

A Velocity Measurement Principle Based on Scaling Parameter Estimation of Chaotic System

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Abstract. Liu Z et al. introduced a simple relation between the target parameters (range and velocity) and the circuit parameters of chaos-generating system in 2007[International Journal of Bifurcation and Chaos, 5(2007)1735-1739]. With this relation, the measurement of the target parameters is transformed into estimation of the circuit parameters from the radar return signals. However, how to estimate the parameters is not given. In this paper, we do further research based on Liu Z and we propose a new principle of getting the target velocity by estimating the scaling parameter of chaos-generating system. First, we derive the relation between the target velocity and the scaling parameter of chaos-generating system, and the circuit parameter estimation is transformed into the scaling parameter estimation. Then a new method for scaling parameter estimation of chaotic system is proposed by exploiting the chaotic synchronization property. Finally, computer simulation results show the effectiveness of the proposed velocity measurement principle in this paper.

Keywords: Chaos synchronization; Parameter estimation; Velocity measurement

1. Introduction

Chaotic signal, generated by nonlinear dynamical circuits and systems, has broadband spectra, noise-like property, good correlation and deterministic character. These characters of chaotic signal have drawn considerable attentions in radar community. Much research has been devoted its application to radar [1-11].

In [11], the authors present a processing scheme of chaotic radar signals by exploiting the generating mechanism of transmitted chaotic signals. They find a simple relation between the target parameters (range and velocity) and the system parameters of chaos-generating circuits. With this relation, the measurement of the target parameters is transformed into estimation of the circuit parameters. But in [11], the authors do not give the way how to estimate the parameters in chaotic system.

In this paper we do further research based on [11] and it is organized as follows. In Section 2 the relation between the target velocity and the circuit parameters of chaos-generating system is given. In Section 3 the relation between the target velocity and the scaling parameter of chaos-generating system is derived. We can estimate the scaling parameter of chaotic system which is easy to accomplish instead of estimating the parameters of chaotic circuit. In Section 4 a new parameter estimation method for chaotic system is proposed. Brief conclusion of this paper is drawn in Section 5.

2. Relating the Target Velocity and the Circuit Parameters

The relation between the target velocity and the system parameters of chaos-generating system has been derived in [11]. Here we will introduce it briefly.

In [11], the authors use Chua's circuit [12] for illustration. Chua's circuit is shown in Fig.1. The differential equation describing the circuit is given by

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$$\begin{cases} C_1 \frac{du_1}{dt} = \frac{1}{R}(u_2 - u_1) - f(u_1) \\ C_2 \frac{du_2}{dt} = \frac{1}{R}(u_1 - u_2) + i_3 \\ L \frac{di_3}{dt} = -u_2 \end{cases} \quad (1)$$

where u_1 and u_2 are the voltage across the capacitors and i_3 is the current in the inductor. R_0 is the internal resistance of the inductor and $f(\cdot)$ is a piecewise-linear function:

$$f(u) = G_b u + 0.5(G_a - G_b)(|u + B_p| - |u - B_p|)$$

Describing the Chua's diode characteristic (Fig.2). In dimensionless form, Eq.(1) is usually written as

$$L(X) = \begin{cases} \dot{x} = \alpha(y - x - f(x)) \\ \dot{y} = x - y + z \\ \dot{z} = -\beta y \end{cases} \quad (2)$$

where $X = [x, y, z]$ and

$$\begin{cases} x = \frac{u_1}{B_p} & y = \frac{u_2}{B_p} & z = \frac{Ri_3}{B_p} \\ a = RG_a & b = RG_b \\ \alpha = \frac{C_2}{C_1} & \beta = \frac{R^2 C_2}{L} \end{cases} \quad (3)$$

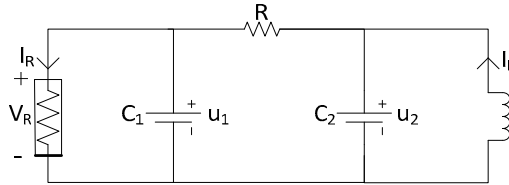


Fig.1 Chua's circuit

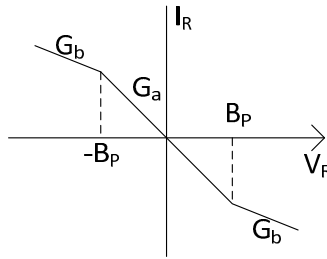


Fig.2 I-V characteristic of Chua's diode

In [11], the author proved that the target velocity v could be determined by the circuit parameters. That is

$$v = \frac{c}{2} \left(1 - \frac{C_2}{\tilde{C}_2}\right) = \frac{c}{2} (1 - s) \quad (4)$$

where c is the light velocity and $C_2 / \tilde{C}_2 = s$. s is the time-scaling scalar and is defined by Eq.(5).

$$s = 1 - (2v / c) \quad (5)$$

In [11], how to estimate the parameter \tilde{C}_2 is not given, in this paper we will do further research based on [11], and give a way to estimate the parameters.

