2010 3rd International Conference on Computer and Electrical Engineering (ICCEE 2010)
IPCSIT vol. 53 (2012) © (2012) IACSIT Press, Singapore
DOI: 10.7763/IPCSIT.2012.V53.No.1.38

Robust Analysis for Dichotomy and Absolutely Stability of the 2D Lure Systems with Parameter Uncertainties

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Abstract. This paper is concerned with the problems of robust analysis for dichotomy and absolutely stability of the two-dimensional (2D) Lure systems with parameter uncertainties. Firstly, based on the Lyapunov equation, time domain criteria for dichotomy and absolutely stability of the 2D Lure systems is obtained. Furthermore, considering the norm-bounded parameter uncertainties, the criteria based on a set of linear matrix inequalities (LMIs) for dichotomy and absolutely stability of the uncertain 2D Lure systems are proposed.

Keywords: 2D (two-dimensional) Lure systems; dichotomy; absolutely stability; robust; LMI (linear matrix inequality)

1. Introduction

Over the past three decades, there has been a growing interest in the study of two-dimensional systems due to their extensive applications and theoretical importance [1-8]. In [1], we find that 2D models can describe systems in image data processing and transformation, water stream heating, thermal process, biomedical imaging, gas absorption, and other areas of 2D digital processing. Naturally, it is an important concern to ensure the stability of 2D systems. So a great number of results on system analysis and control synthesis of 2D systems in both the discrete and continuous contexts have been presented in the literature [2-8]. For the discrete 2D systems, Robert P. Roesser [2] considers the basic concepts, including the general response formula, state transition matrix, Cayley-Hamilton theorem, observability, and controllability. The stability of 2D systems is studied in [3-7], by using the 2D Lyapunov equation. On the other hands, since the uncertainties can not be avoided, the problems of robust stability analysis and synthesis for the uncertain 2D systems have received much attention [8, 9]. Furthermore, the 2D system theory is also frequently used as an analysis tool to some problems, e.g. iterative learning control [10], repetitive process control [11], PI control of discrete linear repetitive process [12] and spatially interconnected systems [13]. All the above results are obtained without considering the 2D systems in nonlinear case and as well known that most of the actual systems are nonlinear. So Xie and Zhang consider the problem of stabilization of the discrete time nonlinear 2D systems based on TS fuzzy model [14].

Except for the absolutely stability, dichotomy is one of the most important properties of nonlinear dynamic systems, for the solutions such as limit cycles and chaos attractors will be excluded in the dichotomous system. One kind of the classic nonlinear systems, i.e. Lure systems, have attracted more and more attention [15-21], because this kind of systems are widely applied in various fields of mechanics and engineering, such as phase synchronization [22].

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In this paper, the problems of dichotomy and absolutely stability for the 2D Lure systems will be investigated via Lyapunov methods. The class of continuous-time 2D systems under consideration are described by the Roesser state-space model with sector restricted nonlinear functions and parameter uncertainties. The parameter uncertainties are assumed to be norm-bounded in the matrices of the system model. The rest of this paper is organized as follows. In section II, some basic definitions, preliminary results and the problem formulation are presented. In section III, the dichotomy and absolutely stability analysis for 2D Lure systems are discussed. In section IV, the robust dichotomy and absolutely stability for uncertain 2D Lure systems are considered. Finally, we conclude the paper in section V.

We introduce the notations used in this paper. $X>0$ (or $X\ge 0$) means the matrix X is symmetric and positive de-finite (or symmetric and positive semi-definite). X^T de- notes the transpose of X . The set of real numbers is denoted by R .

2. Problem Fomulation

Consider a class of 2D linear continuous systems Σ described as follows [7]

$$\begin{bmatrix} \frac{\partial}{\partial t} x^h(t,\tau) \\ \frac{\partial}{\partial \tau} x^v(t,\tau) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x^h(t,\tau) \\ x^v(t,\tau) \end{bmatrix} = Ax \tag{1}$$

$$y(t,\tau) = \begin{bmatrix} c_1 & c_2 \end{bmatrix} \begin{bmatrix} x^h(t,\tau) \\ x^v(t,\tau) \end{bmatrix} = Cx \tag{2}$$

where $x^h \in R^{n_1}$, $x^v \in R^{n_2}$, and $y\in R^m$ are horizontal state vector, vertical state vector, and output, $t\ge 0$, $\tau \ge 0$.

