

# Performance Analysis and Modification of A Decision Feedback DSTBC System

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**Abstract**—To improve the performance of conventional differential space-time block coding scheme (DSTBC), this paper analyzed the performance of a decision feedback (DF) based DSTBC scheme and gave a modification to this system. Through simulation, it is shown that the modified system has about 1dB performance gains over conventional DSTBC scheme and has a lower computational complexity compared with multiple symbol differential detection scheme (MSDD).

**Keywords**-DSTBC;MIMO;MSDD; decision feedback

## 1. Introduction

Space-Time block coding (STBC) has been widely investigated in recent years for its capability of improving the performance over fading channels in wireless communication systems.

STBC scheme with two transmit antennas was first proposed by Alamouti in[1], which supports a maximum likelihood detection scheme based only on linear processing at the receiver. The scheme was generalized by Tarokh[2] from two transmit antennas to an arbitrary number. And it is able to achieve full diversity, which is defined by the product of transmit and receive antenna numbers. Due to its relative simplicity of implementation, STBC has been adopted by 3G standards, such as WCDMA and CDMA2000.

In detection process, the STBC scheme mentioned above needs perfect channel state information (CSI), which holds only in slowly changing channels. In cases when accurate channel estimation is undesirable or infeasible, for example, mobile channel, detection scheme without CSI is preferred. In[3], differential space-time block coding (DSTBC) was first proposed utilizing two transmit antennas where neither the transmitter nor the receiver have the *priori* channel information. In[4], the DSTBC scheme was generalized to more than two transmit antennas.

To reduce the deteriorate effect of fast fading channels,[5]proposed to pass the differentially encoded symbol into a column-wise interleaver, which demonstrates significant gains at the cost of larger processing delay and additional transmission power. In[6], decision feedback (DF) detection was adopted in conjunction with[5]'s scheme, which was shown to be able to improve DSTBC's performance over time-varying flat Rayleigh fading channels.

Compared with coherent detected STBC scheme, conventional DSTBC scheme has a 3dB performance loss due to the doubled noise term in differential decoding process. To narrow the performance gap between DSTBC and STBC, Divsalar and Simon[7] first proposed multi-symbol differential detection (MSDD), whose performance depends on the number of symbols detected simultaneously. To reduce the heavy computational complexity in MSDD decoding process, multiple symbol differential feedback detection was proposed in[8]; a breath-first sorting algorithm, called M-algorithm first proposed in[9] was also used to simplify the searching

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process in MSDD[10]. In[11] the author proposed a DSTBC scheme, which first derives channel information from received data block and then utilizes the channel information in detection process. However, as is analyzed in this paper, the scheme in[11] doesn't improve the performance.

In this paper, the basic system is described in Section II. The performance of a decision feedback based DSTBC system is analyzed and a modification is proposed in Section III. Then simulation is carried out in Section IV, where the performance and computational complexity is compared. Finally, the conclusion is drawn in the last section.

## 2. System Description

### 2.1. Signal Model

Assume that the system operates over a slow flat Rayleigh fading channel, where the data is sent from two transmit antennas to one receive antenna. Under this assumption, signals transmitted during two time slots are grouped into a  $2 \times 2$  matrix  $\mathbf{X}_k$

$$\mathbf{X}_k = \begin{bmatrix} x_{1,k} & -x_{2,k}^* \\ x_{2,k} & x_{1,k}^* \end{bmatrix} \quad (1)$$

