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Application of PSO Algorithm and Surface with Minimum Threat in Path Planning of Aircraft

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Abstract. 3D flight path is projected on a 2D plane based on surface with minimum threat. So path planning of aircraft is mapped from 3D space to 2D plane. With Taylor expansion a polynomial function with finite items is used to approximate the horizon projection of flight path. So the original planning problem is simplified to search the best series of values in the coefficient space of the polynomial function. Threat model and cost model of path are constructed and PSO algorithm is applied to search optimal path. Flow of optimization is introduced and validated with simulation. Results of simulation show that PSO algorithm and surface with minimum threat can effectively solve global optimization of flight path of aircraft.

Keywords: PSO; path planning; surface with minimum threat

1. Introduction

Path planning is the process of generating a path between an initial location and a target location that has optimal performance against specific criteria [1]. It's one of the most important steps of aircraft mission planning. General method of path planning can be divided into three steps: Firstly, a data structure of the flight space is introduced; secondly, considering fuel, air-defense weapon and other factors, an appropriate evaluation model is constructed to calculate the cost of each flight path; finally, a specific planning algorithm is applied to get the optimal flight path in order to meet with the minimum cost.

In order to accelerate convergence of the planning algorithm, path planning is usually carried into execution in horizontal plane and vertical plane respectively. Apparently the simplified searching space is too rough to omit some optimal paths. So based on the concept of "*surface with minimum threat*" (SWMT), this paper projects 3D flight path on a 2D plane. Then optimization of flight path can be carried out on 2D plane. Finally, with the application of particle swarm algorithm (PSO), the optimal projection of flight path can be achieved.

2. Path Planning Model

2.1 SWMT

The concept of "*surface with minimum threat*" was firstly brought forward by Menon [2]. Menon viewed location and extension of threats as a special kind of terrain. The special terrain can be attached to the real terrain. In other words, threats of flight path drive the real terrain up. The foregoing process results in a synthetical terrain in flight space. Sampling of the synthetical terrain elevation produces a discrete digit map $h(x,y)$ for flight path planning. Based on the digit map and probability of collision between aircraft and ground, the optimal flight height h_c can be achieved. Flight height h_c produces minimum threat for aircraft. So the discrete SWMT $H_d(x,y)$ can be expressed as follow:

$$H_d(x,y) = h(x,y) + h_c \tag{1}$$

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$H_d(x,y)$ has to be fitted to achieve continuous and slippery SWMT $H(x,y)$. Once the initial location and target location have been appointed, any curve on SWMT is an available flight path. Each flight path has a projection on a 2D plane. If we find out the optimal projection, with $H(x,y)$ we can get the optimal flight path conversely.

This paper omits the calculation of h_c and surface fitting of $H_d(x,y)$. Following work is based on the hypothesis that $H(x,y)$ has been achieved.

2.2 Simplified optimizing space

Considering above projection $y=y(x)$ is a continuous and differentiable curve on 2D plane, it can be expanded in Taylor's series as follow:

$$y=y(x)=a_0+a_1x+a_2x^2+\cdots+a_nx^n+\cdots \tag{2}$$

In practice the number of item in equation (2) is always finite, so searching optimal flight path is actually searching an optimal curve on 2D (projection). Further the optimal combination of $a_0,a_1\cdots a_n$ is our interim solution. Now that the initial location (x_0,y_0) and target location (x_t,y_t) are appointed, according to equation (2) following equation can be achieved:

$$\begin{cases} y_0=a_0+a_1x_0+a_2x_0^2+\cdots+a_nx_n^n \\ y_t=a_0+a_1x_t+a_2x_t^2+\cdots+a_nx_t^n \end{cases} \tag{3}$$

So according equation (2) and (3), a_0 and a_1 can be expressed with $a_2,a_3\cdots a_n$ as follow:

$$\begin{cases} a_0=f_1(a_2,a_3,\cdots a_n) \\ a_1=f_2(a_2,a_3,\cdots a_n) \end{cases} \tag{4}$$

Now the set composed of $a_2,a_3\cdots a_n$ actually constructs a searching space of optimal flight path.

