

Condition-Based Maintenance Model under the Imperfect Maintenance

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Abstract. According to the imperfect maintenance in the preventive maintenance, a condition-based maintenance model (CBM) under the imperfect maintenance is presented for deteriorating system which is described by the continuous time Markov chain. Parameter of preventive maintenance is defined, determining which one of previous deterioration stages the deteriorating system can be restored to by preventive maintenance. The algorithm is presented to get the solution of steady-state availability, and the joint optimization of inspection rate and the threshold value are investigated for the maximization of system steady-state availability. At last, the feasible of the model is verified through a numerical example.

Keywords: Continuous time Markov chain; condition-based maintenance; imperfect maintenance

1. Introduction

By means of inspection, maintenance and replacement, preventive maintenance can decrease the possibility of system failure and the losses caused by the failure; consequently, it has significance for enhancing the systems efficiency.

Generally, preventive maintenance can be divided two types, i.e. time-based maintenance (TBM) and condition-based maintenance (CBM). For time-based maintenance, preventive maintenance undertaken regularly at preselected intervals. For condition-based maintenance, the action after inspection may be depending on the status of system. Condition-based maintenance can effectively prolong deteriorating system's life.

According to the maintenance effect, there are three preventive maintenance types: major maintenance, minimal maintenance and imperfect maintenance. Major maintenance may restore the system to 'as good as new'; minimal maintenance restores the system to the previous deterioration stage; the effect of imperfect maintenance is worse than the major maintenance but better than the minimal maintenance. Numerous researchers investigate the major and minimal maintenance, e.g. the condition-based maintenance model of major maintenance was given based on Markov theory by Amari [5]; the close-form expressions of system availability when the system undergoes both major and minimal maintenance were derived, based on continuous time Markov chain in [6]. Considering imperfect maintenance is more practical in reality, the increasing number of investigations appears recently concerning imperfect maintenance, which was summarized by Pham and Wang [7]. It is also one of the maintenance research topics that the maintenance policies are optimized, in order to maximize the system availability or minimize the operational cost.

In addition, Poisson failure occurs instantaneously and the failure-rate is a constant in the system deteriorating process, while deterioration grows gradually with time and leads to failure ultimately [8].

In this paper, we consider a system subject to both deterioration and Poisson failure, and give a condition-based model under the imperfect maintenance based on the continuous time Markov chain. The solution of

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steady-state availability is shown, and the joint optimization of inspection rate and the threshold value are investigated for the maximization of system steady-state availability.

2. Description of the Condition-based Maintenance Model

2.1 Notation

k	total number of deterioration stages
i	deterioration stage
n	preventive maintenance threshold value
d	preventive maintenance parameter
λ_d	failure rate
$1/\lambda_m$	mean time between inspections (MTBI)
$1/\mu_{in}$	mean time to carry out maintenance inspection
$1/\mu_m$	mean duration of preventive minimal maintenance
$1/\mu_M$	mean duration of preventive major maintenance
$1/\mu_{im}$	mean duration of preventive imperfect maintenance
$1/\mu_r$	mean time of minimal repairing a Poisson failure
$1/\mu_R$	mean time of major repairing a deterioration failure