The columns of  $\mathbf{X}_k$  are separately sent at two consecutive time slots, and  $x_{i,k}$  ( $i=1,2$ ) stands for symbol transmitted at the  $i$ th transmit antenna. The columns of  $\mathbf{X}_k$  are normalized so that the average signal power transmitted during a time slot is one. The received signal matrix during two time slots can be expressed as

$$\mathbf{R}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{N}_k \quad (2)$$

where  $\mathbf{N}_k$  is the additive white Gaussian noise(AWGN),  $\mathbf{H}_k$  is the fading path gain matrix

$$\mathbf{H}_k = \begin{bmatrix} h_{1,1} & h_{2,1} \\ -h_{2,1}^* & h_{1,1} \end{bmatrix} \quad (3)$$

where  $h_{i,j}$  is Gaussian complex variable, denoting path gain from the  $i$ th transmit antenna to the  $j$ th receive antenna. The channel is modeled as quasi-static, i.e. the path gain matrix  $\mathbf{H}_k$  is assumed to be invariant during a frame. Therefore,  $\mathbf{H}_k$  can be written as  $\mathbf{H}$  in a frame for simplicity.

### 2.2. Differential Encoding

In this paper, BPSK constellation and DSTBC scheme similar to that proposed in [3] is considered. The message matrix transmitted  $\mathbf{X}_k$  is defined by (1), which is a unitary matrix

$$\mathbf{X}_k \mathbf{X}_k^H = \mathbf{I}_{2 \times 2} \quad (4)$$

where the superscript  $(\cdot)^H$  denotes Hermitian transpose, and  $\mathbf{I}_{2 \times 2}$  denotes two-dimensional identity matrix. The initialized transmitted unitary matrix which carries no information is defined as

$$\mathbf{X}_0 = \begin{bmatrix} \sqrt{2} & -\sqrt{2} \\ \sqrt{2} & \sqrt{2} \end{bmatrix} \quad (5)$$

Suppose  $\mathbf{X}_k$  has been transmitted already, and 2 bits  $\mathbf{b}^T(t) = [b(2t), b(2t+1)]$  arrive at the encoder. Then  $\mathbf{b}(t)$  is mapped to a differential coefficient vector  $\mathbf{c}_{k+1}^T = [c_{1,k+1}, c_{2,k+1}]$  according to mapping algorithm given by[3] which is listed in TABLE I. Then the differential coefficient matrix  $\mathbf{C}_{k+1}$  is constructed from  $\mathbf{c}_{k+1}$  as

$$\mathbf{C}_{k+1} = \begin{bmatrix} c_{1,k+1} & -c_{2,k+1}^* \\ c_{2,k+1} & c_{1,k+1}^* \end{bmatrix} \quad (6)$$

Finally the message matrix  $\mathbf{X}_{k+1}$  is constructed from both previous transmitted matrix  $\mathbf{X}_k$  and differential coefficient matrix  $\mathbf{C}_{k+1}$

$$\mathbf{X}_{k+1} = \mathbf{X}_k \mathbf{C}_{k+1} \quad (7)$$

### 2.3. Differential Detection

Rewrite the received signal matrix expressed by (2) as below

$$\mathbf{R}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{N}_k \quad (8)$$

where

$$\begin{aligned}
\mathbf{R}_k &= \begin{bmatrix} r_{1,k} & r_{2,k} \\ -r_{2,k}^* & r_{1,k}^* \end{bmatrix} & \mathbf{H} &= \begin{bmatrix} h_{1,1} & h_{2,1} \\ -h_{2,1}^* & h_{1,1}^* \end{bmatrix} \\
\mathbf{X}_k &= \begin{bmatrix} x_{1,k} & -x_{2,k}^* \\ x_{2,k} & x_{1,k}^* \end{bmatrix} & \mathbf{N}_k &= \begin{bmatrix} n_{1,k} & n_{2,k} \\ -n_{2,k}^* & n_{1,k}^* \end{bmatrix}
\end{aligned} \tag{9}$$

TABLE I. MAPPING BITS TO DIFFERENTIAL COEFFICIENTS

$\mathbf{b}^T(t)$	[0,0]	[0,1]	[1,0]	[1,1]
$\mathbf{c}_{k+1}^T$	[1,0]	[0,1]	[0,-1]	[-1,0]