3. Relating the Target Velocity and the Scaling Parameter

Directly estimating the parameter \tilde{C}_2 in circuit system is not easy. However, estimating \tilde{C}_2 can be transferred to estimate the scaling parameter in the chaotic system. In order to illustrate this, two theorems are given.

Theorem 1 Consider two n- dimensional chaotic systems:

$$\dot{X} = g(X) \quad (6)$$

$$\dot{\tilde{X}} = \lambda g(\tilde{X}) \quad (7)$$

where λ is the scaling parameter. If we let $X(t)$, $\tilde{X}(t)$ denote the solution to Eq.(6) and Eq.(7) respectively. Then we have

$$\tilde{X}(t) = X(\lambda t) \quad (8)$$

Proof: Assume $X(\tau) = X(\lambda t)$, (where $\tau = \lambda t$) according to Eq.(6):

$$\begin{aligned} \dot{X}(\tau) &= \frac{d}{dt} X(\tau) \\ &= \frac{d[X(\tau)]}{d\tau} \frac{d[\tau]}{dt} = \lambda \frac{d[X(\tau)]}{d\tau} = \lambda f(X(\tau)) \end{aligned}$$

That is

$$\dot{X}(\lambda t) = \lambda f(X(\lambda t))$$

According to Eq.(7), we have:

$$\dot{\tilde{X}}(t) = X(\lambda t) .$$

Theorem 2 If radar transmitted signal is generated by \dot{X} defined by Eq.(6), then the returned signal from the point moving target could be simulated by the signal generated by $\dot{\tilde{X}}$ defined by Eq.(7).

Proof: Assume that a point target is located at a distance r_0 at time t_0 , travelling with a linear velocity of v along the line of sight of the radar. Then the range to target at any time t is:

$$r(t) = r_0 + v(t - t_0)$$

Without the loss of generality, we let the initial time $t_0 = 0$. The delay corresponding to the two-way path will be

$$\tau = \frac{2r(t)}{c} = \frac{2r_0}{c} + \frac{2vt}{c}$$

where c is the light velocity. Assume radar transmitted signal is generated by $x_1(t)$ which is one variable of $X(t)$. (where $X(t) = [x_1, x_2, \dots, x_n]$). Then the returned signal $x'_1(t)$ from the moving target is

$$x'_1(t) = x_1(t - \tau) = x_1 \left[\left(1 - \frac{2v}{c} \right) \left(t - \left(\frac{2r_0}{c - 2v} \right) \right) \right]$$

Assume

$$\lambda = 1 - (2v/c) \quad (9)$$

$$\tilde{\tau} = 2r_0 / (c - 2v)$$

$x'_1(t)$ can be rewritten as:

$$x'_1(t) = x_1(\lambda(t - \tilde{\tau}))$$

According to Theorem 1 $x'_1(t)$ is the solution of $\dot{\tilde{X}}$, thus the returned signal from the point moving target could be simulated by the signal generated by $\dot{\tilde{X}}$.

Theorem 1 and theorem 2 indicate that if we could estimate the parameter λ in the chaotic system $\dot{\tilde{X}}$, then we can get the target velocity by Eq.(4) in [11]. ($v = \lambda - 1 - (2v/c)$). Thus the estimating the parameter \tilde{C}_2 in circuit system is transferred to estimate the parameter λ in the chaotic system.

4. A New Method for Estimating the Scaling Parameter

In this section we offer a new parameter estimation method based on driven-response synchronization [13], since driven-response synchronization is easy to accomplish.

The driven chaotic system is shown as Eq.(7), where λ is the estimated parameter. The response system is :

$$\hat{X} = \gamma f(\hat{X}) \quad (10)$$

where γ is a constant and $\gamma \in T = [1 - (2v_{\max}/c), 1 + (2v_{\max}/c)]$, v_{\max} is the top limit of the target velocity.

The step of new method for estimation the parameter of chaotic system is as follows:

- 1) Define sampling length l and sample γ_i ($i=1,2,\dots,N$) in the small interval T , where N is the total sampling number in the small interval. Let $\gamma = \gamma_i$.
- 2) Use $\hat{x}_1(t)$ as the driven signal to driven the system defined by Eq.10.
- 3) Compute the synchronization error $E(\gamma_i)$ by Eq.(11)

$$E(\gamma_i) = \|\tilde{x}_2(t) - \hat{x}_2(t)\|, i = 1, 2, \dots, N \quad (11)$$

- 4) Choose $\hat{\gamma}$ in the small interval T , according to

$$\hat{\gamma} = \arg\{\min(E(\gamma_i))\}. \quad (12)$$

Then we consider the value of $\hat{\gamma}$ is estimated value of λ .

