

An Improved MMSE Turbo Equalization with Matched Filter Based on Hybrid Soft Feedback Interference Cancellation

Yuxin Cheng⁺, Lei Zhang and Haige Xiang

School of Electronics Engineering and Computer Science, Peking University

Beijing, China

Abstract—This paper presents an improved soft feedback interference cancellation scheme which is used in matched filtered minimum mean squared error (MF-MMSE) turbo equalization. In the iterative receiver, a kind of combined prior information and a dynamic updating method are adopted to improve the performance. Simulation results show that the proposed scheme can achieve 0.25 to 0.5 dB performance gain over the original MF-MMSE turbo equalizer.

Keywords—turbo equalization; iterative receiver; combined prior information; dynamic updating

1. Introduction

The turbo receiver which consists of a maximum a posteriori (MAP) equalizer and an MAP decoder was proposed in [1] to improve the performance of digital communication systems suffering from intersymbol interference (ISI). MAP turbo equalizer is the optimal turbo equalizer for minimizing the bit error rate (BER). Unfortunately, its complexity increases exponentially with the length of the channel impulse response (CIR). Another widely used turbo equalizer was proposed in [2] and [3], which employs a time-varying linear minimum mean squared error (MMSE) filter to estimate the transmitted symbols.

A modified MMSE turbo equalizer is proposed in [4], in which a matched filter (MF) is introduced into the original MMSE turbo equalizer, which was called MF-MMSE turbo equalizer. The matched filter is used to preprocess the received symbol sequence. Then the preprocessed symbol sequence is passed to the MMSE filter and takes the place of the symbol sequence received from channel in the original MMSE scheme. This matched filter can concentrate the received symbol sequence and, consequently, shorten the length of the observation vector.

In the original MMSE turbo equalization, the decoder's extrinsic information is passed to the equalizer as the equalizer's prior information. A new approach about calculating prior information which combined the whole extrinsic information and the partial decoder's full information was proposed in [5]. The performance of the MMSE turbo equalization can be improved significantly. In [6], Fabian proposed a dynamic updating scheme to calculate the equalizer's prior information which outperformed traditional schemes. In this paper, an improved scheme which combines Fabian's approach with the combined prior information scheme to derive the equalizer's prior information is proposed. It can obtain better performance.

This paper is organized in the following manner, in section II the system model is introduced; the proposed scheme is described in section III; the EXIT chart analyses and the simulation results are given in Sections IV; finally, Section V concludes the work discussed in this paper.

⁺ Corresponding author.
E-mail address: Chengyx@pku.edu.cn

2. Syntem model

2.1. The MMSE turbo equalizer with a Matched Filter

For simplicity, BPSK modulation is assumed in this paper. Firstly, a block of equiprobable information bits $b_k \in \{+1, -1\}$ of length K is encoded to M code symbols c_m , $m = 1, 2, \dots, M$, by a convolutional encoder. Then an interleaver permutes the c_m and outputs M symbols x_m , $m = 1, 2, \dots, M$, which will be transmitted over an ISI channel. Considering an equivalent discrete multi-path channel model, the channel output can be given as

$$y_m = \sum_{l=0}^{L-1} h_l x_{m-l} + w_m \quad (1)$$

where $\{h_l\}$, $l = 0, 1, \dots, L-1$, are the coefficients of the CIR and $\{w_m\}$, $m = 1, 2, \dots, M+L-1$, are the additive white Gaussian noise (AWGN) with zero-mean and variance σ_w^2 . We assume $\sum_{l=0}^{L-1} |h_l|^2 = 1$.

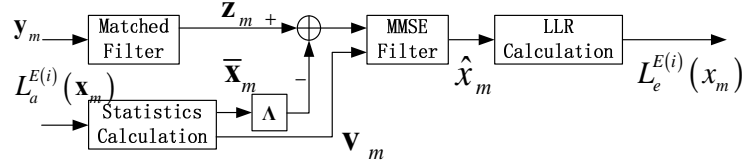


Fig. 1. Block diagram of MF-MMSE equalizer

The structure of the MF-MMSE equalizer is shown in Fig. 1. For the convenience of the derivation of the MF-MMSE equalizer, we delay the output sequence of the matched filter by $L-1$ symbol intervals,