It can be easily proved that

$$\begin{aligned}
\mathbf{X}_k \mathbf{X}_k^H &= \mathbf{I}_{2 \times 2} & \mathbf{H} \mathbf{H}^H &= (|h_{1,1}|^2 + |h_{1,2}|^2) \mathbf{I}_{2 \times 2} \\
\mathbf{X}_k^H \mathbf{X}_{k+1} &= \mathbf{C}_{k+1} & \mathbf{C}_{k+1}^H \mathbf{C}_{k+1} &= \mathbf{I}_{2 \times 2}
\end{aligned} \tag{10}$$

and

$$\begin{aligned}
\mathbf{R}_k^H \mathbf{R}_{k+1} &= (|h_{1,1}|^2 + |h_{1,2}|^2) \mathbf{C}_{k+1} \\
&+ \mathbf{X}_k^H \mathbf{H}^H \mathbf{N}_{k+1} + \mathbf{N}_k^H \mathbf{H} \mathbf{X}_{k+1} + \mathbf{N}_k^H \mathbf{N}_{k+1}
\end{aligned} \tag{11}$$

To demodulate  $\mathbf{C}_{k+1}$  in a maximum-likelihood sense, one uses (11) and obtains the estimation

$$\begin{aligned}
\tilde{\mathbf{C}}_{k+1} &= \arg \min_{\mathbf{C}_{k+1} \in \mathbf{V}} \left\{ \left\| \mathbf{C}_{k+1} - \mathbf{R}_k^H \mathbf{R}_{k+1} \right\|^2 \right\} \\
&= \arg \max_{\mathbf{C}_{k+1} \in \mathbf{V}} \left\{ \text{Re tr} \left( \mathbf{R}_{k+1}^H \mathbf{R}_k \mathbf{C}_{k+1} \right) \right\}
\end{aligned} \tag{12}$$

where  $\mathbf{V}$  is the set containing all possible  $\mathbf{C}_{k+1}$ ,  $\text{Re}(\cdot)$  is the real part operator and  $\text{tr}(\cdot)$  is the trace of a matrix.

Based on these basic system components, a decision feedback assisted system is described in the next section.

### 3. Performance Analysis And Modification of a DF-Based DSTBC System

A decision feedback assisted detection scheme divides the detection process into two stages[11]

In the first stage the system operates like a conventional DSTBC system for  $k \leq N$ . Having obtained  $\tilde{\mathbf{C}}_{k+1}$  from (12), the estimate of  $\mathbf{X}_{k+1}$  is calculated by

$$\tilde{\mathbf{X}}_{k+1} = \tilde{\mathbf{X}}_k \tilde{\mathbf{C}}_{k+1} \tag{13}$$

and the estimate of fading path gain matrix  $\mathbf{H}$  is

$$\tilde{\mathbf{H}} = \mathbf{R}_{k+1} \tilde{\mathbf{X}}_{k+1}^H \tag{14}$$

Note that  $\tilde{\mathbf{H}}$  is the estimate from one data block. In order to reduce the interference from additive noise, the estimate of  $\mathbf{H}$  is averaged as follows:

$$\tilde{\mathbf{H}}_{\text{avg}} = \lambda(k) \tilde{\mathbf{H}}_{\text{avg}} + [1 - \lambda(k)] \tilde{\mathbf{H}} \tag{15}$$

where  $\lambda(k)$  is the forgetting factor. If uniform average is needed, then  $\lambda(k)$  is set to

$$\lambda(k) = \frac{k-1}{k} \tag{16}$$

In the second stage, system utilizes the channel information obtain from the first stage. Using the average estimate of fading path gain matrix  $\tilde{\mathbf{H}}_{\text{avg}}$ , one can obtain the information of transmitted matrix  $\mathbf{X}_{k+l}$  from  $\mathbf{R}_{k+l}$

$$\hat{\mathbf{X}}_{k+l} = \tilde{\mathbf{H}}_{\text{avg}}^H \mathbf{R}_{k+l} \tag{17}$$