2.3 Threat model of flight path

For aircraft, air-defense radar and missile are two main threats [3]. Their models are constructed as follow.

Radar equation is necessary for analyzing detection performance of radar. Consider the classic radar equation as follow [4]:

$$P_r=\frac{P_tG^2\lambda^2\sigma F^4}{(4\pi)^3C_bLR_{rd}^4} \tag{5}$$

Where P_r is the echo power, P_t is the emission power, G is the antenna gain, λ is the wavelength, C_b is the matching coefficient between waveform and filter, L is the loss factor. Parameters above describe intrinsic performance of radar. R_{rd} is the distance between radar and target, σ is radar cross-section of target. In order to simplify the analysis, we hypothesize σ as a constant. Radar detection probability decreases when R_{rd} increases. The result is the decrease of radar threat. Thus the radar threat J_{ri} to path segment i can be calculated according to the following formula [5]:

$$J_{ri}=\frac{L_i}{3}\bullet\sum_{j=1}^{n}\left(\frac{1}{d_{ij1/4}^4}+\frac{1}{d_{ij1/2}^4}+\frac{1}{d_{ij3/4}^4}\right) \tag{6}$$

Where L_i is the length of path segment i, n is the amount of radars, $d_{ij1/4}$, $d_{ij1/2}$, $d_{ij3/4}$ is respectively defined as the distance between radar j and 1/4, 1/2, 3/4 of path segment i.

$R_{\max}$ and $R_{\min}$ are respectively defined as maximal ceiling and minimum ceiling of air-defense missile. The threat of missile battlefield j to point k is defined as J_{dkj}, which can be calculated as follow:

$$J_{dkj}=\begin{cases} 0,\ R>R_{\max}\ or\ R<R_{\min} \\ \dfrac{R_{\max}-R}{R_{\max}},\ R_{\min}\le R\le R_{\max} \end{cases} \tag{7}$$

Where R is the distance between missile battlefield j and point k. The threat of missile battlefields to path segment i is defined as J_{di}, which can be calculated as follow:

$$J_{di} = \frac{L_i}{3}\sum_{j=1}^{m}\left(J_{dij1/4} + J_{dij1/2} + J_{dij3/4}\right) \tag{8}$$

Where m is the amount of missile battlefields, $J_{dij1/4}$, $J_{dij1/2}$, $J_{dij3/4}$ respectively expresses the threat of missile battlefield j to 1/4, 1/2, 3/4 of path segment i.

2.4 Cost model of flight path

The objective of path planning is not only avoiding opposing detection and attack, but also avoiding dangerous terrain. SWMT has eliminated the adverse influence of dangerous terrain, so we express the cost model of flight path as follow:

$$J = \sum_{i=1}^{k} \omega_1 L_i + \omega_2 J_{ri} + \omega_3 J_{di} \tag{9}$$

Where ω_1, ω_2 and ω_3 are weight coefficients.

3. Path Planning based on PSO Algorithm

3.1 Introduction of PSO algorithm

Kenny and Eberhart put forward PSO as a swarm intelligent algorithm in 1995 [6]. Initially PSO produces some random particles. Each particle has two attributes: fitness value and velocity vector. Fitness value is calculated with optimization function. Velocity vector determines moving orientation and distance of the particle. PSO is actually an optimizing iterative course in which all particles are continuously chasing the optimal particle. After every cycle, each particle can update individual optimal location vector (local extremum) and the swarm can update global optimal location vector (global extremum). Updating procedure is expressed as follow:

$$\begin{cases} v_{t+1} = c_0 v_t + c_1\left(pbest_t - x_t\right) + c_2\left(gbest_t - x_t\right) \\ x_{t+1} = x_t + k \cdot v_{t+1} \end{cases} \tag{10}$$

Where v_t is current velocity vector. x_t is current location vector. $pbest_t$ is current individual optimal location vector. $gbest_t$ is current global optimal location. c_0, c_1 and c_2 are all cognitive coefficients of the swarm. k is a constringent coefficient. With few parameters, PSO is simple theoretically and practically.