$$z_m = \sum_{l=0}^{L-1} h_l^* y_{m+l} \quad (2)$$

Substituting (1) into (2), we have

$$\begin{aligned} z_m &= \sum_{l=0}^{L-1} h_l^* y_{m+l} = \sum_{l=0}^{L-1} h_l^* \left(\sum_{l'=0}^{L-1} h_{l'} x_{m+l-l'} + w_{m+l} \right) \\ &= \sum_{u=-L+1}^{L-1} \sum_{l=0}^{L-1} h_l^* h_{l-u} x_{m+u} + \sum_{l=0}^{L-1} h_l^* w_{m+l} \\ &= \sum_{u=-L+1}^{L-1} \lambda_u x_{m+u} + \sum_{l=0}^{L-1} h_l^* w_{m+l} \end{aligned} \quad (3)$$

where $\lambda_u = \sum_{l=0}^{L-1} h_l^* h_{l-u}$, $u = -L+1, -L+2, \dots, L-1$.

In the MF-MMSE equalizer, to estimate the m -th transmitted symbol x_m , the matched filter output symbol z_m with its subsequent M_1 symbols and preceding M_2 symbols are considered together as the observation vector of the MMSE filter. The observation vector is defined as $\mathbf{z}_m = [z_{m-M_2} \ z_{m-M_2+1} \ \dots \ z_{m+M_1}]^T$, whose length is M_{wd} , which is given as

$$M_{wd} = M_1 + M_2 + 1 \quad (4)$$

Based on the MMSE criterion [2], the estimate \hat{x}_m of x_m can be written as

$$\hat{x}_m = E(x_m) + \text{Cov}(x_m, \mathbf{z}_m) \text{Cov}(\mathbf{z}_m, \mathbf{z}_m)^{-1} (\mathbf{z}_m - E(\mathbf{z}_m)) \quad (5)$$

In order to calculate (5), we must obtain the mean vector $\bar{\mathbf{x}}_m$ and the variance vector \mathbf{v}_m . Because the symbol probability is derived by (6)

$$P(x_n = \alpha_j) = \frac{1}{2} \left(1 + \alpha_j \tanh \left(\frac{L_a^E(x_n)}{2} \right) \right) \quad (6)$$

where $\alpha_j \in \{+1, -1\}$

Hence, prior LLRs $L_a^E(c_n)$, $m - M_2 - L + 1 \leq n \leq m + M_1 + L - 1$, are used to compute the mean and the variance of the transmitted symbols by (7) and (8), respectively.

$$\bar{x}_n = E(x_n) = \sum_{j=0}^1 \alpha_j P_a^E(x_n = \alpha_j) \quad (7)$$

$$= \tanh(L_a^E(x_n)/2)$$

$$v_n = \text{Cov}(x_n, x_n) = 1 - |\bar{x}_n|^2 \quad (8)$$

Using \bar{x}_n and \bar{v}_n , we can obtain $\bar{\mathbf{x}}_m = [\bar{x}_{m-M_2-L+1}, \dots, \bar{x}_m, \dots, \bar{x}_{m+M_1+L-1}]^T$ and $\mathbf{v}_m = [v_{m-M_2-L+1}, \dots, v_m, \dots, v_{m+M_1+L-1}]^T$.

With \hat{x}_m , we can derive the extrinsic information of the equalizer according to [2] and [3].

2.2. The structure of conventional turbo equalizer

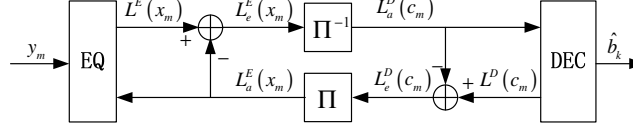


Fig.2. The conventional turbo equalizer structure

The conventional turbo receiver structure is shown in Fig. 2, which consists primarily of an MF-MMSE equalizer and an MAP decoder. We denote prior information, full information and extrinsic information of the turbo equalizer as $L_a^E(x_m)$, $L^E(x_m)$ and $L_e^E(x_m)$, respectively, where superscript 'E' stands for the equalizer. And prior information, full information and extrinsic information belonging to the decoder are defined as $L_a^D(c_m)$, $L^D(c_m)$ and $L_e^D(c_m)$, respectively, where superscript 'D' means the decoder. The relationship among prior information, full information and extrinsic information can be expressed as $L_e^E(x_m) = L^E(x_m) - L_a^E(x_m)$ and $L_e^D(c_m) = L^D(c_m) - L_a^D(c_m)$.

In conventional turbo equalization, the extrinsic information $L_e^E(x_m)$ is deinterleaved to $L_a^D(c_m)$ and passed to the decoder as its prior information and the extrinsic LLR $L_e^D(c_m)$ is interleaved to $L_a^E(x_m)$ and fed to the equalizer as its prior information for the next iteration.

