ranking for security. Consequently, as a part of a power system security assessment, a continuous system monitoring becomes necessary to detect critical situations as soon as possible. If the security margin is determined using deterministic criteria, the resulting degree of security may or may not he optimal depending on the operating conditions. While probabilistic methods have been used extensively in power system planning and operating, they have so far not been widely applied in security analysis. Because unplanned outages in power systems have stochastic phenomena, determining their ranking requires a probabilistic approach.

There are comparatively a few applications of probabilistic analysis to voltage security assessment using PLF. In [2] the general problem of dynamic security assessment is treated probabilistically. In [3] the expected voltage instability proximity index at a load bus is calculated taking into account element forced outages and the probability that the voltage stability worst state in a system will occur. In [4] the advantages probabilistic load flow is utilized to calculate for the assessment of voltage instability. We focus on security with respect to bus voltage magnitude and line flows in the system. In this paper the application of PLF analysis to bus voltage security assessment is investigated.

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It proposes the method using PLF solution for the security assessment of bus voltages, so that the probabilistic distribution function or PDF of bus voltages readily provide probabilities of threshold violations for a whole planning period, reflecting the random variation of loads, generation uncertainties, dispatching effects and outages [5, 6]. Consequently, as a part of a power system security assessment, monitoring bus voltages in system become necessary to detect dangerous situations as soon as possible. The advantage of the proposed method is that all possible contingencies are taken into account without complex convolutions and provides the densities and distributions of the bus voltages and line flows in a single run. Also, system operator can utilize the information of security assessment to improve reliability of power systems using the proposed method in this paper.

2. Theoretical Background

2.1. Complex Random Variables

Formulation of the problem using *complex random variables* mitigates the complexity of considering real and imaginary components separately in order to obtain the true picture of the system under consideration.

A CRV can be expressed in the form Z=X + jY where both X and Y are real random variables. If Z has a probability mass function f (Z), then it can also be defined in the same probability space with a joint probability mass function f(x, y). If Z**,** takes up three different values say, $Z_1 = x_1+jy_1$, $Z_2 = x_2+jy_2$ and $Z_3 = x_3+jy_3$ with probabilities of occurrence p_1, p_2 and p_3 respectively. Then $p_i = f(X=x_i, Y= y_i)$ for $Z_i = x_i + jy_i$. It may be noted that the real random variables X and Y occur as one event i.e., X always occurs along with Y and obviously $p_1+ p_2+ p_3 =1$. This can be shown as in Fig 1.

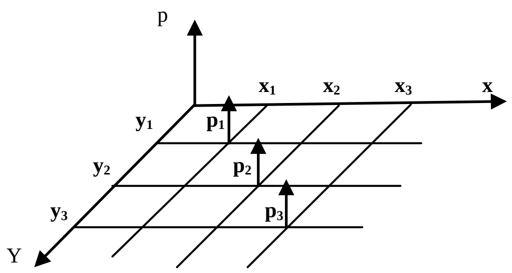

Fig 1 Discrete probability distribution of CRV Z

2.2. Calculation of moments and cumulants

The moment-generating function of a random variable is by definition [7] the integral,

$$M(t) = \int_{-\infty}^{\infty} f(x)\, e^{tx}\, dx \qquad (1)$$

Where, f(x) is the PDF of a random variable X. It is well known that if all moments are finite, the moment-generating function assumes a Maclaurin series expansion

$$M(t) = \sum_{n=0}^{\infty} m_n t^n/n! \qquad (2)$$

Where, the raw moments are,

$$m_n = \int_{-\infty}^{\infty} x^n f(x)\, dx \qquad (3)$$

For the discrete PDF of a CRV given in Fig 1, the t^{th} moment m_t is given by,

$$m_t = E[Z^t] = \sum_{i=1}^{n} Z_i^{\,t} p_i \qquad t=1, 2, 3 \qquad (4)$$

Where, Z_i is a CRV, and p_i its probability mass.

The cumulants are linear combinations of the statistical moments about an arbitrary point.

Two important properties of moments and cumulants are used for the addition and product of independent CRV's.