3.2 Procedure of optimizing flight path

When PSO is applied to path planning, each available flight path projection is viewed as a particle. The procedure of optimizing flight path is as followers:

(a) Appoint some constants: T, $\left(x_0, y_0, z_0\right)$ and $\left(x_t, y_t, z_t\right)$. T is iterative amount. $\left(x_0, y_0, z_0\right)$ is coordinate of initial location. $\left(x_t, y_t, z_t\right)$ is coordinate of target location.

(b) Initialize c_0, c_1, c_2, and k.

(c) Generate n flight path projection on $X-Y$ plane random. Each projection is divided into m segments. Namely there are $m+1$ points on each projection. If m is large enough, each segment can be processed as a straight line.

(d) With assumed SWMT, calculate H coordinate of each point on projection i to achieve corresponding waypoint. Connect these waypoints to achieve flight path i.

(e) $t \leftarrow 0$, $pbest_0^i$ is same as flight path i.

(f) According to equation (9), calculate fit_0^i as initial fitness value of flight path i. The flight path with maximum fitness value is $gbest_0$.

(g) $t \leftarrow 1$.

(h) $i \leftarrow 1$.

(i) Achieve new projection i according to equation ̃ı. Referring to step (d), then corresponding flight path i is updated.

(j) According to equation (6), calculate fit_t^i as new fitness value of flight path i.

(k) If $fit_i^t < fit_i^{t-1}$, $pbest_t^i$ replace $pbest_t^i$; If $fit_i^t \geq fit_i^{t-1}$ $pbest_t^i$ is same as $pbest_t^i$.

(l) $i \leftarrow i+1$

(m) If $i \le n$, repeat steps (i) through (l).

(n) The flight path with maximum new achieved fitness value is $gbest_t$

(o) $t \leftarrow t+1$

(p) If $t \le T$, repeat steps (h) through (o).

(q) $gbest_T$ is the optimal flight path.

4. Simulation Cases

In simulation cases below, digit map is a grid map of $100km \times 100km$. Grid interval is $10km$. T is 100. k is 0.01. c_0, c_1 and c_2 are all random numbers between 0 and 1. Initially 20 path projections are generated. SWMT equation is hypothesized as follow:

$$H(x, y) = -0.048x^2 + 0.046y^2 + 1200 \tag{9}$$

To validate the algorithm, diverse initial location, target location, threat location and amount of threat are chosen. We omit missile threat in three simulation cases below. These simulation cases are programmed with MATLAB. Some simulation results are shown in Figure1 through Figure3 where red solid curves represent optimal flight path and blue triangles represent radar threat.

Case 1: Initial location coordinate is $(-100, 0, 90)$, target location coordinate is $(100, 0, 90)$.

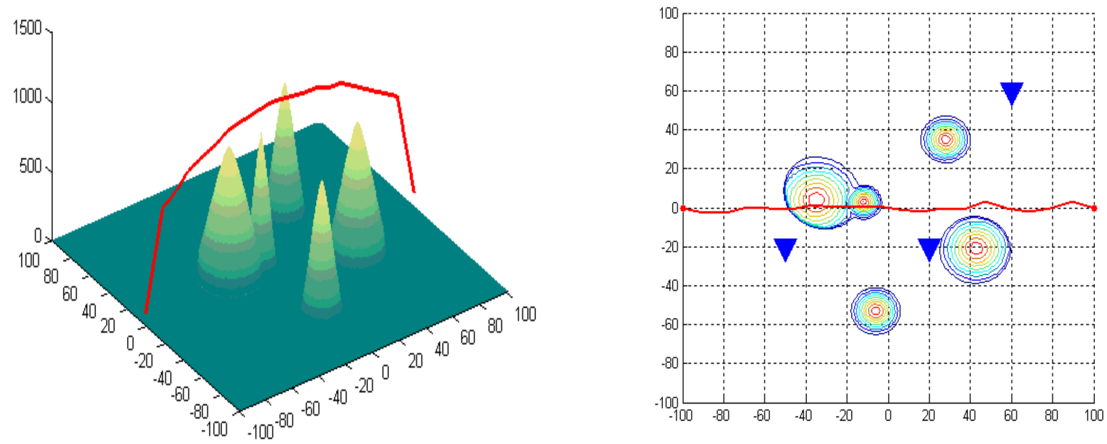

Figure 1. Perspective view and top view of Case 1

Case 2: Initial location coordinate is $(-80, -80, 90)$, target location coordinate is $(80, 80, 90)$.