3. Hybrid Soft Feedback Interference Concelation

In this section, we will combine the improved prior information scheme in [5] with Fabian's dynamic updating approach to obtain the equalizer's prior information which is further used in (6) and (7).

3.1. The Combined Prior Information

In the conventional turbo receiver,

$$L_a^E(x_n) = \Pi[L_e^D(c_k)] \quad (9)$$

According [5], the revised prior information of the MF-MMSE turbo equalizer is given in (12), which consists of the whole decoder's extrinsic information and the partial decoder's prior information which is also the equalizer's extrinsic information.

$$\begin{aligned} L_a^E(x_n) &= \Pi(\eta \cdot L_a^D(c_k) + L_e^D(c_k)) \\ &= \eta \cdot L_a^D(c_n) + L_e^D(c_n) \end{aligned} \quad (10)$$

In (7), η is the factor of the combined prior information. When $\eta=0$, the combined prior information is conventional extrinsic information and when $\eta=1$, the combined prior information is decoder's full information.

3.2. Dynamic Updating the Prior Information

From (3), the observation vector consists of both causal symbol and anticausal part of transmitted symbol x_m . According [5], when applying soft feedback cancelation, the causal symbol's estimator can be derived by the equalizer's extrinsic information in the current iteration and the anticausal symbol's estimator can be derived by the decoder's extrinsic information in the last iteration.

According to (10) and the above discussion, the symbol probability in (6) used in equalizer can be revised as (11),

$$\tilde{P}(x_n = \alpha_j) = \begin{cases} \frac{1}{2} \left(1 + a_j \cdot \tanh \left(\frac{\eta \cdot L_e^{E(i)}(x_n) + L_e^{D(i-1)}(c_n)}{2} \right) \right), & 1 \leq n \leq m-1 \\ \frac{1}{2} \left(1 + a_j \cdot \tanh \left(\frac{\eta \cdot L_e^{E(i-1)}(c_n) + L_e^{D(i-1)}(c_n)}{2} \right) \right), & m+1 \leq n \leq M \end{cases} \quad (11)$$

where i is the iteration times and $x_n = \Pi(c_k)$.

4. EXIT Analysis and Simulations Results

In this section, the analysis and the simulation conditions listed below are the same as that designated in [2].

- the data sequence length is $M=65536$
- a rate $R=1/2$ recursive systematic convolutional code with the generator $[1,5/7]_{oct}$ is considered as the channel code with BCJR decoding algorithm
- S -random interleaver with $S = 0.5\sqrt{0.5 \cdot M}$ is adopted
- binary phase shift keying (BPSK) is considered;
- the observation vector length M_{wd} which is defined by (4)
- a length-5 ISI channel with $\mathbf{h} = [0.227 \ 0.46 \ 0.688 \ 0.46 \ 0.227]^T$ is defined in [7]

the channel state information including channel impulse response (CIR) and the noise variance σ^2 is assumed to be well known at the receiver

EXIT charts of proposed scheme with observation vector length equal to 3 and 9 are shown in Fig. 3. Little figures of BER curves are also shown in the upper left corner. It shows that under the observation vector length $M_{wd}=3$, the proposed scheme can achieve the BER performance in AWGN channel after 9 iteration times. When $M_{wd}=9$, it can also reach the BER performance after 10 iteration times.