In general [8],

$$m_t\left(\prod_{i}^{n} Z_i\right) = \prod_{i}^{n} m_t(Z_i) \quad t = 1,2,3\ldots\ldots \tag{5}$$

$$k_t\left(\sum_{i}^{n} Z_i\right) = \sum_{i}^{n} k_t(Z_i) \quad t = 1,2,3\ldots\ldots \tag{6}$$

The moments and cumulants of the admittance matrix Y_{bus} are calculated using transmission line data. Similarly, moments of bus voltages and injected powers at buses are calculated.

$$k_t(S_i) = k_t(S_G) - k_t(S_D) \tag{7}$$

For a P-Q bus, the moments of bus voltages are calculated iteratively such that the (u+1) th iteration gives the moment of voltage at bus 'i' as,

$$m_t\left(V_i^{(u+1)}\right) = m_t\left[(P_i - jQ_i)/(V_i^{(u)*} - \sum_{i=1}^{k-i} Y_{ki} V_i^{(u+1)} - \sum_{i=k+1}^{n} Y_{ki} V_i^{(u)}\right] / (m_t Y_{ii}) \tag{8}$$

The iterative process is continued till the change in magnitude of bus voltage, $|\Delta V_i^{(u+1)}|$ between two consecutive iterations is less than a certain tolerance for all bus voltages. For P-V bus, if reactive power constraints are satisfied, moments of bus voltage are calculated accordingly. The line flows are calculated with the final bus voltages and the given line admittances and line charging using equation given below:

$$m_t(P_{ik} - jQ_{ik}) = m_t\left(V_i^*(V_i - V_k)\, y_{ik} + V_i^* V_i\right) y_{ik} \tag{9}$$

2.3. Edgeworth's Gram Charlier Expansion

For any random variable ξ with an unknown distribution and finite moments, if μ denotes the mean value and σ the standard deviation then using the standardized RV, $Z = (\xi - \mu) / \sigma$, the PDF denoted by f (z), and CDF denoted by F(z) can be given as [9],

$$f(z) = \varphi(z) + c_1 \varphi'(z)/1! + c_2 \varphi''(z)/2! + c_3 \varphi^{(3)}(z)/3! + c_4 \varphi^{(4)}(z)/4! + c_5 \varphi^{(5)}(z)/5! + \ldots. \tag{10}$$

$$F(z) = \Phi(z) + c_1 \Phi'(z)/1! + c_2 \Phi''(z)/2! + c_3 \Phi^{(3)}(z)/3! + c_4 \Phi^{(4)}(z)/4! + c_5 \Phi^{(5)}(z)/5! + \ldots \tag{11}$$

$\varphi(z)$ and $\Phi(z)$ represent the PDF and CDF of the normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$.The constant coefficients are given in terms of central moments as,

$$\begin{aligned} &c_o = 1;\ c_1 = c_2 = 0;\ c3 = -\beta_3/\sigma^3;\\ &c_4 = \beta_4/\sigma^4 - 3;\ c_5 = -\beta_5/\sigma^5 + 10\,\beta_3/\sigma^{3;}\\ &c_6 = \beta_6/\sigma^6 - 15\,\beta_4/\sigma^4 + 30\ldots \end{aligned} \tag{12}$$

and,

$$\begin{aligned} &\beta_1 = 0;\ \beta_2 = k_2 = \sigma^2;\ \beta_3 = k_3;\\ &\beta_4 = k_4 + 3\,k_2^2;\ \beta_5 = k_5 + 10\,k_2k_3;\\ &\beta_6 = k_6 + 15\,k_2k_4 + 10\,k_3^2 + 15\,k_2^3\ldots.. \end{aligned} \tag{13}$$

Using (10) and (11) the PDF and CDF can be expressed by the Gram-Charlier expansion as,

$$f(z) = \varphi(z) - G_1\,\varphi^{(3)}(z)/6 + G_2\,\varphi^{(4)}(z)/24 - G_3\,\varphi^{(5)}(z)/120 + (G_4 + 10G_1^2)\,\varphi^{(6)}(z)/720 \ldots\ldots \tag{14}$$

$$F(z)=\Phi(z) - G_1 \Phi^{(3)}(z)/6 + G_2 \Phi^{(4)}(z)/24 - G_3 \Phi^{(5)}(z)/120 + (G_4+10G_1^2) \Phi^{(6)}(z)/720 \ldots \quad (15)$$

Where, $\varphi(z) = (1/\sqrt{(2\pi)}) \exp(-z^2/2)$ and $\varphi^{(i)}(z) = d^i \varphi(z) / d^i z$

The parameters G_1 and G_2 determine the skewness and peakedness of the distribution. These are defined in terms of the cumulants of the RV, ξ as follows

$$G_1 = k_3(Z) = k_3(\xi)/ [k_2(\xi)]^{3/2} \qquad G_2 = k_4(Z) = k_3(\xi)/ [k_2(\xi)]^2$$
$$G_3 = k_5(Z) = k_3(\xi)/ [k_2(\xi)]^{5/2} \qquad G_4 = k_6(Z) = k_3(\xi)/ [k_2(\xi)]^3 \quad (16)$$

The mean and variance of ξ are expressed as, $k_1(\xi) = \mu$ and $k_2(\xi) = \sigma^2$.

