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Research on SNR Estimation of Signals in Galileo SAR System

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Abstract. Considering the characteristics of Galileo Search and Rescue signal, we present a new SNR estimation algorithm based on Maximum Likelihood Estimation (MLE) criterion. This new algorithm reduces two dimensions search of frequency and initial phase for one dimension search of phase. Estimation error of this new algorithm is less than 0.5*dB* when actual SNR is -5*dB* and can be controlled within 0.3*dB* when actual SNR ranges from 0 to 20*dB*. Theoretical analysis and measurement results have proved the effectiveness of the algorithm and it has been applied in the Galileo MEOLUT station.

Keywords: SNR estimation, Maximum likelihood estimation, phase search

1. Introduction

Galileo Search and Rescue (SAR) system mainly includes five parts: Beacon equipments of users, Satellite system, MEOLUT (Medium-altitude Earth Orbit Local User Terminal) station, Mission Control Centre (MCC) and SAR team [1]. Beacon equipments send beacon signals when users are in distress. Satellite system receives and transmits beacon signals to MEOLUT station. In MEOLUT station, the function of Beacon signal detection, beacon message extraction and positioning are completed, and the results are sent to MCC and SAR team finally.

The same beacon signal through various satellite transponders will be received by different antennas of MEOLUT station and sent to corresponding signal processing channels. The processing channels output several beacon messages. If we estimate SNR of the processing channels accurately, we can make weighted combining of output beacon messages which have the same source message according to the SNR estimation. And it will promote Bit Error Ratio (BER) performance of beacon message extraction [3].Therefore, the SNR estimation of beacon signals is the key to extract beacon message accurately and reduce the BER.

SNR estimation algorithm based on FFT is motioned in [4], but the sample rate must be over 4 times of signal frequency if we want fine estimation precision. And the algorithm only applies to high SNR, there before; it cannot meet the background requirements. Reference [5] shows we can get SNR estimation through signal and noise projection expression by using signal projection algorithm. But it's very complex and the realizability is poor. The algorithm with fine SNR estimation precision through signal and noise power estimation based on signal sub_space decomposition is presented in [6]. But the algorithm means to structure received signal correlation matrix and Singular Value Decomposition. Thus, the calculation amount is so large that it's inapplicable. Reference [3] presents and compares three realizable algorithms. But they have poor estimation performances when actual SNR is below 0*dB*.

Considering the real-time requirements of SAR signal processing, we wish to get accurate SNR estimation with the cost of short time and small calculation. This paper mainly discusses a new algorithm of SNR estimation in Galileo SAR system .And in this paper; we introduce the structure of Galileo SAR signal in section 2. Section 3 presents the new algorithm of SNR estimation based on MLE criterion. Section 4

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demonstrates good estimation performance with simulation and measurement results. Finally, we conclude this paper

2. Beacon Signal Model

The means of the period of COSPAS-SARSAT signal sent by Galileo SAR Beacon equipments is 50*s* (47.5~52.5*s*) [7]. The format of Galileo SAR beacon signal has two different structures: short message and long message, as shown in Fig 1.

160*ms*(64bit)	15bit	9bit	1bit	87bit/119bit
Unmodulated Carrier	bit SYNC	Frame SYNC	Short/long symbol	Information Data

Fig 1 Beacon Signal Format

The length of beacon signal (measured at 90% of the power point) is 440*ms*±1% for short message and 520*ms*±1% for long message. The initial 160*ms*±1% of beacon signal shall consist of an unmodulated carrier. For short message, the following 280*ms*±1% of the signal shall contain a 112-bit message at a bit rate of 400*bps*±1%. For long message, the following 360*ms*±1% of the signal shall contain a 144-bit message at a bit rate of 400*bps*±1%. In following messages, before actual information data, a bit-synchronization pattern consisting of '1' shall occupy in the first 15-bit positions, and then, a frame synchronization pattern consisting of 9 bits shall occupy bit positions 16 through 24. The 25th bit is the symbol whether users' information data is short or long. '0' means short message and '1' means long message. The final bits (87 bits for short message, 119 bit for long message) are information data of users.

It is concluded from beacon signal format that, we cannot use the bit-frame synchronization to estimate SNR because the length of bit-frame synchronization is only 60*ms* and it's too short. If we use users' information data to estimate SNR, we must complete the function of demodulation and decoding first. Considering that the beacon signal contains unmodulated carrier with the length of 160*ms*, and the SNR estimation of unmodulated carrier need only frequency and little prior information, we can estimate the SNR of unmodulated carrier and regard it as the SNR of the whole beacon signal.