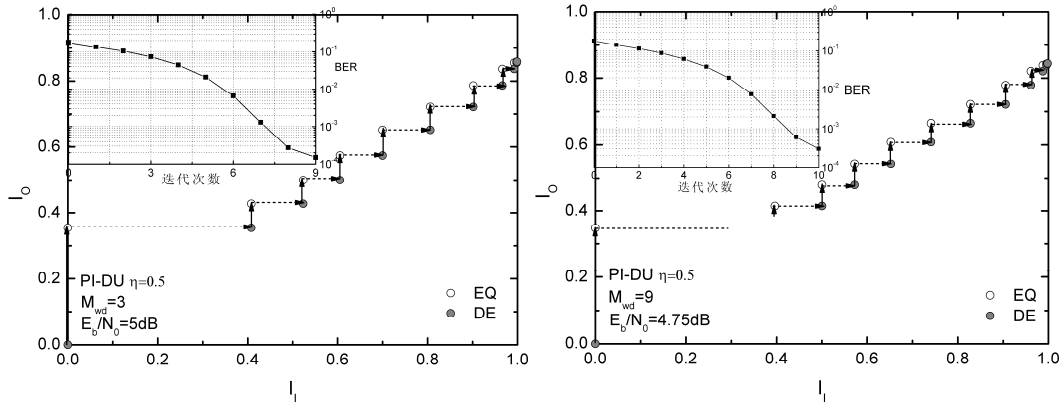


Fig.3. EXIT chart of proposed scheme with $M_{wd}=3$ and $M_{wd}=9$

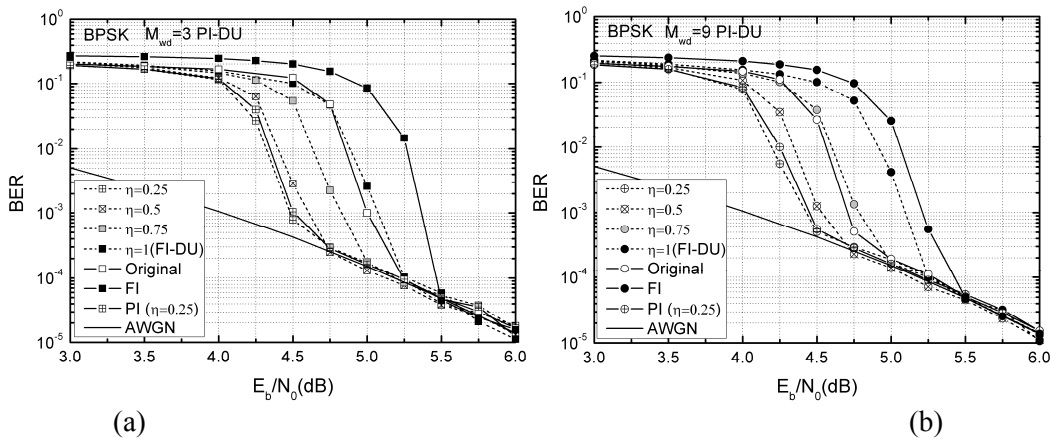


Fig.4. BER performance of the original MF-MMSE turbo equalization proposed schemes with different η .

Fig. 4(a) and (b) shows the BER performance of the original and proposed schemes with different η under the observation vector length $M_{wd}=3$ and $M_{wd}=9$, respectively. When $\eta=0$, the proposed scheme degrades to the original MF-MMSE turbo equalization. When $\eta=1$, it means the combined prior information adopts decoder's full information (FI). The scheme "FI-DU" means the combined prior information adopts decoder's full information (FI) with dynamic updating. While "FI" means the combined prior information adopts decoder's full information (FI) without dynamic updating. These two figures show that the proposed scheme with $\eta=0.25$ achieves the best BER performance both in $M_{wd}=3$ and $M_{wd}=9$ which can achieve the performance gain about 0.5dB in $M_{wd}=3$ and 0.25dB in $M_{wd}=9$ over the original MF-MMSE turbo equalizer.

5. Conclusions

In this paper, a hybrid soft feedback interference cancellation scheme which adopts both the combined prior information scheme and Fabian's dynamic updating approach to derive the equalizer's prior information is proposed. Simulation results demonstrate that the proposed scheme can improve the performance of MF-MMSE turbo equalization.

6. References

- [1] Catherine D., Michel J., Claude B., et al, "Iterative correction of intersymbol interference: Turbo equalization," European Transaction on Telecommunication, pp. 507-511, 1995, 6(5).
- [2] Michael T., Ralf K., Andrew C. S, "Turbo equalization: principles and new results," IEEE Transaction on Communication, pp. 754-767, May, 2002.
- [3] M. Tuchler, A. C. Singer, and R. Koetter, "Minimum mean squared error equalization using a priori information," IEEE Transactions On Signal Processing, pp. 673-683, March, 2002.
- [4] Lei Zhang, Yong Shang, Yuxin Cheng, Haige Xiang, "MMSE Turbo Equalization based on Soft ISI Cancellation," Signal Processing, pp. 1812-1820, September, 2009
- [5] ZHANG Lei, CHENG Haichuan, CHENG Yuxin, JANG Wei, XIANG Haige, "MMSE Turbo Equalization with Partial Prior Information," ACTA Scientiarum Naturalium Universitatis Pekinensis, Vol 45, Sep, 2009, pp. 773-775.
- [6] Fabian V., Sven H, "Low complexity turbo equalization based on soft feedback interference cancellation," IEEE Communication Letter, pp. 586-588, July, 2005.
- [7] J. G. Proakis, "Digital Communications," 4th Edition ed. New York: McGraw-Hill, pp. 631, 2001.