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Reliability of A Generalized Two-dimension System

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Abstract. A generalized two-dimension system consists of $m \times n$ components, and fails if and only if k or more components in any pattern are failed. This system can be applied to many actual engineering systems, such as temperature monitoring systems for important chemical reactor, video monitoring systems etc. All existing consecutive-(r,s)-out-of-(m,n) system and r-within-consecutive-(r,s)-out-of-(m,n) system are special cases of this new system. An effective method is proposed for obtaining the system reliability by using Monte Carlo methods. Three numerical examples based on different patterns are given to show that the proposed method is effective and flexible.

Keywords: reliability, generalized two-dimension system, monte carlo methods.

1. Introduction

The consecutive-k-out-of-n: F system [1] is a famous reliability system. The characteristics of this system are high reliability and low costs. So many researches about its reliability were published [2-4]. Salvia & Lasher [5] firstly extended the consecutive-k-out-of-n: F system to two-dimension situation in 1990s. The first two-dimension system was named as k^2/n^2: F system, which contains n^2 components in a square, fails if and only if a sub-square consisting of exact k^2 failed components is appeared ($2 \le k \le n-1$). After that, Boehme et al. [6] proposed a more general two-dimension system. It is the linear connected-(r,s)-out-of-(m,n): F system, which consists of m rows and n columns components. Its failure criterion is that at least r ($1 \le r \le m$) rows and s ($1 \le s \le n$) columns components are failed in the system. Another more general two-dimension system is the k-within-consecutive-(r,s)-out-of-(m,n): F system, which was proposed by Papastavridis and Koutras [7]. This system consists of m rows and n columns components ordered in a rectangular region. Its failure criterion is that there are at least k ($1 \le k \le rs$) failed components in a subregion with r ($1 \le r \le m$) rows and s ($1 \le s \le n$) columns. Engineering concrete instances of above two-dimension systems include alarm systems [8], liquid crystal screen of a computer [9] etc.

In order to obtain the exact reliability of two-dimension systems, some researches were published. By using finite Markov chain imbedding approach, Zhao & Cui [10-12] constructed exact reliability formulas for some special two-dimension systems. By using the SDP method, Zuo[13] presented the reliability formulas of general cases. A YM algorithm which is more efficient than the SDP method is proposed by Yamamoto & Miyakawa[14] to obtain the reliability of the linear connected-(r,s)-out-of-(m,n): F system. Lin and Zuo [15] proposed a recursive algorithm for the exact reliability of the k-within-consecutive-(r,s)-out- of-(m,n): F system. By using the finite Markov chain imbedding approach, Zhao *et al.*[16] proposed a unified formula for the reliability of the linear connected-(r,s)-out-of-(m,n): F system, Chang and Huang [16] propose efficient methods to evaluate the reliability of k-within-consecutive-(r,s)-out- of-(m,n): F system. Obviously, the finite Markov chain imbedding approach is a efficient method for evaluating reliability of two-dimension systems.

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Because it is difficult to obtain the exact reliability of two-dimension systems, many researches[17-18] focused on proposing some methods to get the bounds of the two-dimension system reliability.

A review about the studies and new developments of two-dimension systems can be obtained from Yamamoto & Akiba[19].

In this paper, a more generalized two-dimension system is proposed. The new system consists of $m \times n$ components which ordered in a region with m rows and n columns, and fails if and only if k or more components in a sub-region with any pattern $\mathbf{\Phi}$ are failed. It is named as k-within-consecutive-$\mathbf{\Phi}$-out-of-$(m,n):F$ system. For example, $k=14$, $m=10$, $n=12$, $\mathbf{\Phi}$ is a 6×6 matrix with a gap (2×2 matrix) in the top left corner. If the state of the system is as figure 1, it is failed. If the state of the system is as figure 2, it is working. In this paper, An effective method is proposed for obtaining the system reliability by using Monte Carlo methods. Finally, three numerical examples based on different patterns are given to show that the proposed method is effective and flexible.

