

Estimation of Temperature Rise in Stator Winding and Rotor Magnet of PMSM Based on EKF

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Abstract—This paper proposed an estimation approach of temperature rise in stator winding and rotor magnet of a surfaced-mounted permanent magnet (PM) AC motor. By using the 3rd-order extended Kalman filter (EKF) algorithm in rotating reference frame, stator winding resistance and rotor PM flux-linkage can be estimated independently. After comparing the estimation results of resistance and flux-linkage to the rated values measured at 20°C respectively, the corresponding temperature rise in stator winding and rotor magnet can be calculated according to some certain rules. The proposed method was confirmed to be correct and effective by the simulation results from MATLAB/SIMULINK.

Keywords-PMSM; extended Kalman filter; temperature rise estimation

1. Introduction

Thermal condition monitoring and analysis is quite important and necessary for motor, especially under the circumstances of dynamic operation. Currently, several methods are able to fulfill the task[1-3]. Take the thermal equivalent circuit for example, problems related to temperature field can be transferred to calculation of thermal paths with lumped parameters through employing the thermal equivalent circuit organized by simplified thermal source and thermal resistance. Despite of small duty of calculation and approximate estimation of average temperature in iron core as well as winding, the estimation error is relatively large and it is not capable of real-time implementation. In addition, some computational methods are once proposed, such as FEM, equivalent thermal network, etc. However, those methods do not work until the internal structure of motor is well known. Furthermore, it is complicated to establish and analyze the corresponding model. Installation of thermistor or thermocouple during the manufacture process is another way to monitor dynamic thermal environment of motor, which is normally just applied to estimate the temperature of stator. So, it is difficult to measure the temperature of rotor directly. In the control system of AC PMSM, EKF is often applied to identify position and speed of rotor as well as the variation of stator winding resistance and rotor PM flux-linkage[4-8]. According to the relationship between winding resistance and temperature, the variation of temperature in stator can be evaluated by the variation of winding resistance in certain scope. Besides, the variation of PM flux-linkage also reflects the variation of temperature in rotor in terms of the relationship curve between flux and temperature. Therefore, temperature of motor can be estimated and observed without installing extra equipment or understanding motor structure well. Consequently, using EKF

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to evaluate the variation of winding resistance and PM flux-linkage is an economical, convenient and effective method to estimate temperature of motor indirectly.

2. Principle of temperature estimation

Considering the temperature performance of conductance coefficient for copper, temperature of stator can be derived indirectly by comparing the estimation value of winding resistance to the rated value. In the same way, temperature of rotor can be derived indirectly by comparing the estimation value of PM flux-linkage to the rated value due to the temperature sensitivity of ferromagnetic material.

2.1. Estimation of Temperature Rise in Stator Winding

As a nonlinear and dramatically coupled system with multiple variables, the mathematical model of AC PMSM in rotating reference frame can be described as following after some appropriate simplification and assumption:

$$\begin{cases} u_d = r i_d + L \frac{di_d}{dt} - \omega L i_q \\ u_q = r i_q + L \frac{di_q}{dt} + \omega L i_d + \omega \lambda \end{cases} \quad (1)$$

where u_d , u_q , i_d , i_q are the d-axis and q-axis voltages and currents, respectively. λ is the PM flux-linkage, r is the stator phase resistance, L is the winding inductance, ω is the angular speed of rotor.

In reality, the change of winding resistance caused by temperature variation is much slower than the change of electrical quantities. Assume winding resistance remains unchanged while analyzing motor model, then the estimation model for winding resistance can be described as following:

$$\begin{cases} \frac{dr}{dt} = 0 \\ \frac{di_d}{dt} = -\frac{r}{L} i_d + \omega i_q + \frac{u_d}{L} \\ \frac{di_q}{dt} = -\frac{r}{L} i_q - \omega i_d - \frac{\lambda}{L} \omega + \frac{u_q}{L} \end{cases} \quad (2)$$

where state vector $x = [r \ i_d \ i_q]^T$, output vector $y = [i_d \ i_q]^T$, and an abstract expression of (2) is given by:

$$\begin{cases} \dot{x} = f(x, u) \\ y = h(x) \end{cases} \quad (3)$$

After linearization by Jacobian matrix and discretization, state equation of (3) is described as following:

$$\begin{cases} x_{k+1} = \Gamma(x_k) \delta_k + w_k \\ y_k = \Delta(x_k) \delta_k + v_k \end{cases} \quad (4)$$

where Γ matrix and Δ matrix are expressed as:

$$\Gamma(x_k) = I + T_s \cdot \left. \frac{\partial f(x, u)}{\partial x} \right|_{x=x_k} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{T_s}{L} i_d & 1 - \frac{T_s}{L} r & \omega T_s \\ -\frac{T_s}{L} i_q & -\omega T_s & 1 - \frac{T_s}{L} r \end{bmatrix} \quad (5)$$

$$\Delta(x_k) = \left. \frac{\partial h(x)}{\partial x} \right|_{x=x_k} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