3. New Algorithms for Snr Estimation based on MLE

We get baseband beacon signal by processing IF (intermediate frequency) signal received by MEOLUT station [8].At the same time, we get rough estimation of carrier frequency offset f_d which contains beacon signal frequency inaccuracy and Doppler Effect. After Digital Down Conversion and Low-Pass Filtering, baseband pure carrier can be expressed as:

$$\begin{aligned} r(t) &= As(t) + w(t) \\ &= A\exp\left[j2\pi\left(f_{d\_res}t + \frac{1}{2}f_d' t^2 \right) + j\phi_0 \right] + w(t) \end{aligned} \tag{1}$$

In (1), A is amplitude of signal, f_{d_res} is residual frequency, f_d' is Doppler rate-of-change, ϕ_0 is initial phase (can be any value), $w(t)$ is baseband noise.

The pure carrier can be represented in form of the sum of signal and noise as:

$$r_k = \sqrt{S}m_k + \sqrt{N}z_k \tag{2}$$

Where m_k is complex carrier, z_k is complex Gaussian white noise with mean of 0, S is signal power and N is noise power.

Divide the complex received signal r_k to real part and image part:

$$\begin{aligned} r_k &= r_{I_k} + r_{Q_k} \\ &= \sqrt{S}(m_{I_k} + jm_{Q_k}) + \sqrt{N}(z_{I_k} + jz_{Q_k}) \end{aligned} \tag{3}$$

Thus, the real part and the image part of r_k are respectively as follows

$$\begin{cases} r_{I_k} = \sqrt{S}m_{I_k} + \sqrt{N}z_{I_k} \\ r_{Q_k} = \sqrt{S}m_{Q_k} + \sqrt{N}z_{Q_k} \end{cases} \tag{4}$$

We use $\gamma_{I_k} = \sqrt{N} z_{I_k}, \gamma_{Q_k} = \sqrt{N} z_{Q_k}$ to describe in-phase component and quadrature component of the complex noise. Since the mean of them are both 0 and the variance are both $N/2$, and they are independent, the joint probability density function of γ_{I_k} and γ_{Q_k} is:

$$f\left(\gamma_{I_k}, \gamma_{Q_k}\right) = \frac{1}{\pi N} e^{-\left(\gamma_{I_k}^2 + \gamma_{Q_k}^2\right)/N} \tag{5}$$

According to (4) and (5), the joint probability density function of the in-phase component and the quadrature component is

$$f\left(\gamma_{I_k}, \gamma_{Q_k} \middle| S, N\right) = \frac{1}{\pi N} \exp\left\{-\left[\frac{\left(r_{I_k} - \sqrt{S} m_{I_k}\right)^2 + \left(r_{Q_k} - \sqrt{S} m_{Q_k}\right)^2}{N}\right]\right\} \tag{5}$$

Therefore, the joint probability density function of received sequence is

$$\begin{aligned} f\left(r_I, r_Q \middle| S, N\right) &= \prod_{k=0}^{K-1} f(\gamma_{I_k}, \gamma_{Q_k} | S, N) \\ &= (\pi N)^{-K} \exp\left\{-\frac{1}{N}\left[\sum_{k=0}^{K-1}(r_{I_k} - \sqrt{S} m_{I_k})^2 + \sum_{k=0}^{K-1}(r_{Q_k} - \sqrt{S} m_{Q_k})^2\right]\right\} \end{aligned} \tag{6}$$

Where K is the length of received sequence, and

$$r_I = \left\{r_{I_0}, r_{I_1}, \cdots, r_{I_{K-1}}\right\}, r_Q = \left\{r_{Q_0}, r_{Q_1}, \cdots, r_{Q_{K-1}}\right\}$$

We can get logarithmic likelihood function $\Gamma(S, N)$ from (6) as follows

$$\begin{aligned} \Gamma(S,N) &= \ln f(r_I, r_Q | S, N) \\ &= -K\ln(\pi N) - \frac{1}{N}\left[\sum_{k=0}^{K-1}(r_{I_k} - \sqrt{S} m_{I_k})^2 + \sum_{k=0}^{K-1}(r_{Q_k} - \sqrt{S} m_{Q_k})^2\right] \end{aligned} \tag{7}$$