Using Hermite polynomials and rearranging the Edgeworth expression to restore the monotonicity of the approximations and more importantly to improve the approximation of the unknown distribution [10], the above equations can be expressed as,

$$f(z) = \varphi(z)\{1+ G_1(z^3- 3z)/6 + G_2(z^4-6z^2+3)/24 +G_1^2(z^6-15z^4+45z^2-15)/72\} \ldots \quad (17)$$

$$F(z) = \Phi(z) + \varphi(z)\{G_1(z^2-1)/6 + G_2(z^3-3z)/24 + G_1^2(z^5 -10z^3 +15z)/72\} \ldots \quad (18)$$

The PLF algorithm thus progresses as follows:

Step1. Given the probabilistic description of generation, transmission lines and load, calculate the moments and then cumulants of injected power S_i and V_i using equations (4)-(6) and (7), assuming that V, and S, are independent CRV's.

Step2. Compute the moments and cumulants of all line flows.

Step3. Calculate the Gram-Charlier expansion coefficients G_1, G2, G_3 and G_4 using equation (16).

Step4. The PDF and CDF of bus voltages and line flows can then be obtained using equations (17) and (18).

3. Case Studies

The IEEE RTS 7-Bus test system given in Fig 2 and the IEEE 24 bus system in Fig 6 are both used to assess bus voltages security and line flow overload using PLF. The historical failure data of each bus and transmission line are as shown in Table 1 and Table 2 while bus 4 having constant availability is assumed as the Slack bus in the test system.

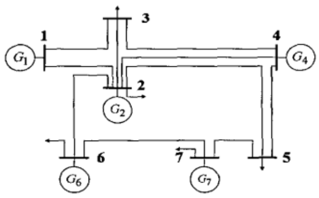

Fig 2: Single line diagram of the IEEE RTS 7-bus test system

Table 1: Bus Parameters for the IEEE RTS 7-Bus system

Bus	*Voltage (pu)*	*Load MVA)*	*Generation (MVA)*	*Type*	p_i
1	1.01 + j 0	-	200 - j 20	PV	0.91
2	1 + j 0	40 + j 20	100 - j 25	PV	0.88
3	1 + j 0	300 + j 50	-	PQ	0.96
4	1.02 + j 0	80 + j30	125.67-j187.24	Slack	1.00
5	1 + j 0	130 + j 40	-	PQ	0.89
6	1.05 + j 0	200 + j 40	370+j116.7	PV	0.87
7	0.98 + j 0	200 + j 80	181+ j 60	PV	0.89

Table2: Branch Data for the IEEE RTS 7-Bus System

Branch	*From p-bus*	*To q-bus*	*Impedance Z_{pq} (pu)*	*Lin.ch. adm. B/2 (pu)*	p_i	q_i	*Rating MVA*
1	1	2	0.02 + j 0.06	0.01	0.85	0.15	170
2	1	3	0.06 + j 0.24	0.02	0.86	0.14	130
3	2	3	0.04 + j 0.18	0.02	0.95	0.05	130
4	2	4	0.06 + j 0.18	0.03	0.83	0.17	100
5	2	5	0.04 + j 0.12	0.02	0.87	0.13	100
6	2	6	0.03 + j 0.06	0.02	0.93	0.07	160
7	3	4	0.01 + j 0.03	0.01	0.87	0.13	200
8	4	5	0.08 + j 0.24	0.06	0.92	0.08	100
9	6	7	0.04 + j 0.12	0.08	0.89	0.11	160
10	7	5	0.02 + j 0.06	0.02	0.88	0.12	120

The cumulants are calculated from the moments of each bus voltage and line flow. The Edgeworth type of Gram-Charlier series is then used to approximate the PDFs and CDFs of the bus voltages and line flows. Fig.3 and 4 shows PDFs and CDFs of bus voltages of the RTS-7 bus system. Fig 5 shows the CDF of all 10 line flows.