According to physical laws, resistance of winding has something to do with the resistivity ρ of copper, while the resistivity ρ is related to temperature T and temperature coefficient α_ρ of copper. The relationship equation is:

$$\rho_T = \rho_{20} \times [1 + \alpha_\rho (T - 20)] \quad (7)$$

where $\alpha_\rho \approx 0.004/^\circ\text{C}$ for copper, ρ_{20} is the resistivity of copper at 20°C , ρ_T is the resistivity of copper at $T^\circ\text{C}$. Then the resistance of copper at $T^\circ\text{C}$ is given by:

$$r_T = r_{20} \times [1 + \alpha_\rho (T - 20)] \quad (8)$$

Therefore, the relationship between temperature of stator winding and resistance at corresponding temperature can be deduced as:

$$T = 20 + [r_T / r_{20} - 1] / \alpha_\rho \quad (9)$$

2.2. Estimation of Temperature in Rotor Magnet

Estimation of rotor PM flux-linkage can be realized by employing stochastic filtering algorithm. Based on the same principle of stator temperature estimation, assume PM flux-linkage remains unchanged while analyzing motor model, then estimation model for PM flux-linkage is derived:

$$\begin{cases} \frac{d\lambda}{dt} = 0 \\ \frac{di_d}{dt} = -\frac{r}{L} i_d + \omega i_q + \frac{u_d}{L} \\ \frac{di_q}{dt} = -\frac{r}{L} i_q - \frac{\lambda}{L} \omega - \omega i_d + \frac{u_q}{L} \end{cases} \quad (10)$$

where state vector $x = [\lambda \ i_d \ i_q]^T$, output vector $y = [i_d \ i_q]^T$. After linearization by Jacobian matrix and discretization, Γ matrix and Δ matrix are given by:

$$\Gamma(x_k) = I + T_s \cdot \left. \frac{\partial f(x, u)}{\partial x} \right|_{x=x_k}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 - \frac{r}{L} T_s & \omega T_s \\ -\frac{\omega}{L} T_s & -\omega T_s & 1 - \frac{r}{L} T_s \end{bmatrix} \quad (11)$$

$$\Delta(x_k) = \left. \frac{\partial h(x)}{\partial x} \right|_{x=x_k} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (12)$$

The demagnetization characteristics of permanent magnets, i.e. the coercivity, the remanence and the knee point also vary with temperature. For example, the remanence B_r decreases with temperature. B_r at $T^\circ\text{C}$ is:

$$B_{rT} = B_{r20} \times [1 + \alpha_{Br}(T - 20)] \quad (13)$$

where B_{r20} is the remanence at 20°C , α_{Br} is temperature coefficient of the remanence, for NdFeB magnetic material $\alpha_{Br} \approx -0.001/^\circ\text{C}$. The PM flux-linkage is proportional to B_r if the influence of temperature on magnetic saturation is negligible, while the permanent magnet will not be irreversibly demagnetized because of temperature rise. Therefore, according to (13), the calculation formulation of flux-linkage is expressed by:

$$\lambda = \lambda_{20} \times [1 + \alpha_{Br}(T - 20)] \quad (14)$$

Consequently, temperature of rotor magnet is derived:

$$T = 20 + [\lambda / \lambda_{20} - 1] / \alpha_{Br} \quad (15)$$

2.3. Recursive Algorithm of EKF

As well known, EKF is an optimal recursive algorithm. During the implementation process, the recursive equations are given by:

$$\begin{cases} \hat{x}_{k+1/k} = f(\hat{x}_{k/k}, u_k) \\ P_{k+1/k} = \Gamma_k P_{k/k} \Gamma_k^T + Q \end{cases} \quad (16)$$

$$\begin{cases} K_{k+1} = P_{k+1/k} \Delta_k^T [\Delta_k P_{k+1/k} \Delta_k^T + R]^{-1} \\ \hat{x}_{k+1/k+1} = \hat{x}_{k+1/k} + K_{k+1} (y_{k+1} - h_k(\hat{x}_{k+1/k})) \\ P_{k+1/k+1} = (I - K_{k+1} \Delta_k) P_{k+1/k} \end{cases} \quad (17)$$

where the subscript $k+1/k$ denotes prediction value and $k+1/k+1$ means eventual estimated value. Q and R are covariance matrix of process noise and measurement noise, respectively. Γ is the Jacobian matrix of partial derivatives of f with respect to x , i.e.

$$\Gamma_k = \left. \frac{\partial f(x, u)}{\partial x} \right|_{x=\hat{x}_k} \quad (18)$$

Δ is the Jacobian matrix of partial derivatives of h with respect to x , i.e.

$$\Delta_k = \left. \frac{\partial h(x,u)}{\partial x} \right|_{x=\hat{x}_k} \quad (19)$$

Besides, several initial values are needed for the implementation of algorithm. Different initial values have different influence on the convergence speed.

$$\hat{x}_{0/0} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$P_{0/0} = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 20 \end{bmatrix} \quad Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3. Simulation experiment and Estimation results

The vector control system of PMSM and identification models for different parameters are established in MATLAB/SIMULINK. Some parameters of the motor and the control system are listed in Tab. I . From Fig.1 to Fig.4, waveforms of rotor position and speed, waveforms of voltage and current, identification results of winding resistance and PM flux-linkage as well as temperature rise in stator winding and rotor magnet are given.