2.2 Assumptions

- Deterioration grows gradually with time and leads to failure ultimately. After inspection, preventive maintenance depending on the status of system can prolong deteriorating system's life.
- Periodically the system is removed from operation for inspection; each inspection reveals the system deterioration state perfectly.
- Following an inspection based on the current value of i , one of the decisions is selected:
 - If $0 \leq i \leq n$: no maintenance action;
 - If $n+1 \leq i \leq k$: preventive maintenance is performed.
- The effect of preventive maintenance is imperfect maintenance. Imperfect maintenance may restore the system to the one state between 'as good-as-new' and 'as bad-as-old', which can be defined as preventive maintenance parameter, i.e. the larger this parameter is, the better preventive maintenance effect shows.
- During an inspection or maintenance, the system is neither operating nor deteriorating.
- The system has a deterioration failure immediately following the completion of k stages of deterioration. Following a deterioration failure, a corrective maintenance is performed so that the system is returned to an as good-as-new condition.
- The duration of each deterioration stage follows an exponentially distributed with rate λ_d , The uptime between successive inspection is exponentially distributed with rate λ_m .
- When Poisson failure occurs, minimal repair is performed to restore the system to the operate state it was in just before failure; When deterioration failure occurs, major repair is performed to restore the system to "as good as new".
- The duration and cost of preventive maintenance depend on maintenance parameter. Popularly, the better preventive maintenance effect shows, the more maintenance time and cost may be expended. All maintenance duration are exponentially distributed: $1/\mu_m < 1/\mu_{im} < 1/\mu_M < 1/\mu_R$.
- Optimizing maintenance and repair policies for maximal system steady-state availability.

3. Mathematical Formulations

3.1 State space description

- $s(i,0)$: System is operating in the deterioration stage i , $0 \leq i \leq k$;
- $s(i,1)$: System is under inspection in the deterioration stage i , $0 \leq i \leq k$;
- $s(i,2)$: System is under the preventive maintenance after inspection in the deterioration stage i , $n+1 \leq i \leq k$
- $s(F)$: System has failed due to deterioration, and it is under corrective maintenance.

As shown in Fig.1, all states and their transition probabilities may constitute a Markov Chain, the state probabilities can be defined as follows:

$$\pi_{i,j} = P(i,j) \quad i = 0,1,\dots,k; j = 1,2 \quad \pi_F = P(F)$$

3.2 Mean duration of preventive imperfect maintenance

To describe the imperfect maintenance, the mean duration of preventive imperfect maintenance should be between the mean duration of preventive major maintenance and minimal maintenance, i.e. $1/\mu_m < 1/\mu_{im} < 1/\mu_M$. We define preventive maintenance parameter d , to quantitatively describe the mean duration of preventive imperfect maintenance. After the preventive maintenance, system can be restore to stage $n+1-d$ from stage $i = n+1$, we can know $d \leq n+1$ because $n+1-d \geq 0$. The mean duration of preventive imperfect maintenance can be defined as follows:

$$\frac{1}{\mu_{im}} = \frac{1}{\mu_m} + \frac{d-1}{n+1} \left(\frac{1}{\mu_M} - \frac{1}{\mu_m} \right) \quad (1)$$

3.3 The state probabilities and steady-state availability under system deterioration failure

According to the relationships among parameters k , d and n , we need analyze the condition-based model under six conditions as follows:

$$\left\{ \begin{array}{l} \text{case 1: } k-2d+1 < n < k-d \\ \text{case 2: } n > k-d \\ \text{case 3: } n = k-d \\ \text{case 4: } n = k-2d+1 \\ \text{case 5: } n = k-2d \\ \text{case 6: } n < k-2d \end{array} \right.$$

As shown in Fig.1, we take an example for case 1 to illuminate how to get the solution of steady-state availability. The Markov chain looks difficult to solve analytically even