Seeking partial derivatives for S and N separately in (7) and let them equal to 0, the MLE of signal power and noise power are as follows:

$$\begin{cases} \hat{S}_{ML} = \left[\dfrac{\frac{1}{K}\sum_{k=0}^{K-1}(r_{I_k} m_{I_k} + r_{Q_k} m_{Q_k})}{\frac{1}{K}\sum_{k=0}^{K-1}[(m_{I_k})^2 + (m_{Q_k})^2]}\right]^2 \\ \hat{N}_{ML} = \dfrac{1}{K}\sum_{k=0}^{K-1}(r_{I_k}^2 + r_{Q_k}^2) - \hat{S}_{ML}\dfrac{1}{K}[(m_{I_k})^2 + (m_{Q_k})^2] \end{cases} \tag{8}$$

If m_{I_k}, m_{Q_k} are normalized,

$$(m_{I_k})^2 + (m_{Q_k})^2 = 1 \tag{9}$$

Expression (8) will be simplified as

$$\begin{cases} \hat{S}_{ML} = \left[\dfrac{1}{K}\sum_{k=0}^{K-1}\mathrm{Re}\left\{r_k^* m_k\right\}\right]^2 \\ \hat{N}_{ML} = \dfrac{1}{K}\sum_{k=0}^{K-1}|r_k|^2 - \left[\dfrac{1}{K}\sum_{k=0}^{K-1}\mathrm{Re}\left\{r_k^* m_k\right\}\right]^2 \end{cases} \tag{10}$$

Thus, the estimated value of SNR is

$$\begin{aligned} S\hat{N}R_{ML} &= \frac{\hat{S}_{ML}}{\hat{N}_{ML}} \\ &= \frac{\left[\frac{1}{K}\sum_{k=0}^{K-1}\mathrm{Re}\left\{r_k^* m_k\right\}\right]^2}{\frac{1}{K}\sum_{k=0}^{K-1}|r_k|^2 - \left[\frac{1}{K}\sum_{k=0}^{K-1}\mathrm{Re}\left\{r_k^* m_k\right\}\right]^2} \end{aligned} \tag{11}$$

Where $\mathrm{Re}\{\cdot\}$ means to get real part.

We know from (11) that the essence of SNR estimation based on MLE criterion is to calculate cross-correlation by using different local signals and received signal. We will get the best SNR estimation when the square of the mean of the cross-correlation is max. According to (1) and (11), we should search all the possible values of residual frequency and initial phase when generating local signals. But the calculation amount of the two dimensions search is large, so we simplify it.

After process before, the rmse of $\overline{f_d}$ is

$$\overline{f_d}_{rmse} \le 0.33Hz \tag{12}$$

Thus, the range of f_{d_res} is

$$-3\overline{f_d}_{rmse} \le f_{d\_rem} \le 3\overline{f_d}_{rmse} \tag{13}$$

While the Doppler rate-of-change is

$$-0.7Hz/s \le f_d^{'} \le 0.7Hz/s \tag{14}$$

Therefore, the max phase offset caused by f_{d_rem} and $f_d^{'}$ within 160*ms* is given as follows

$$\theta = \theta_1 + \theta_2 \tag{15}$$

Where θ_1 is the max phase offset caused by f_{d_rem} and θ_2 is the max phase offset caused by $f_d^{'}$, and

$$\begin{cases} \theta_1 = 2\pi f_{d\_rem}\tau \\ \theta_2 = 2\pi \frac{1}{2} f_d^{'}\tau^2 \end{cases} \tag{16}$$

Where $\tau = 160ms$.

Take (13), (14) and (16) into (15) ,we will have

$$-1.1rad \le \theta \le 1.1rad \tag{17}$$

Since the range of θ is narrow, and if we consider the arbitrariness of initial phase ϕ_0, the periodicity and symmetry of cosine signals, we will have

$$\begin{cases} \cos(\pm\frac{\pi}{2}) \le \cos(\theta+\phi_0) \le \cos(0) \\ \sin(-\frac{\pi}{2}) \le \sin(\theta+\phi_0) \le \sin(\frac{\pi}{2}) \end{cases} \tag{18}$$

So, we believe that the range of all phase offset caused by the residual frequency f_{d_res} and the initial phase ϕ_0 is

$$-\frac{\pi}{2} \le \theta_{offset} \le \frac{\pi}{2} \tag{19}$$

Therefore, we only need to search the phase within the range of θ_{offset} with a certain step when generating local signals for estimating SNR based on MLE criterion.