TABLE I. PARAMETERS OF MOTOR AND CONTROL SYSTEM

Rated Current	4(A)
Rated Speed	400(rpm)
Pole Pair	5
Winding Resistance	1(Ω)
Flux-linkage	0.0776(Wb)
Winding Inductance	3.366e-3(H)
Inertia	0.8e-5
Load Torque	1.5(N·m)
DC Voltage of Inverter	36(V)

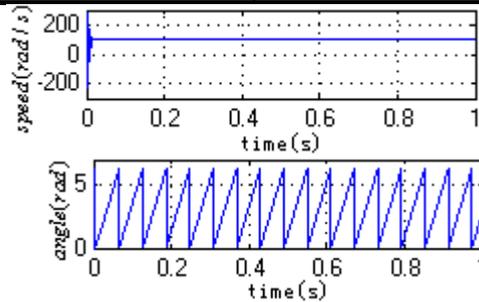
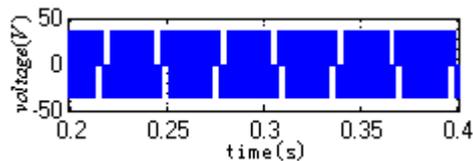


Fig.1. Speed and angle of rotor



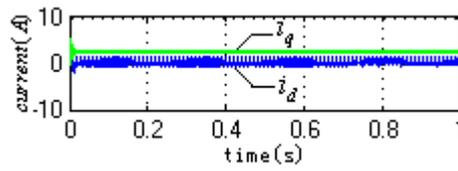


Fig.2. Waveforms of voltage and current

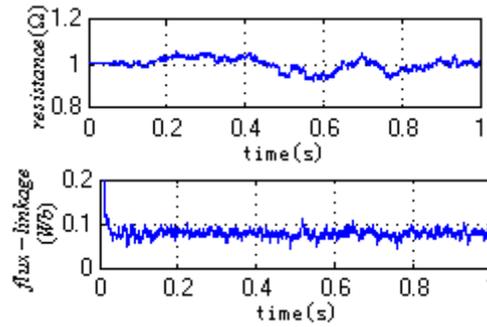


Fig.3. Identification results of winding resistance and PM flux-linkage

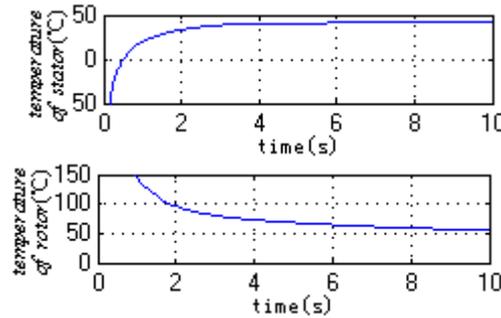


Fig.4. Estimation results of temperature rise in stator winding and rotor magnet

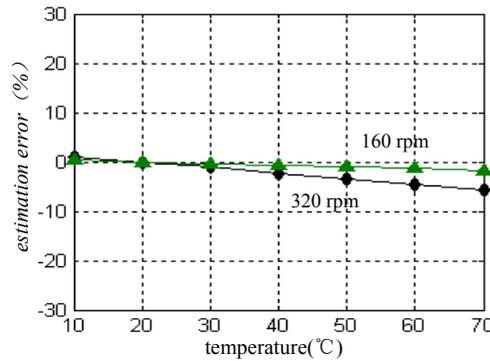


Fig.5. Estimation error of temperature rise in stator winding at different speed

It is noticed that temperature variation influences on winding resistance and PM flux-linkage simultaneously. In this paper, winding resistance and PM flux-linkage are estimated independently without considering the interactive influence. Actually, the temperature coefficient of resistance is nearly 3 times larger than that of flux-linkage. However, error analysis and modification of estimation results are still necessary. With consideration of extreme situation that the temperature of motor rises to 50°C at short notice, the estimation error of resistance is less than -5.8% by using the proposed method when no large temperature difference exists between stator and rotor. Therefore, the estimation error of temperature should also be restricted in certain scope. If the variation of flux-linkage can be timely modified while identifying resistance, then the estimation error will get smaller. Fig.5 shows the estimation error of stator temperature at different speed. It demonstrates that the estimation error gets smaller when the speed is slower than the rated value. When the speed is about half of the rated value, the estimation error is approximately -1.7%.

4. Conclusion

Using extended Kalman filter to estimate temperature of motor is a meaningful discovery. In this paper, 3rd-order EKF algorithm is employed to identify winding resistance and PM flux-linkage. Then, temperature rises in stator winding and rotor magnet are estimated indirectly, which helps monitoring thermal condition of motor and contributes to lower fault rate, better control performance and longer motor life. The theoretical analysis and estimation results are confirmed to be correct and valid by simulation results from MATLAB/SIMULINK.

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