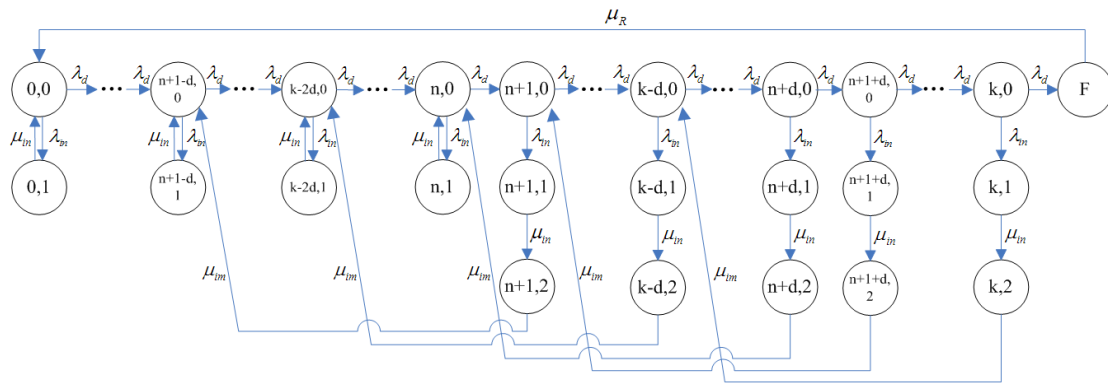


Figure 1. The continuous time Markov chain model of deteriorating system under imperfect maintenance

for this simplified model. So we use the rule that the Markov process tends towards stationary distributing when $t \rightarrow \infty$ in [9], and get equations as follows:

- if $0 \leq i \leq n-d$

$$\pi_{i,0} = \pi_{0,0}$$

- if $n-d+1 \leq i \leq n$

$$\lambda_d \cdot \pi_{i-1,0} + \mu_m \cdot \pi_{i+d,2} = \lambda_d \cdot \pi_{i,0}$$

$$\mu_m \cdot \pi_{i+d,2} = \lambda_{in} \cdot \pi_{i+d,0}$$

- if $n+1 \leq i \leq k-d$

$$\lambda_d \cdot \pi_{i-1,0} + \mu_m \cdot \pi_{i+d,2} = (\lambda_d + \lambda_{in}) \cdot \pi_{i,0}$$

$$\mu_m \cdot \pi_{i+d,2} = \lambda_{in} \cdot \pi_{i+d,0}$$

- if $k-d+1 \leq i \leq k$:

$$\lambda_d \cdot \pi_{i-1,0} = (\lambda_d + \lambda_{in}) \cdot \pi_{i,0}$$

- At fail state F we have:

$$\mu_R \cdot \pi_F = \lambda_d \cdot \pi_{k,0} = \lambda_d \cdot \pi_{0,0}$$

The closed-form solution in terms of $\pi_{0,0}$ is therefore

$$\pi_{i,0} = \begin{cases} \left(1 + \frac{\lambda_{in}}{\lambda_d} \sum_{j=1}^{i-n+d} E_{n+j}\right) \pi_{0,0} & n-d+1 \leq i \leq k-2d \\ \left(E_n - \frac{\lambda_{in}}{\lambda_d} \cdot \sum_{j=0}^{n-i-1} w^{k-n-d+j}\right) \pi_{0,0} & k-2d+1 \leq i < n \\ E_i \cdot \pi_{0,0} & n \leq i \leq k-d \\ w^{k-i} \cdot \pi_{0,0} & k-d+1 \leq i \leq k \end{cases} \quad (2)$$

$$\pi_F = \frac{\lambda_d}{\mu_R} \pi_{0,0} \quad (3)$$

where

$$w = \frac{\lambda_d + \lambda_{in}}{\lambda_d}, \quad E_i = w^{k-i} \left[-\frac{\lambda_{in}}{\lambda_d} (k-d-i) w^{-d-1} + 1 \right].$$

The group of preventive maintenance state $\pi_{i,1}$ and $\pi_{i,2}$ is

$$\begin{cases} \pi_{i,1} = \frac{\lambda_{im}}{\mu_{im}} \pi_{i,0} & 0 \leq i \leq k \\ \pi_{i,2} = \frac{\lambda_{im}}{\mu_{im}} \pi_{i,0} & n+1 \leq i \leq k \end{cases} \quad (4)$$

The sum of all state probabilities equals to 1:

$$\sum_{i=0}^k \pi_{i,0} + \sum_{i=0}^k \pi_{i,1} + \sum_{i=n+1}^k \pi_{i,2} + \pi_F = 1 \quad (5)$$

