

Elimination of Algorithm's Initial Value Error by Optimizing the Maneuver Direction in Airborne Passive Location

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Abstract. Single airborne passive bearing-only multi-point cross location is studied and the nonlinear least squares estimate model is built up in this paper whose applying background is locating remote slow offing targets from fast airborne observer. The paper studies the relationship of the location error and the target position, in the same time this paper also analyses the influence of the initial guess on accuracy. Through mathematical calculation an expression of the optimal maneuver direction and a figure depicted the optimal maneuver direction under different distance are given. The simulation results show that the approach obviously improves the precision and stabilization of the system of single airborne passive bearing-only cross location.

Keywords: Passive location; cross location; nonlinear least squares; location error

1. Introduction

The applying background of this paper is locating remote slow offing targets from fast airborne observer. Now passive radars have drawn increasing interest in military applications, and many people pay much more attention to passive location technique now. Airborne passive location systems have the advantage of single observation station, far operating distance, high concealment and rapid maneuver ability[1]. Airborne passive location is a rapid development problem in passive location field recently.

The purpose of this paper is to analyze the influence and elimination of algorithm's initial value in airborne passive location. Through studying the relation of the location error and the target position, the paper analyses the method of finding the initial guess, makes a mathematics deduction to get the expression of the optimal maneuver direction under the different distance of the target when the distance of the plane movement is fixed. The result of the simulation experiment indicates that the approach obviously improves the precision and stabilization of the location for single airborne passive bearing-only multi-point cross location.

2. Problem Formulation

Single aircraft with a direction-finding location system makes bearing measurements at different points in its trajectory. The target is a remote slow offing, on account of the high speed of the aircraft, we can approximately consider the offing is stationary. This hypothesis will not bring serious error.

The intersection of two bearing lines provides an estimate of the location of the transmitter. In the presence of noise, more than two bearing lines will not intersect at a single point[2]. The problem geometry is depicted in Fig.1. However, the appropriate processing allows an improved estimate of the transmitter position.

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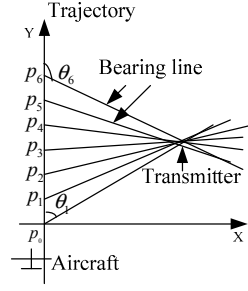


Figure 1. Bearing lines from aircraft positions.

Fig.1 shows the geometry of the passive position location system. A stationary emitter located at $X=[x_t, y_t]^T$ in Cartesian coordinates. A set of N measurements $\theta_i, i=1,2,\dots,N$, is collected by a moving aircraft in straight trajectory at various positions (x_{pi}, y_{pi}) . In the absence of random measurement errors, θ_i is equal to a known function $H(x_{pi}, y_{pi})$. In the presence of additive error:

$$\theta_i = H(x_{pi}, y_{pi}) + \Delta_i, i = 1, 2, \dots, N \quad (1)$$

These N equations can be written as a single equation for N -dimensional column vectors:

$$Z = H(X) + \Delta \quad (2)$$

where:

$$H(X) = \begin{Bmatrix} \arctan \frac{x_t - x_{p1}}{y_t - y_{p1}} \\ \arctan \frac{x_t - x_{p2}}{y_t - y_{p2}} \\ \dots \\ \arctan \frac{x_t - x_{pn}}{y_t - y_{pn}} \end{Bmatrix} \quad (3)$$

In general, $H(X)$ is a nonlinear vector function. To determine a reasonably simple estimator, $H(X)$ can be linearized by expanding it in a Taylor series about a reference point specified by the vector x_0 and retain the first two terms.

$$Z = H(X_0) + G(X_0)(X - X_0) \quad (4)$$

where X and X_0 are $n \times 1$ column vectors and G is the $N \times n$ matrix of derivatives evaluated at X_0 :

$$G(X_0) = \frac{\partial H}{\partial X} \Big|_{X=X_0} = \begin{Bmatrix} \frac{y_0 - y_{p1}}{r_i^2} & -\frac{x_0 - x_{p1}}{r_i^2} \\ \frac{y_0 - y_{p2}}{r_i^2} & -\frac{x_0 - x_{p1}}{r_i^2} \\ \dots & \dots \\ \frac{y_0 - y_{pn}}{r_i^2} & -\frac{x_0 - x_{pn}}{r_i^2} \end{Bmatrix} \quad (5)$$

where: $r_i^2 = (x_0 - x_{pi})^2 + (y_0 - y_{pi})^2, (i = 1, 2, \dots, n)$

Each row of this matrix is the gradient vector of one of the components of $H(X)$. The vector X_0 could be an estimate of X determined from a previous iteration of the estimation procedure or based upon a priori information.