Take (13), (14) into (16) and we have

$$\theta_1 \approx 18\theta_2 \tag{20}$$

Thus, the influence of θ_2 can be ignored. For $\overline{f_d}_{rmse} \le 0.33Hz$, if we search the phase with the step of θ_{step} which can truly reflect the phase change, we will obtain an accurate estimate of SNR. And θ_{step} is

$$\theta_{step} = 2\pi\left(\frac{\overline{f_d}_{rmse}}{10}\right)\tau \tag{21}$$

We generate normalized complex cosine signal sequence by using different phase when taking the phase search. And then, calculate (11). The max SNR result is the SNR estimation based on MLE.

This new SNR estimation algorithm applies to low frequency and narrow Doppler rate-of-change. In this condition, what we need to do is to adjust the phase search range and the step according to frequency and Doppler rate-of-change. In other words, the algorithm has generality and extensibility.

4. Simulation and Measurement Results

4.1 Monte carlo simulation

Simulation conditions: The received signal is complex baseband beacon signal with sample rate of 200*kHz*. Set Doppler frequency as 26*kHz*. Set Doppler rate as 0.7*Hz/s* and set length of input sequence as 1024. We take 2 Monte Carlo simulations to prove the effectiveness of the algorithm.

In Simulation 1, we search frequency and initial phase. The frequency search range is $-1 \sim 1Hz$ and the step is $1/40\,Hz$ while the range of initial phase search is $-\pi/2 \sim \pi/2 rad$ and the step is $\pi/80$ rad.

In Simulation 2, we only search phase with the range of $-\pi/2 \sim \pi/2rad$ and the step of $\pi/80\,rad$.

Both of the Monte Carlo simulations are taken 10000 times. Observing the errors of estimation SNR, the results are shown in Fig 2 and Fig 3.

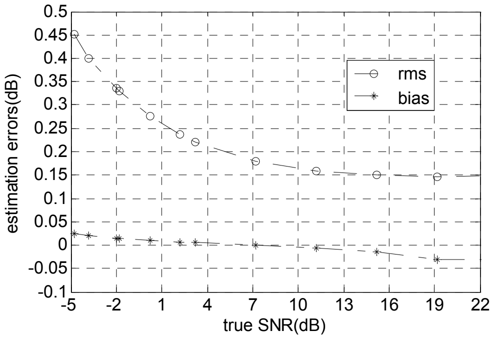

Fig 2　SNR errors with frequency and initial phase search

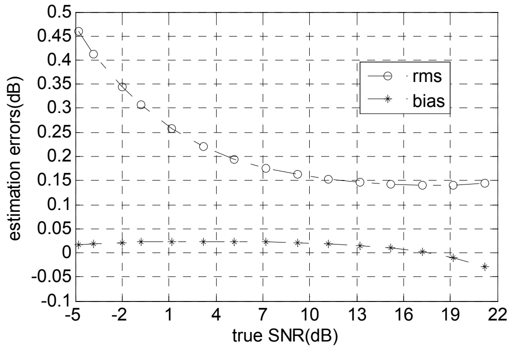

Fig 3　SNR errors with phase search

The estimation results shows that the performance of SNR estimation with phase search is almost the same as that with frequency and initial phase search. The estimation error with phase search is less than $0.5dB$ when the input SNR is $-5dB$, and less than $0.3dB$ when the input SNR is in range of $0 \sim 20dB$. The SNR estimation performance is very good and it can meet the precision.

4.2 Measurement results

This algorithm is implemented in a FPGA of xc5vsx50t-2ff1136 and the utilization of its resources is shown in Tab 1. It is seen that the algorithm needs very few resources. The time that SNR estimation takes is about $0.2ms$ when FPGA works at the frequency of $40MHz$.

Tab 1　Utilization of FPGA resources

Name	Used Num	percents
Slice	734	2%
RAM	1	1%
DSP48Es	5	1%

The algorithm has been realized in Galileo MEOLUT station. The statistical performance of SNR estimation for 10000 times with different input SNR is shown in Tab II and the measurement results are totally the same as the Monte Carlo simulation results.

Tab 2　Measurement SNR errors

True SNR (dB)	Bias (dB)	Rms (dB)
-5	0.041	0.452

−3	0.035	0.373
0	0.026	0.293
4	0.012	0.213
9	0.011	0.188
15	0.008	0.144
20	−0.002	0.142

5. Conclusions

This paper presents a new SNR estimation algorithm based on MLE criterion by considering the characteristics of Galileo SAR signal. Theoretical analysis and measurement results show that the estimation error is less than 0.5*dB* when actual SNR is -5*dB* and can be controlled within 0.3*dB* when actual SNR ranges from 0 to 20*dB*. The algorithm has been applied in the Galileo MEOLUT station，and it has generality and extensibility.

6. References

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