The closed-form solution of $\pi_{0,0}$ and steady state availability A_s is then

$$\begin{aligned} \pi_{0,0} &= \left[\sum_{i=0}^{n-d} \left(1 + \frac{\lambda_{in}}{\mu_{in}}\right) + \sum_{i=n-d+1}^{k-2d} \left(1 + \frac{\lambda_{in}}{\mu_{in}}\right) \left(1 + \frac{\lambda_{in}}{\lambda_d} \sum_{j=1}^{i-n+d} E_{n+j}\right) \right. \\ &\quad + \sum_{i=k-2d+1}^{n-1} \left(1 + \frac{\lambda_{in}}{\mu_{in}}\right) \left(E_n - \frac{\lambda_{in}}{\lambda_d} \cdot \sum_{j=0}^{n-i-1} w^{k-n-d+j}\right) \\ &\quad + \left(1 + \frac{\lambda_{in}}{\mu_{in}}\right) E_n + \sum_{i=n+1}^{k-d} \left(1 + \frac{\lambda_{im}}{\mu_{im}} + \frac{\lambda_{in}}{\mu_{in}}\right) E_i \\ &\quad \left. + \sum_{i=k-d+1}^k \left(1 + \frac{\lambda_{in}}{\mu_{in}} + \frac{\lambda_{im}}{\mu_{im}}\right) w^{k-i} + \frac{\lambda_d}{\mu_R} \right]^{-1} \\ A_s &= \sum_{i=0}^k \pi_{i,0} = \sum_{i=0}^{n-d} \pi_{0,0} + \sum_{i=n-d+1}^{k-2d} \left(1 + \frac{\lambda_{in}}{\lambda_d} \sum_{j=1}^{i-n+d} E_{n+j}\right) \pi_{0,0} \\ &\quad + \sum_{i=k-2d+1}^{n-1} \left(E_n - \frac{\lambda_{in}}{\lambda_d} \cdot \sum_{j=0}^{n-i-1} w^{k-n-d+j}\right) \pi_{0,0} + E_n \cdot \pi_{0,0} \\ &\quad + \sum_{i=n+1}^{k-d} E_i \pi_{0,0} + \sum_{i=k-d+1}^k w^{k-i} \pi_{0,0} \end{aligned} \quad (6)$$

$$\quad (7)$$

Similarly, we can get answers for case 2 to 6. The final result is listed as follows:

$$\pi_{0,0} = \begin{cases} \left[a \cdot f + \sum_{i=n-d+1}^{k-2d} ab_i + \sum_{i=k-2d+1}^{n-1} a \left(E_n - \frac{\lambda_{in}}{\lambda_d} \cdot \sum_{j=0}^{n-i-1} w^{k-n-d+j} \right) + aE_n + \sum_{i=n+1}^{k-d} cE_i + \sum_{i=k-d+1}^k cw^{k-i} + e \right]^{-1} & k-2d+1 < n < k-d \\ \left[a \cdot f + \sum_{i=n-d+1}^{k-d} a \left(1 + \frac{\lambda_{in}}{\lambda_d} \sum_{j=0}^{i-n+d-1} w^{k-i-d+j} \right) + \sum_{i=k-d+1}^n aw^{k-n} + \sum_{i=n+1}^k cw^{k-i} + e \right]^{-1} & n > k-d \\ \left[a \cdot f + \sum_{i=n-d+1}^n a \left(1 + \frac{\lambda_{in}}{\lambda_d} \sum_{j=0}^{i-n+d-1} w^{k-n-1-j} \right) + \sum_{i=n+1}^k cw^{k-i} + e \right]^{-1} & n = k-d \\ \left[a \cdot f + \sum_{i=n-d+1}^{n-1} ab_i + aE_n + \sum_{i=n+1}^{k-d} cE_i + \sum_{i=k-d+1}^k cw^{k-i} + e \right]^{-1} & n = k-2d+1 \\ \left[a \cdot f + \sum_{i=n-d+1}^n ab_i + \sum_{i=n+1}^{k-d} cE_i + \sum_{i=k-d+1}^k cw^{k-i} + e \right]^{-1} & n = k-2d \\ \left[a \cdot f + \sum_{i=n-d+1}^n ab_i + \sum_{i=n+1}^{k-2d} c \left(w^{-i} b_i + (1-w^{-1}) \sum_{j=1}^{i-n-d} w^{i-n-d-j} E_{n+d+j} \right) + \sum_{i=k-2d+1}^{k-d} cE_i + \sum_{i=k-d+1}^k cw^{k-i} + e \right]^{-1} & n < k-2d \end{cases} \quad (8)$$