The least squares estimator is

$$\hat{X} = (G^T G)^{-1} G^T (Z - H(X_0)) + X_0 \quad (6)$$

The expression shows the estimator is the function of the initial value, The convergence of these algorithms is influenced by their initial values. When the initialization error is large, the performance of these algorithms degrades significantly.

3. Estimator Accuracy

The main factors which cause the error of the single airborne BO multi-point cross location system are the errors of the bearing measurement, the large estimation bias from the pseudo linear and the initial guess error [3]. The convergence of these algorithms is influenced by their initial guess. When the initial guess error is large, the performance of these algorithms degrades significantly. However, even the initial guess can make algorithm convergent, there is still error between location result and the target's real position. In this paper the reasons which lead to this error are analyzed with geometry theory. When the aircraft are moving in line, it may be possible to use the known receiver trajectories to enhance the accuracy of the transmitter location.

In general, it can be used to select the initial guess by the intersection point of the first and the second bearing lines. But the cut angle of these two lines is small, so if the intersection point is used as the initial guess, the error must be large, the performance of the least squares estimator degrades significantly. Therefore we choose the intersection point of the first and the last bearing lines as the initial guess. In this paper the initial guess is optimized by finding the optimal maneuver direction of the aircraft.

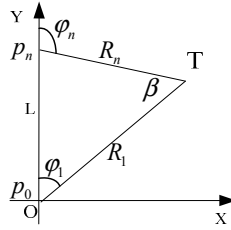


Figure 2. Cross location of the first and last airborne point.

Let T denote the target, the coordinates of which are denoted by (t_x, t_y) . The first and last points of the aircraft lie in p_0 and p_n , with coordinates denoted by (x_{p0}, y_{p0}) and (x_{pn}, y_{pn}) . The distance between p_0 and p_n is L, as illustrated in Fig. 2. Suppose φ_1, φ_n are azimuth angles measured by these airborne passive sensors, and their measurement errors are denoted by $\Delta\varphi_1, \Delta\varphi_n$, which are affected by additive, gaussian-distributed white noise components with zero mean values and standard deviations $\sigma_{\varphi_1}^2, \sigma_{\varphi_n}^2$, respectively. β is the cut angle satisfies $\beta = \varphi_n - \varphi_1$. Thus using triangulation method to get the target location (t_x, t_y)

$$t_x = L \frac{\sin \varphi_1 \sin \varphi_n}{\sin \beta} \quad (7)$$

$$t_y = L \frac{\cos \varphi_1 \sin \varphi_n}{\sin \beta} \quad (8)$$

On the basis of the knowledge of mathematics analysis, the relationship of the function error and the variable errors can be approximately denoted by differential coefficient.

$$\delta_x = \frac{\partial m_x}{\partial \varphi_1} \delta_{\varphi_1} + \frac{\partial m_x}{\partial \varphi_n} \delta_{\varphi_n} \quad (9)$$

$$\delta_y = \frac{\partial m_y}{\partial \varphi_1} \delta_{\varphi_1} + \frac{\partial m_y}{\partial \varphi_n} \delta_{\varphi_n} \quad (10)$$

The location error is defined as $\delta_X = [\delta_x, \delta_y]^T$, the bearing measurement error is defined as $\delta_\Phi = [\delta_{\varphi_1}, \delta_{\varphi_n}]^T$, we can obtain the expression $\delta_X = A\delta_\Phi$, where A is the Jacobian matrix.

$$A = \begin{pmatrix} \frac{\partial m_x}{\partial \varphi_1} & \frac{\partial m_x}{\partial \varphi_n} \\ \frac{\partial m_y}{\partial \varphi_1} & \frac{\partial m_y}{\partial \varphi_n} \end{pmatrix} = \frac{L}{\sin^2 \beta} \begin{pmatrix} \sin^2 \varphi_n & -\sin^2 \varphi_1 \\ \sin \varphi_n \cos \varphi_n & -\sin \varphi_1 \cos \varphi_1 \end{pmatrix} \quad (11)$$