Then[11] proposed to demodulate  $\mathbf{C}_{k+l}$  as follows

$$\begin{aligned}
\tilde{\mathbf{C}}_{k+l} &= \arg \min_{\mathbf{C}_{k+l} \in \mathbf{V}} \left\{ \left\| \mathbf{C}_{k+l} - \hat{\mathbf{X}}_{k+l}^H \hat{\mathbf{X}}_{k+l-1} \right\|^2 \right\} \\
&= \arg \max_{\mathbf{C}_{k+l} \in \mathbf{V}} \left\{ \text{Re tr} \left( \hat{\mathbf{X}}_{k+l}^H \hat{\mathbf{X}}_{k+l-1} \mathbf{C}_{k+l} \right) \right\}
\end{aligned} \tag{18}$$

However, in effect, the scheme described by (18) doesn't improve performance compared with the conventional DSTBC scheme. Consider an extreme case where the channel matrix  $\mathbf{H}$  is perfectly estimated, i.e.  $\tilde{\mathbf{H}}_{\text{avg}} = \mathbf{H}$ . Then  $\hat{\mathbf{X}}_{k+l-1}$  and  $\hat{\mathbf{X}}_{k+l}$  can be expressed as

$$\begin{aligned}
\hat{\mathbf{X}}_{k+l} &= \mathbf{H}^H \mathbf{R}_{k+l} \\
&= \left( |h_{1,1}|^2 + |h_{1,2}|^2 \right) \mathbf{X}_{k+l} + \mathbf{H}^H \mathbf{N}_{k+l} \\
\hat{\mathbf{X}}_{k+l-1} &= \mathbf{H}^H \mathbf{R}_{k+l-1} \\
&= \left( |h_{1,1}|^2 + |h_{1,2}|^2 \right) \mathbf{X}_{k+l-1} + \mathbf{H}^H \mathbf{N}_{k+l-1}
\end{aligned} \tag{19}$$

Then expand the product term  $\hat{\mathbf{X}}_{k+l-1}^H \hat{\mathbf{X}}_{k+l}$  in (18) as below

$$\begin{aligned}
\hat{\mathbf{X}}_{k+l-1}^H \hat{\mathbf{X}}_{k+l} &= \left( \left( |h_{1,1}|^2 + |h_{1,2}|^2 \right) \mathbf{X}_{k+l-1}^H + \mathbf{N}_{k+l-1}^H \mathbf{H} \right) \\
&\quad \times \left( \left( |h_{1,1}|^2 + |h_{1,2}|^2 \right) \mathbf{X}_{k+l} + \mathbf{H}^H \mathbf{N}_{k+l} \right) \\
&= \left( |h_{1,1}|^2 + |h_{1,2}|^2 \right)^2 \mathbf{C}_{k+l} \\
&\quad + \left( |h_{1,1}|^2 + |h_{1,2}|^2 \right) \left( \mathbf{X}_{k+l-1}^H \mathbf{H}^H \mathbf{N}_{k+l} + \mathbf{N}_{k+l-1}^H \mathbf{H} \mathbf{X}_{k+l} \right) \\
&\quad + \left( |h_{1,1}|^2 + |h_{1,2}|^2 \right) \mathbf{N}_{k+l-1}^H \mathbf{N}_{k+l}
\end{aligned} \tag{20}$$

Compare (20) with (11), one can find that the result of (20) is just only the result of (11) multiplied by a coefficient  $|h_{1,1}|^2 + |h_{1,2}|^2$ . Obviously, this won't yield any performance improvement in the searching process described by (18).

As seen in Fig.1, (18) performs the same as conventional DSTBC, which is analyzed above. If assume that the previously demodulated block  $\hat{\mathbf{X}}_{k+l-1}$  is always correct, i.e.  $\mathbf{X}_{k+l-1}$  is substituted for  $\hat{\mathbf{X}}_{k+l-1}$  in (13) and (18), then the performance given in [11] can be attained, which is the dashed line in Fig.1. The dashed line shows about 2dB gain over conventional DSTBC. However, it requires perfect estimate of  $\mathbf{X}_{k+l-1}$ , which is hard to be satisfied in actual system.