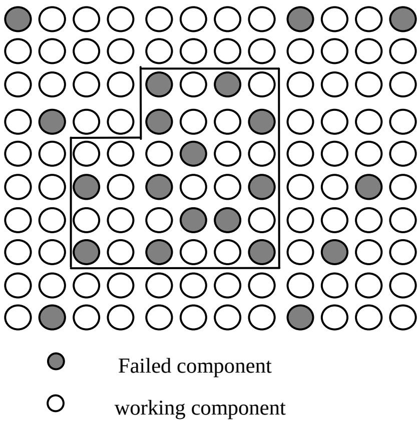

Fig. 1: Failed system

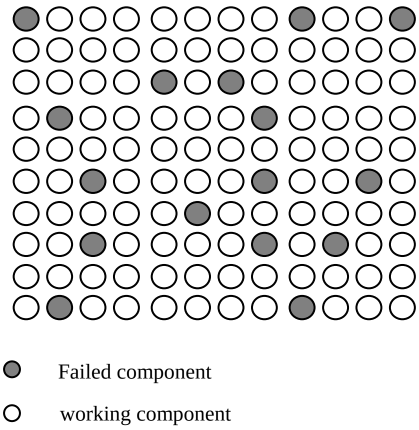

Fig. 2: Working system

2. Notation

In order to make the analysis easy to describe, some notations are given as follows.

(i, j): component located on the *i*-th row and the *j*-th column.

X_{ij}: the state of component (i, j). $X_{ij}=0$, if component (i, j) is working; $X_{ij}=1$, if component (i, j) is failed.

$p_{ij} \equiv 1-q_{ij}$: the reliability of component (i, j), that is to say $p_{ij}=P(X_{ij}=0)$.

$N_{(r,s)}^{(i_1,j_1)} = \prod_{i=i_1}^{i_1+r-1} \prod_{j=j_1}^{j_1+s-1} X_{ij}$: the number of failed components in a region with r rows and s columns (component (i_1, j_1) is at the top left corner), where $1 \le i_1 \le m-r+1$, $1 \le j_1 \le n-s+1$.

$S_{(m,n)}$: the state of system. $S_{(m,n)} = 1$, if the system is failed; $S_{(m,n)} = 0$, if the system is working.

3. Approximation of Reliability

3.1. Assumptions

a) Each component and the system either function or fail.

b) All components are mutually statistically independent.

3.2. Algorithm description

3.2.1. Inputting parameters

In this step, the parameters about the system construction m and n, failure criterion $\mathbf{\Phi}$, component reliability p_{ij}, iteration count $iternum$, Num_iter=0, Num_fs=0 etc. should be input.

3.2.2. Generating a system

a) Generating one component states

For each component (i, j) of system, generating an uniform random variable v_{ij}. The component state $X_{ij} = 0$, if $v_{ij} \le p_{ij}$. The component state $X_{ij} = 1$, if $v_{ij} > p_{ij}$.

b) Changing component number

If $i \le m$ and $j < n$, $i = i$ and $j = j + 1$; if $i < m$ and $j = n$, $j = 1$ and $i = i + 1$. Return to step *a)*. Recording the component states by using an element of matrix with m rows and n columns.

if $i = m$ and $j = n$, turn to step *3)*.

3.2.3. Judging the system state

Judging the system state according to failure criterion $\mathbf{\Phi}$. For example, because the failure criterion of two-dimension system in Fig.2 is that if and only if $k = 14$ or more components in a region which is a 6×6 matrix with a gap (2×2 matrix) in the top left corner and the following inequality is tenable $\max\limits_{\substack{1 \le i_1 \le m-r+1 \\ 1 \le j_1 \le n-s+1}} (N_{(6,6)}^{(i_1, j_1)} - N_{(2,2)}^{(i_1, j_1)}) < k$, the system state $S_{(m,n)} = 0$. because the following inequality is tenable $\max\limits_{\substack{1 \le i_1 \le m-r+1 \\ 1 \le j_1 \le n-s+1}} (N_{(6,6)}^{(i_1, j_1)} - N_{(2,2)}^{(i_1, j_1)}) \ge k$ in Fig.1, the system state $S_{(m,n)} = 1$.

3.2.4. Recording some frequencies

Current iteration number Num_iter= Num_iter+1; failed system number Num_fs= Num_fs+S(m,n).

3.2.5. Repeating step 2), 3) and 4)

Repeating step 2), 3) and 4) until Num_iter=iternum.