The reason of using Eq.(12) is according to that if $E \rightarrow 0$ then $|\hat{\theta} - \theta| \rightarrow 0$ in [3], where E is the synchronization error, θ is the estimated parameter and its estimation value is $\hat{\theta}$.

5. Numerical Simulation

In order to verify the effectiveness of the theory in this paper, simulations have been done in this section.

Assume two leaving targets with the velocity $v_1=30m/s$, $v_2=150m/s$ respectively. According to Eq.(5).

$$s_1 = 1 - (2v_1/c) = 1 - 2 \times 10^{-7}$$

$$s_2 = 1 - (2v_2/c) = 1 - 10^{-6}$$

According to the theory in section 3 we could use the signal generated by Eq.(7) to simulate the returned signal from the moving target when the transmitted signal is generated by Eq.(6). Since this paper is a further research of [11], we also use Chua's chaotic system for illustration. That is, $f(x)$ in Eq.(6) is Chua's chaotic system. We choose the parameter values in Chua's chaotic system as $\alpha = 9$, $\beta = 14.286$. Next, the method proposed in section 4 is used to estimate the parameter s_i ($i=1,2$). Since usually $v \leq 600m/s$, we let $v_{\max} = 600m/s$. Thus the small interval $T = [1 - 4 \times 10^{-6}, 1 + 4 \times 10^{-6}]$. We let the sampling length $l = 10^{-8}$. Using the method in section 4 we get $\hat{s}_1 = 1 - 2.1 \times 10^{-7}$, $\hat{s}_2 = 1 - 0.98 \times 10^{-6}$ and the synchronization error simulation is shown in Fig.1 and Fig.2.

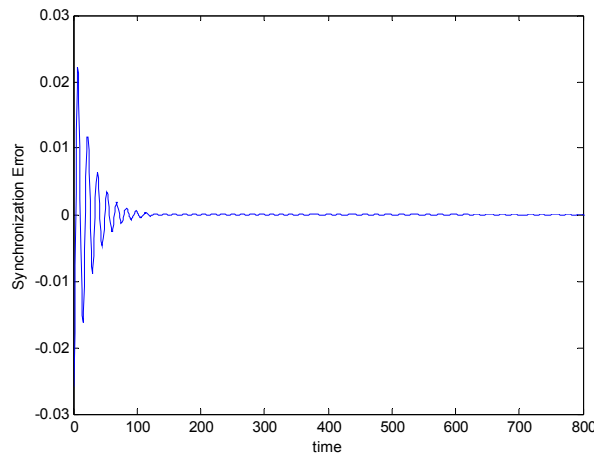


Fig.2 The synchronization Error of the driven-response system, when the driven system with parameter s_1 and the response system with parameter \hat{s}_1

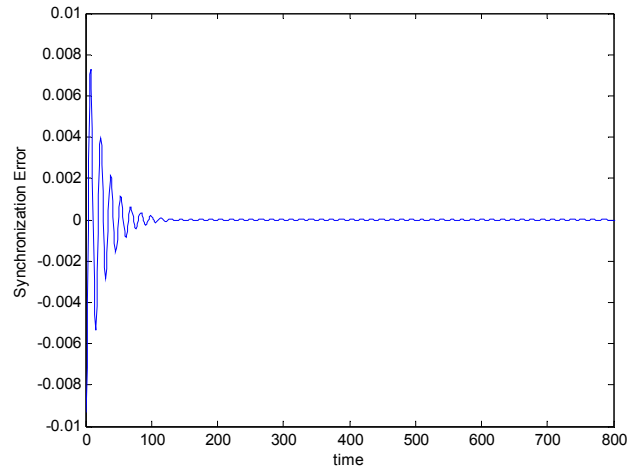


Fig.3 The synchronization Error of the driven-response system, when the driven system with parameter s_2 and the response system with parameter \hat{s}_2

According to Eq.(4) we use $\hat{v} = \frac{c}{2}(1 - \hat{s})$ to get $\hat{v}_1 = 31.5m/s$, $\hat{v}_2 = 147m/s$. The estimation error is small.

So the proposed method to get the target velocity is effective.

6. Conclusion

In this paper we derived the relation between the scaling parameter of chaotic system and the target velocity. What is more, a new method for parameter estimation of chaotic system is proposed. We can get the target velocity by estimating the scaling parameter of returned signal. Computer simulation shows the effectiveness of the proposed method in this paper.

7. Acknowledgment

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8. Reference

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