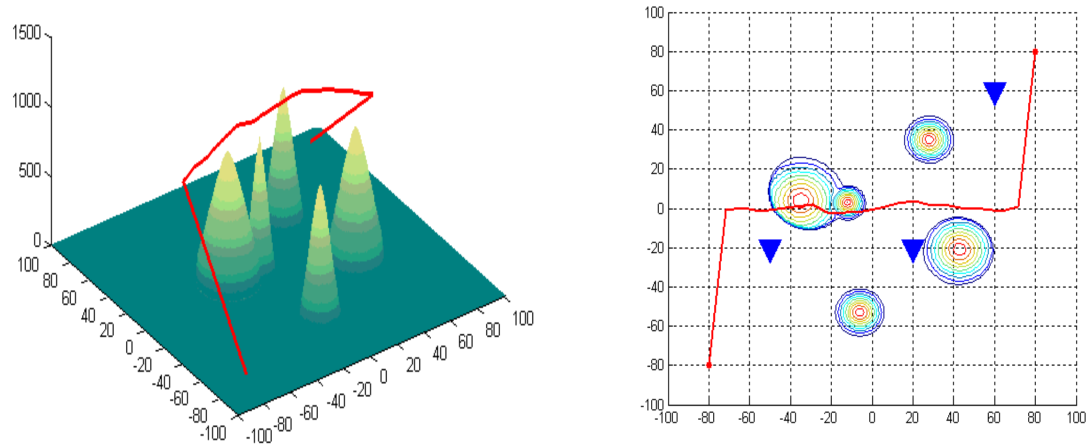

Figure 2. Perspective view and top view of Case 2

Case 3: The coordinates of initial location and target location are same as Case 2, but a new radar threat is added.

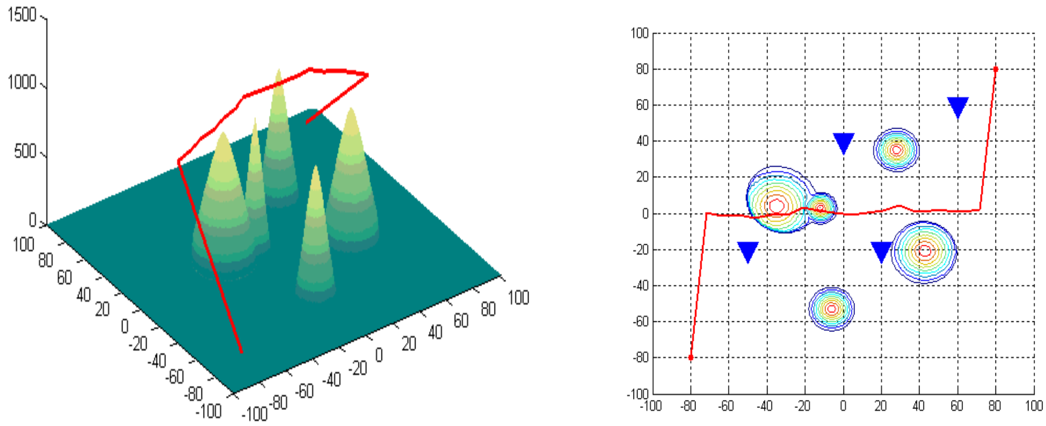

Figure 3. Perspective view and top view of Case 3

As the results show, each simulation case can achieve a smooth curve as optimal flight path. The optimal flight paths all avoid threats of terrain, radar and missile effectively.

5. Conclusion

With the application of SWMT in the study on path planning of aircraft, this paper transforms the searching space of optimal solution from entire 3D space to a surface. SWMT avoids blindness of optimizing availably. After sequential iterations, PSO algorithm finally achieves a smooth flight path in the compressed searching space. Cooperation between SWMT and PSO algorithm provides a new method in path planning of aircraft. But the maneuverability constraints (maximum rotation angle, available overload, etc) of aircraft aren't considered in this paper, this is our following work.

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