Extending the usual Lure systems to 2D case, the continuous-time 2D Lure system $\bar{\Sigma}$ is proposed to represent one kind of continuous-time nonlinear 2D systems

$$\begin{aligned} \begin{bmatrix} \frac{\partial}{\partial t} x^h(t,\tau) \\ \frac{\partial}{\partial \tau} x^v(t,\tau) \end{bmatrix} &= \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x^h(t,\tau) \\ x^v(t,\tau) \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \begin{bmatrix} \phi_1(\sigma_1) \\ \phi_2(\sigma_2) \end{bmatrix} \\ &= Ax + B\phi \end{aligned} \tag{3}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{bmatrix} x^h(t,\tau) \\ x^v(t,\tau) \end{bmatrix} = Cx \tag{4}$$

where $x^h \in R^{n_1}$, $x^v \in R^{n_2}$, and $\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} \in R^m$ are horizontal state vector, and vertical state vector, and output, $t\ge 0$, $\tau \ge 0$. A , B , and C are matrices of appropriate dimensions. The nonlinear functions $\phi_1(\sigma_1)$ and $\phi_2(\sigma_2)$ satisfies

$$0 \le \sigma_1 \phi_1(\sigma_1) \le \mu_1 \sigma_1^2 \tag{5}$$

$$0 \le \sigma_2 \phi_2(\sigma_2) \le \mu_2 \sigma_2^2 \tag{6}$$

Assumption 1: A has no imaginary eigenvalue, (A , B) is controllable, (A , C) is observable and $K(0) = -CA^{-1}B$ is nonsingular.

Assumption 1: $\phi_1(\sigma_1)$ and $\phi_2(\sigma_2)$ are local lipschitz continuous and possess a finite number of isolated zeros.

Remark 1: Any equilibrium (x_{eq} , σ_{eq}) of system $\bar{\Sigma}$ satisfies $Ax_{eq} = -B\phi(\sigma_{eq})$ and $\sigma_{eq} = Cx_{eq}$. By the non-singularity of A , it follows that $x_{eq} = -A^{-1}B\phi(\sigma_{eq})$ .Since $K(0) = -CA^{-1}B$ is nonsingular, $\sigma_{eq} = -CA^{-1}B\phi(\sigma_{eq})$. Therefore, the equilibrium set $\mathbf{E}$ of system $\bar{\Sigma}$ is

$$\mathbf{E} = \{(x_{eq}, \sigma_{eq}) \mid x_{eq} = -A^{-1}B\phi(\sigma_{eq}), \sigma_{eq} = -CA^{-1}B\phi(\sigma_{eq})\}.$$

For the system 2D Lure system $\bar{\Sigma}$, the definitions of the dichotomy and absolutely stability are given.

Definition1: System $\bar{\Sigma}$ is said to be dichotomous if every its bounded solution is convergent.

Definition 2: System $\bar{\Sigma}$ is said to be absolutely stable if all its solutions are convergent to one equilibrium point.

Remark 1: The definition1 and definition 2 are similar to the dichotomy definition and gradient-like definition for the usual Lure system [15].

In the following, we introduce Lemma 1 and Lemma 2 which will be used in the proof of our main results in section III and section IV.

Lemma 1 [9]**:** Let D , E and F be real matrices of appro-priate dimensions such that $F^T F \le I$, then, for any scalar $\varepsilon > 0$,

$$DFE + E^T F^T D^T \le \varepsilon DD^T + \varepsilon^{-1} E^T E \tag{8}$$

Lemma 2 [24]**:** (Schur complement)**:** Σ_1 , Σ_2 and Σ_3 are all real symmetric matrices, where $\Sigma_1 = \Sigma_1^T$, and $\Sigma_2 > 0$, $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$, if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \tag{9}$$

or when $\Sigma_2 = \Sigma_2^T$, $\Sigma_1 > 0$,

$$\begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0 \tag{10}$$

3. Dicotomy and Absolutly Stablity of 2d Lure systems

In this section, the sufficient conditions for dichotomy and absolutely stability of the 2D continuous Lure systems are presented.

Theorem 1: Under the assumption 1 and assumption 2, the 2D continuous Lure system $\overline{\Sigma}$ is dichotomous, if there exist real numbers λ_1 and λ_2 , $q_1 \ge 0$ and $q_2 \ge 0$, symmetric matrices W_1 and W_2 , such that

$$\begin{bmatrix} A^T\widetilde{W}_1 + \widetilde{W}_1 A + A^T \widetilde{W}_2 + \widetilde{W}_2 A & \widetilde{W}_1 B + \widetilde{W}_2 B + A^T C^T P^T + \frac{1}{2}\overline{C}^T Q \widetilde{\mu} \\ B^T \widetilde{W}_1 + B^T \widetilde{W}_2 + PCA + \frac{1}{2}\widetilde{\mu} Q \overline{C} & PCB + B^T C^T P^T - Q \end{bmatrix} < 0 \tag{11}$$

where $\widetilde{W}_1 = \begin{bmatrix} W_1 & 0 \\ 0 & 0 \end{bmatrix}$, $\widetilde{W}_2 = \begin{bmatrix} 0 & 0 \\ 0 & W_1 \end{bmatrix}$, $P = \mathrm{diag}(\lambda_1, \lambda_2)$, $Q = \mathrm{diag}(q_1, q_2)$, $\widetilde{\mu} = \mathrm{diag}(\mu_1, \mu_2)$.