If the bearing measurement errors are independent random variables with variances $\sigma_{\varphi_i}^2, i=1,2$, then

$$P_r = E[\delta_\Phi, \delta_\Phi'] = \begin{pmatrix} \sigma_{\varphi_1}^2 & 0 \\ 0 & \sigma_{\varphi_n}^2 \end{pmatrix} \quad (12)$$

P_x is the covariance matrix of the location error in cartesian coordinates

$$\begin{aligned} P_x &= E[\delta_X, \delta_X'] = E[A\delta_\Phi, \delta_\Phi' A'] = AP_r A' \\ &= \begin{pmatrix} \sigma_x^2 & \rho_{xy} \sigma_{\varphi_1} \sigma_{\varphi_n} \\ \rho_{xy} \sigma_{\varphi_1} \sigma_{\varphi_n} & \sigma_y^2 \end{pmatrix} \end{aligned} \quad (13)$$

Direct calculation using (11), (12) and (13) establishes that the elements of the covariance matrix of P_x are

$$\sigma_x^2 = \frac{L^2}{\sin^4 \beta} (\sin^4 \varphi_n \sigma_{\varphi_1}^2 + \sin^4 \varphi_1 \sigma_{\varphi_n}^2) \quad (14)$$

$$\sigma_y^2 = \frac{L^2}{\sin^4 \beta} (\sin^2 \varphi_n \cos^2 \varphi_n \sigma_{\varphi_1}^2 + \sin^2 \varphi_1 \cos^2 \varphi_1 \sigma_{\varphi_n}^2) \quad (15)$$

Since we suppose that the bearing measurement error is additive, zero-mean gaussian-distributed white noise, then (16) indicates that \hat{X} is a Gaussian random vector. Its probability density is

$$\begin{aligned} f(x, y) &= \frac{1}{2\pi(1-\rho^2)^{1/2} r_{xx}^{1/2} r_{yy}^{1/2}} \exp \left\{ \frac{-1}{2(1-\rho^2)} \right. \\ &\left. f(x, y) = \times \left[\frac{(x-x_t)^2}{r_{xx}} - 2\rho \frac{(x-x_t)(y-y_t)}{r_{xx}^{1/2} r_{yy}^{1/2}} + \frac{(y-y_t)^2}{r_{yy}} \right] \right\} \end{aligned} \quad (16)$$

where $\rho(-1 < \rho < 1)$ denotes the correlation coefficient and (x_t, y_t) is the true coordinate of the target. So the ambiguous location region is an ellipse. For the sake of simplicity, the location accuracy is denoted by GDOP (geometric dilution of precision), which is defined as the ratio of the root-mean-square position error to the root-mean-square ranging error, then

$$\begin{aligned} GDOP &= (\sigma_x^2 + \sigma_y^2)^{0.5} \\ &= \frac{L}{\sin^2(\varphi_n - \varphi_1)} (\sin^2 \varphi_n \sigma_{\varphi_1}^2 + \sin^2 \varphi_1 \sigma_{\varphi_n}^2)^{0.5} \end{aligned} \quad (17)$$

Then (17) indicates that the GDOP bears relation to the baseline length L, standard deviations $\sigma_{\varphi_1}^2, \sigma_{\varphi_n}^2$ and the bearing measurement φ_1, φ_n . Further more the GDOP is in direct proportion to the baseline length L. We suppose $\sigma_{\varphi_1}^2 = \sigma_{\varphi_n}^2 = \sigma_\varphi^2$, the GDOP is in direct proportion to σ_φ^2 . So the distribute of the GDOP is decided by the

location of the target. Through studying the relation of the location error and the target position, analyses the method of finding the initial guess, makes a mathematics deduction to get the expression of the relationship of the optimal maneuver direction and the distance of the target when the distance of the plane movement are fixed.

4. Optimal Maneuver Direction

We suppose $\sigma_{\varphi_1}^2 = \sigma_{\varphi_n}^2 = \sigma_{\varphi}^2$, from (17) one knows that when (18) arrives to the minimum, the minimum value of GDOP can be obtained.

$$w = \frac{\sin^2 \varphi_n + \sin^2 \varphi_1}{\sin^4(\varphi_n - \varphi_1)} \quad (18)$$

We suppose that the distance of the aircraft maneuver is a fixed value L . The Scope of the initial distance of the aircraft and the emitter can be got through the priori information or the power of the signal from the emitter. In this section, we make a mathematics deduction to get the expression of the relationship of the optimal maneuver direction and the distance of the target when the distance of the aircraft maneuver is fixed.