$$A_s = \sum_{i=0}^k \pi_{i,0} = \begin{cases} \left(f + \sum_{i=n-d+1}^{k-2d} b_i + \sum_{i=k-2d+1}^{n-1} \left(E_n - \frac{\lambda_m}{\lambda_d} \cdot \sum_{j=0}^{n-i-1} w^{k-n-d+j} \right) + E_n + \sum_{i=n+1}^{k-d} E_i + \sum_{i=k-d+1}^k w^{k-i} \right) \pi_{0,0} & k-2d+1 < n < k-d \\ \left(f + \sum_{i=n-d+1}^{k-d} \left(1 + \frac{\lambda_m}{\lambda_d} \sum_{j=0}^{i-n+d-1} w^{k-i-d+j} \right) + (n-k+d)w^{k-n} + \sum_{i=n+1}^k w^{k-i} \right) \pi_{0,0} & n > k-d \\ \left(f + \sum_{i=n-d+1}^n \left(1 + \frac{\lambda_m}{\lambda_d} \sum_{j=0}^{i-n+d-1} w^{k-n-1-j} \right) + \sum_{i=n+1}^k w^{k-i} \right) \pi_{0,0} & n = k-d \\ \left(f + \sum_{i=n-d+1}^{n-1} b_i + E_n + \sum_{i=n+1}^{k-d} E_i + \sum_{i=k-d+1}^k w^{k-i} \right) \pi_{0,0} & n = k-2d+1 \\ \left(f + \sum_{i=n-d+1}^n b_i + \sum_{i=n+1}^{k-d} E_i + \sum_{i=k-d+1}^k w^{k-i} \right) \pi_{0,0} & n = k-2d \\ \left(f + \sum_{i=n-d+1}^n b_i + \sum_{i=n+1}^{k-2d} \left(w^{-i} b_i + (1-w^{-1}) \sum_{j=1}^{i-n-d} w^{j-n-d-j} E_{n+d+j} \right) + \sum_{i=k-2d+1}^{k-d} E_i + \sum_{i=k-d+1}^k w^{k-i} \right) \pi_{0,0} & n < k-2d \end{cases} \quad (9)$$

where

$$\begin{cases} a = 1 + \frac{\lambda_m}{\mu_m}, \quad b_i = 1 + \frac{\lambda_m}{\lambda_d} \sum_{j=1}^{i-n+d} E_{n+j}, \quad c = 1 + \frac{\lambda_m}{\mu_m} + \frac{\lambda_m}{\mu_m} \\ e = \frac{\lambda_d}{\mu_R}, \quad f = n-d+1 \end{cases} \quad (10)$$

3.4 The state probabilities and steady-state availability under both system deterioration failure and Poisson failure

Suppose the device also experiences Poisson failures with rate λ_{pf} [6]. Upon the occurrence of a Poisson failure, a minimal repair is carried out which restore the device to the state it was in just before the failure. To the Markov chain model in Fig.1, one Poisson failure state $s(i,3)$ is added corresponding to each state $s(i,0)$, with failure rate λ_{pf} and repair rate μ_r . Then we have

$$\pi_{i,3} = \frac{\lambda_{pf}}{\mu_r} \pi_{i,0}, \quad 0 \leq i < k \quad (11)$$