Return to the fundamental reason why the performance of DSTBC is 3dB inferior to that of STBC. The term  $\mathbf{N}_k^H \mathbf{N}_{k+1}$  in (11) can be neglected at high SNR, which is much smaller compared with the other terms. The two remaining noise terms double the noise power which results in 3dB performance loss [3]. Therefore, if the  $\hat{\mathbf{X}}_{k+l}$  in (20) is multiplied by a matrix that is more 'clean' than  $\hat{\mathbf{X}}_{k+l-1}$ , performance improvement can be expected.

Based on this idea, the detection process can be modified as

$$\begin{aligned}
\tilde{\mathbf{C}}_{k+l} &= \arg \min_{\mathbf{C}_{k+l} \in \mathbf{V}} \left\{ \left\| \mathbf{C}_{k+l} - \tilde{\mathbf{X}}_{k+l-1}^H \hat{\mathbf{X}}_{k+l} \right\|^2 \right\} \\
&= \arg \max_{\mathbf{C}_{k+l} \in \mathbf{V}} \left\{ \text{Re tr} \left( \hat{\mathbf{X}}_{k+l}^H \tilde{\mathbf{X}}_{k+l-1} \mathbf{C}_{k+l} \right) \right\}
\end{aligned} \tag{21}$$

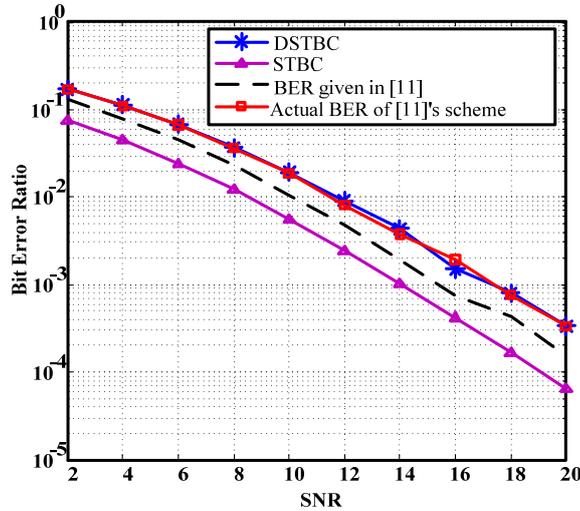


Fig.1. Performance of DSTBC, STBC, scheme in [11](actual BER, marked with square) and performance given in [11](impracticable, dashed line).

Finally, the modified scheme can be concluded as following two stages:

- a) System works like conventional DSTBC scheme; average channel information is obtained using (13) (14) and (15) from  $N$  data blocks
- b) For  $k > N$ , based on the average estimate of path gain matrix  $\tilde{\mathbf{H}}_{\text{avg}}$ , get  $\hat{\mathbf{X}}_{k+l}$  from received data block  $\mathbf{R}_{k+l}$  using (17). Then both  $\tilde{\mathbf{X}}_{k+l-1}$  from (13) and  $\hat{\mathbf{X}}_{k+l}$  is used to estimate the differential coefficient matrix  $\tilde{\mathbf{C}}_{k+l}$  using (21). Note that in the second stage, the  $\tilde{\mathbf{H}}_{\text{avg}}$  is also renewed for each received block using (15).

## 4. Simulation

In this section, the performance of the modified scheme is evaluated by simulation.

The Rayleigh fading channel is assumed to be quasi-static that the path gain matrix  $\mathbf{H}$  keeps invariant over a frame of 130 symbols. The system uses BPSK constellation and 2 transmit antennas together with 1 receive antenna. Note that, using transmit matrix given in [4], this scheme can be generalized to multiple transmit antennas; using linear combination approach, this scheme can be generalized to multiple receive antennas by summing up all the receive matrix from each receive antenna.