Proof: With the Lyapunov equation chosen in [7] and Chapter 8 of [23], we can choose the Lyapunov equation as follows

$$V = V_1 + V_2 + 2\sum_{i=1}^{2} \lambda_i \int_0^{\sigma_i} \phi_i(a) da \tag{12}$$

where

$$V_1 = \begin{bmatrix} x^h(t,\tau) \\ x^v(t,\tau) \end{bmatrix}^T \widetilde{W}_1 \begin{bmatrix} x^h(t,\tau) \\ x^v(t,\tau) \end{bmatrix}, \ V_2 = \begin{bmatrix} x^h(t,\tau) \\ x^v(t,\tau) \end{bmatrix}^T \widetilde{W}_2 \begin{bmatrix} x^h(t,\tau) \\ x^v(t,\tau) \end{bmatrix}.$$

The derivation of the Lyapunov function is

$$\begin{aligned} \dot{V} &= \dot{V}_1 + \dot{V}_2 + 2\sum_{i=1}^{2} \lambda_i \phi_i(\sigma_i) C(Ax + B\phi) \\ &= (Ax + B\phi)^T \widetilde{W}_1 x + x^T \widetilde{W}_1 (Ax + B\phi) + (Ax + B\phi)^T \widetilde{W}_2 x + \\ &\quad x^T \widetilde{W}_2 (Ax + B\phi) + 2\sum_{i=1}^{2} \lambda_i \phi_i(\sigma_i) C(Ax + B\phi) \end{aligned}$$

and

$$\dot{V} = \begin{bmatrix} x \\ \phi \end{bmatrix}^T \begin{bmatrix} A^T\widetilde{W}_1 + \widetilde{W}_1 A + A^T \widetilde{W}_2 + \widetilde{W}_2 A & \widetilde{W}_1 B + \widetilde{W}_2 B + A^T C^T P^T \\ B^T \widetilde{W}_1 + B^T \widetilde{W}_2 + PCA & PCB + B^T C^T P^T \end{bmatrix} \begin{bmatrix} x \\ \phi \end{bmatrix}$$

According to the nonlinear functions $\phi_1(\sigma_1)$ and $\phi_2(\sigma_2)$,

we have

$$q_1 \phi_1(\sigma_1)(\phi_1(\sigma_1) - \mu_1 \sigma_1) \le 0 \tag{13}$$

$$q_2 \phi_2(\sigma_2)(\phi_2(\sigma_2) - \mu_2 \sigma_2) \le 0 \tag{14}$$

and

$$\begin{bmatrix} x \\ \phi \end{bmatrix}^T \begin{bmatrix} 0 & -\frac{1}{2}C^T Q\tilde{\mu} \\ -\frac{1}{2}\tilde{\mu}QC & Q \end{bmatrix} \begin{bmatrix} x \\ \phi \end{bmatrix} \le 0 \tag{15}$$

Let

$$\dot{V} < \begin{bmatrix} x \\ \phi \end{bmatrix}^T \begin{bmatrix} 0 & -\frac{1}{2}C^T Q\tilde{\mu} \\ -\frac{1}{2}\tilde{\mu}QC & Q \end{bmatrix} \begin{bmatrix} x \\ \phi \end{bmatrix} \le 0 \tag{16}$$

Then we can have the inequality (11) of Theorem 1.

From [18] and definition 1, we have that if $\dot{V} < 0$, then every bounded solution of the systems are convergent, i.e. the Theorem 1.

Corollary 1: Under the assumption 1 and assumption 2, the 2D continuous Lure system $\overline{\Sigma}$ is absolutely stable, if there exist real numbers $\lambda_1 \ge 0$, $\lambda_2 \ge 0$, $q_1 \ge 0$ and $q_2 \ge 0$, symmetric matrices $W_1 > 0$ and $W_2 > 0$, and the inequality (11) satisfies.

Proof: the proof can be obtained easily with Lyapunov theory [23] and definition 2.

4. Robust Dichotomy and Absolutly Stability of 2D Lure Systems

In section III, the sufficient conditions for the dichotomy and absolutely stability of the nominal 2D Lure systems have been given in Theorem 1. Based on this, considering the parameter uncertainties, the sufficient conditions for the robust dichotomy and absolutely stability of the 2D continuous Lure systems are presented in the following.