The sum of all state probabilities equals to 1:

$$\sum_{i=0}^k \pi_{i,0} + \sum_{i=0}^k \pi_{i,1} + \sum_{i=n+1}^k \pi_{i,2} + \sum_{i=0}^k \pi_{i,3} + \pi_F = 1 \quad (12)$$

The closed-form solution of $\pi_{0,0}$ and steady state availability A_s can be shown as (8)(9), and all parameters are as same as (10) except

$$a = 1 + \frac{\lambda_m}{\mu_m} + \frac{\lambda_{pf}}{\mu_r}, \quad c = 1 + \frac{\lambda_m}{\mu_m} + \frac{\lambda_m}{\mu_m} + \frac{\lambda_{pf}}{\mu_r}. \quad (13)$$

4. Optimization

In reality, maintenance effect may depend on the maintenance conditions and practical demands. The frequency of inspection and maintenance would influence system steady-state availability and cost rate per unit time. Therefore, joint optimization of inspection rate and the threshold value should be investigated for the maximization of system steady-state availability, i.e. finding (λ_m^*, n^*) to maximize $A_s = f(\lambda_m, n)$. Equation (8) (9) need be utilized iteratively to acquire the optimal values (λ_m^*, n^*) .

Example: Considering an engine subject to both system deterioration failure and Poisson failure as a hypothetical example. The deterioration of the engine is classified into five stages where the last stage is a failed stage. It is observed that the mean time between two successive deterioration stages is 333.3 hours when no maintenances performed. The mean time to inspect and analyze engine condition is about 0.5 hour. On average, the corrective maintenance, preventive major maintenance and preventive minimal maintenance take 40 hours, 5 hours and 2 hours, respectively. The mean time between two successive Poisson failures is 33.3 hours, the minimal repair takes 1 hours. Currently, condition monitoring is performed on average once in every $1/\lambda_m$ hour. Preventive maintenance is initiated when the condition monitoring identifies the engine deteriorated as beyond the n th level. The preventive maintenance restores the engine to the stage $i-2$, supposing it is in the stage i before maintenance. Hence, the parameters of the system are as follows.

$$k = 5, \quad \lambda_d = 0.003, \quad \mu_m = 2, \quad \mu_R = 0.025, \quad \lambda_{pf} = 0.03, \quad \mu_r = 1, \quad \mu_M = 0.2, \quad \mu_n = 0.5, \quad d = 2$$

Therefore, from (8)(9), we have $A_s = 0.95748$, and the optimal inspection rate and the threshold value is $\lambda_m^* = 0.0108$, $n^* = 2$, as shown in Fig. 2.

5. Conclusion

In this paper, we have presented a condition-based maintenance model for the imperfect maintenance under both system deterioration failure and Poisson failure, which is described by the continuous time Markov chain. Also, we have shown polynomial equation solve the optimal inspection rate and the threshold value for the maximization of system steady-state availability. In our future works, we will analyze the inspection policies for the preventive maintenance, e.g. periodical inspection and sequential inspection.

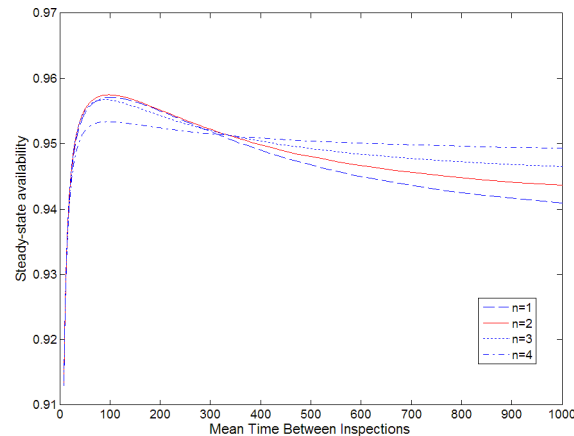


Figure 2. The relationship between steady-state availability and Mean Time Between Inspections.

6. References

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