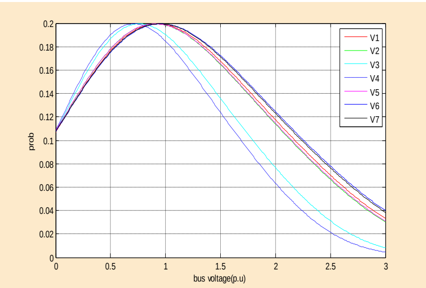

Fig 3: PDF of bus voltages

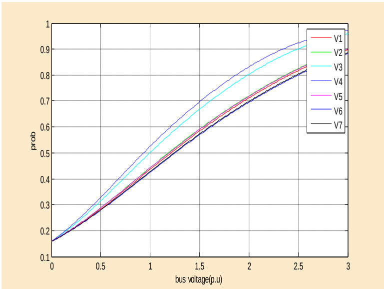

Fig 4: CDF of bus voltages

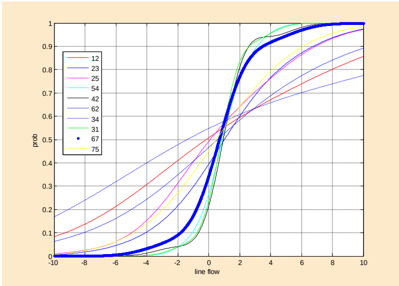

Fig 5: CDF of line flows

When the range of bus voltages by system operator is determined to be secure within a particular range of operation taken here as 0.98 pu - 1.04 pu, the probability of each bus voltage operating within this security limit is are shown in Table 3. The table also gives the ranking based on probability from lowest to highest, calculated by proposed method.

Table 4: Comparison of Worst Case ranking of probabilities of the bus voltages with bus unavailability

Ranking of prob. (voltage within limits)	*Voltage being within limits by applying rearranged Edgeworth expansion*		*Calculated bus unavailability considering line probabilities*	
	Prob	*Bus*	*Prob*	*Bus*
1	0.0277	**1**	0.021000	**1**
2	0.0292	**6**	0.013200	**7**
3	0.0293	**7**	0.007700	**6**
4	0.0325	**5**	0.001248	**5**
5	0.0342	**3**	0.000910	**3**
6	0.0423	**2**	0.000012	**2**

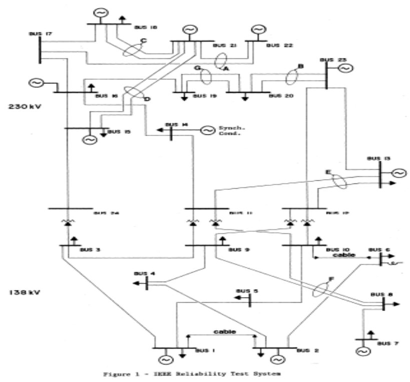

Fig 6: Single line diagram of the IEEE 24bus system

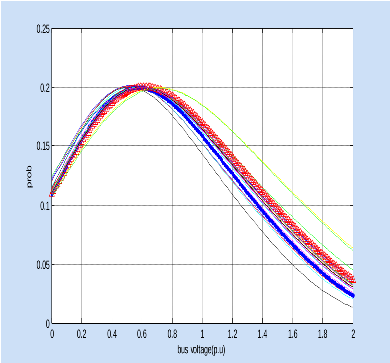

Fig 7: PDF of bus voltages

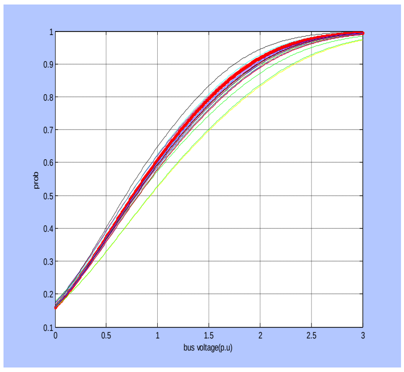

Fig 8: CDF of bus voltages

4. Conclusion

The bus voltages in a power system are required to be maintained within such as a range that they satisfy the requirements of electric customers and are stable under any disturbance.

The proposed PLF method transforms the load flow problem from deterministic to probabilistic formulation. The results obtained by probabilistic approach define the range of variation of load flow output quantities i.e. active and reactive power, bus voltages and line flows, bus voltages for security assessment in this paper. Finally, control room operators also can utilize the information obtained from the security assessment to improve reliability of power systems. The uncertainty of each component under practical operating condition and failure rate of historical data in power systems is utilized the PLF for bus voltage security assessment.

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