Consider the continuous-time 2D Lure system with parameter uncertainties $\overline{\Sigma}_\Delta$

$$\begin{bmatrix} \frac{\partial}{\partial t}x^h(t,\tau) \\ \frac{\partial}{\partial \tau}x^v(t,\tau) \end{bmatrix} = (A+\Delta A)x + B\phi \tag{17}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{bmatrix} x^h(t,\tau) \\ x^v(t,\tau) \end{bmatrix} = Cx \tag{19}$$

where $x^h \in R^{n_1}$, $x^v \in R^{n_2}$, and $\begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} \in R^m$ are horizontal state vector, vertical state vector, and output, $t \ge 0$, $\tau \ge 0$. The nonlinear functions $\phi_1(\sigma_1)$ and $\phi_2(\sigma_2)$ satisfies

$$0 \le \sigma_1\phi_1(\sigma_1) \le \mu_1\sigma_1^2 \tag{20}$$

$$0 \le \sigma_2\phi_2(\sigma_2) \le \mu_2\sigma_2^2 \tag{21}$$

And the ΔA is unknown matrix representing parameter uncertainty, and is assumed to be of the form

$$\Delta A = MFN \tag{22}$$

where M and N are known real constant matrices and F is an unknown matrix satisfying

$$F^T F \le I \tag{23}$$

Theorem 2: Under the assumption 1 and assumption 2, the uncertain 2D continuous Lure system $\overline{\Sigma}_\Delta$ is dichotomous, if there exist real numbers λ_1, λ_2, $q_1 \ge 0$, $q_2 \ge 0$, and $\varepsilon > 0$, symmetric matrices W_1 and W_2, such that

$$\begin{bmatrix} A^T\widetilde{W}_1 + \widetilde{W}_1 A + A^T\widetilde{W}_2 + \widetilde{W}_2 A + \varepsilon N^T N & \widetilde{W}_1 B + \widetilde{W}_2 B + A^T C^T P^T + \frac{1}{2}\overline{C}^T Q\tilde{\mu} & \widetilde{W}_1 M + \widetilde{W}_2 M \\ B^T\widetilde{W}_1 + B^T\widetilde{W}_2 + PCA + \frac{1}{2}\tilde{\mu}Q\overline{C} & PCB + B^T C^T P^T - Q & PCM \\ M^T\widetilde{W}_1 + M^T\widetilde{W}_2 & M^T C^T P^T & -\varepsilon \end{bmatrix} < 0 \tag{24}$$

Proof: According to the Theorem 1, the continuous-time 2D Lure system with parameter uncertainties $\overline{\Sigma}_\Delta$ is dichotomous if

$$\begin{bmatrix} A_\Delta{}^T \widetilde{W}_1 + \widetilde{W}_1 A_\Delta + A_\Delta{}^T \widetilde{W}_2 + \widetilde{W}_2 A_\Delta & \widetilde{W}_1 B + \widetilde{W}_2 B + A_\Delta{}^T C^T P^T + \frac{1}{2}\overline{C}^T Q \widetilde{\mu} \\ B^T \widetilde{W}_1 + B^T \widetilde{W}_2 + PCA_\Delta + \frac{1}{2}\widetilde{\mu} Q \overline{C} & PCB + B^T C^T P^T - Q \end{bmatrix} < 0 \tag{25}$$

where $A_\Delta = A + \Delta A$.

Then with equation (22), Lemma 1 and Lemma 2, we can have inequality (24) from inequality (25) easily.

Corollary 2: Under the assumption 1 and assumption 2, the uncertain 2D continuous Lure system $\overline{\Sigma}_\Delta$ is absolutely stable, if there exist real numbers $\lambda_1 \ge 0$, $\lambda_2 \ge 0$, $q_1 \ge 0$, $q_2 \ge 0$, and $\varepsilon > 0$, symmetric matrices $W_1 > 0$ and $W_2 > 0$, such that the inequality (24) satisfies.

Proof: the proof can be obtained easily with Lyapunov theory, [23] and definition 2.

5. Coclusions

In this paper, the problems of robust dichotomy and absolutely stability for uncertain 2D Lure systems have been considered. In terms of an LMI, the sufficient conditions for the dichotomy and absolutely stability of 2D Lure systems are obtained by a set of LMIs. When these LMIs are feasible, then the 2D systems are dichotomous or absolutely stability.

6. Acknowledgment

The work is supported by the Doctoral Foundation of Beijing University of Technology of P. R. China under Grant X00002211200902.

7. References

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