Fig.2 illustrates the influence of the block number  $N$  used in the first stage. As is shown, the performance using 5 blocks is superior to that using 10 blocks. In this two-stage scheme, system performs the same as conventional DSTBC in the first stage and the improvement exists in the second stage. As a result, a better performance improvement can be expected with a moderately smaller  $N$ .

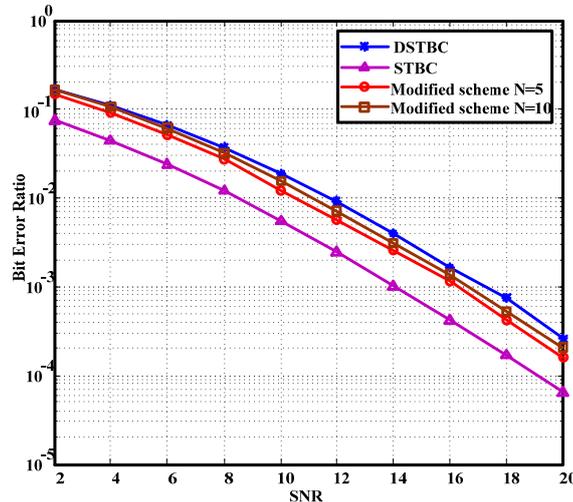


Fig.2. Performance of DSTBC, STBC and modified scheme using different number of blocks to estimate channel matrix.  $N = 5, 10$

Fig.3 compares the performance of the modified scheme with multi-symbol detection scheme (MSDD). MSDD has been known for filling gap between differential detection and coherent detection[7]. Instead of symbol-by-symbol detection as in conventional differential detection, the basic idea of MSDD is to extend the observation interval and make a joint maximum likelihood sequence detection of  $M$  blocks. As seen in Fig.3, the modified scheme with  $N = 5$  performs slightly better than MSDD with  $M = 6$ . And compared with conventional DSTBC, the modified scheme provides about 1dB gains.

However, the computational complexity of MSDD will grow exponentially with the number of observation length  $M$  and the signal constellation. For example, under the condition of BPSK constellation and observation interval  $M = 6$ ,  $2^{2^{(6-1)}} = 1024$  searching paths are needed. In[9], a breadth-first sorting algorithm called M-algorithm is proposed to simplify the searching process, which needs 192 searching paths[10] For comparison, the modified scheme only needs search  $4 \times (6-1) = 20$  times to detect these blocks.

## 5. Conclusion

In this paper, based on the idea of utilizing derived channel information from received data, a modification is proposed for a decision feedback assisted DSTBC system. From simulation it was observed that the proposed scheme has about 1dB gains over conventional DSTBC. Its computational complexity is

much lower than that of MSDD scheme while holds a similar BER performance. And also note that it is a nontrivial task to extend this scheme to a more transmit and receive antennas case.

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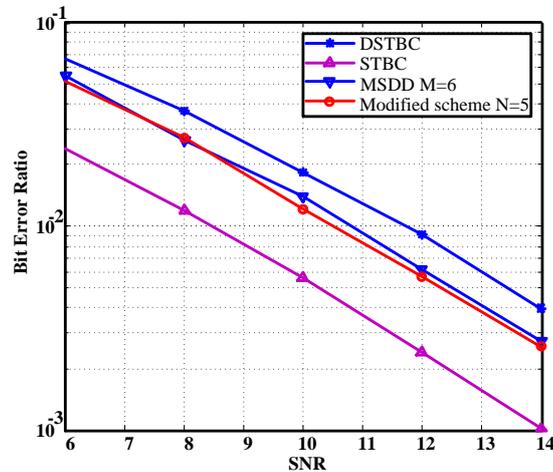


Fig.3. Performance of DSTBC, STBC, MSDD and modified scheme with parameter  $N = 5$

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