On the basis of law of sines:

$$\frac{R_n}{\sin \varphi_1} = \frac{R_1}{\sin \varphi_n} = \frac{L}{\sin(\varphi_n - \varphi_1)} \quad (19)$$

Then

$$w = \frac{(R_1^2 + L^2 - 2R_1L \cos \varphi_1)(2R_1^2 + L^2 - 2R_1L \cos \varphi_1)}{\sin^2 \varphi_1} \quad (20)$$

Direct calculation using (20) establishes the derivative of w and makes the derivative equal zero, then

$$\begin{aligned} \frac{\partial w}{\partial \varphi_1} &= (A \cos^2 \varphi_1 + B \cos \varphi_1 + C) / \sin^3 \varphi_1 \\ \frac{\partial y}{\partial \varphi_1} &= 0 \end{aligned} \quad (21)$$

where

$$\begin{aligned} A &= 6R^3L + 4RL^3 \\ B &= -(4R^4 + 2L^4 + 14R^2L^2) \\ C &= 6R^3L + 4RL^3 \end{aligned}$$

When bearing measurements φ_1, φ_n satisfy $\varphi_1 = \varphi_2$ or $\varphi_1 = \varphi_2 + \pi$; the target cannot be located. So in this paper we suppose the target is not on the baseline, then $\sin \varphi_1 \neq 0$. So the solution of φ_1 is:

$$A \cos^2 \varphi_1 + B \cos \varphi_1 + C = 0 \quad (22)$$

Because of $(-B + \sqrt{B^2 - 4AC}) / 2A > 1$, Thus the solution of (22) is

$$\varphi_1 = \arccos((-B - \sqrt{B^2 - 4AC}) / 2A) \quad (23)$$

5. Simulation

Suppose the bearing measurement error is additive, zero-mean gaussian-distributed white noise, the standard deviation of bearing measurement error is 0.1° , the error of the every measurement is statistical

independence. Fig.1 shows the optimal maneuver direction under the different distance of the target when the distance of the aircraft maneuver is fixed. Suppose the velocity of the aircraft is 250m/s, the interval time of the bearing measurement is 5s, the overall time of 6 min. So the distance of the aircraft maneuver is 90km. If the true position of the target offing is (173, 100). The initial distance of the aircraft and the emitter is 200 km. We can get the optimal maneuver direction is 52° from Fig.3. When the maneuver direction values are from 1° to 89° , calculate the estimator error by the least squares estimator. Fig.4. shows the contrast of the estimator error under the different maneuver direction. The optimal maneuver direction also is 52° , it's consistent with the mathematical calculation result.

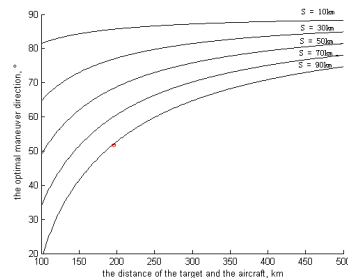


Figure 3. Optimal maneuver direction under different distance.

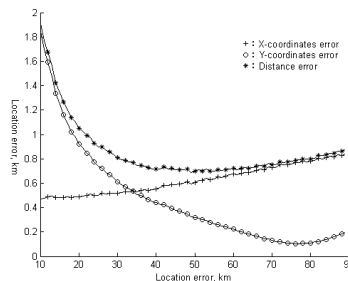


Figure 4. Location error under different maneuver direction.

6. Conclusion

The position of the target is influence to the accuracy of the single airborne passive bearing-only multi-point cross location very obviously. Because the aircraft is able to maneuver quickly, we can optimize the maneuver direction to improve accuracy of location. Through mathematical calculation the expression of the optimal maneuver direction is given in this paper. In the same time, fig.3 can be used to get the optimal maneuver direction intuitively. The simulation results show that the expression of the mathematical calculation is correct, and the approach obviously improves the precision and stabilization of the system of single airborne passive bearing-only cross location.

7. References

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