3.2.6. Computing approximation of reliability

Approximation of system reliability can be obtained according to (1).

$$\text{Reliability} = 1 - \frac{\text{Num\_fs}}{\text{Num\_iter}} \tag{1}$$

In order to show above algorithm more clearly, the procedure for obtaining the reliability approximation of a k-within-consecutive-(r,s)-out-of-(m,n): F system is shown as Fig.3.

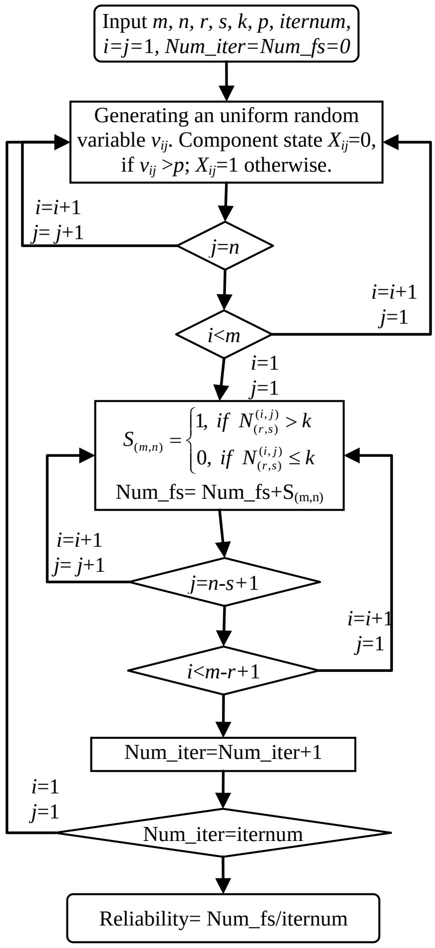

Fig. 3: Procedure for obtaining the reliability approximation of a k -within-consecutive-(r,s)-out-of-(m,n): F system

4. Numerical Results and Discussions

To show the performance of our proposed algorithm, we provide some numerical results which were obtained from our Matlab programs. Example 1 shows that our algorithm is effective for k -within-consecutive-(r,s)-out-of-(m,n): F system with i.i.d. components. Example 2 shows that our algorithm is effective for k -within-consecutive- (r,s) -out-of- (m,n) : F system with independent but no-identical distributed components. Example 3 shows that our algorithm is effective for two-dimension system with any failure pattern. All numerical examples in this paper are carried out on a computer (2.93GHz 2 Duo CPU, 2.00 GB memory, Window 2003) in Matlab 7.9.0.

4.1. Example 1: k -within-consecutive-(r,s)-out-of-(m,n): F system with i.i.d. components

For a k -within-consecutive- (r,s) -out-of- (m,n) : F system with i.i.d. components, reliability approximations with different iteration count are as show in Fig4, where k=5, r=4, s=2, m=10, n=100, p=0.90. The middle break represents the real value of system reliability. It can be seen obviously that reliability approximations gradually approach the real reliability value with the increasing of iteration count.

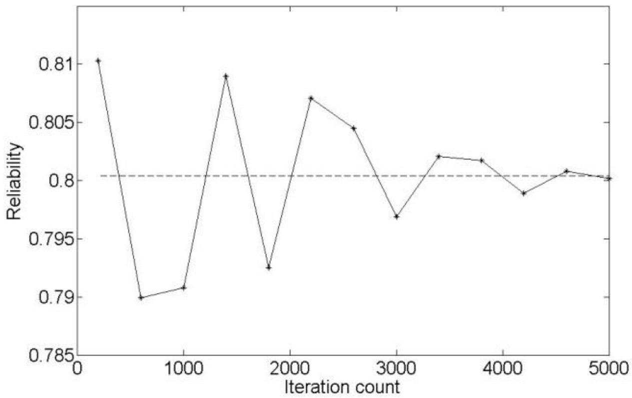

Fig. 4: Reliability approximation with different iteration count

4.2. Example 2: k -within-consecutive-(r,s)-out-of-(m,n): F system with independent but no-identical distributed components

For a k -within-consecutive-(r,s)-out-of-(m,n): F system with independent but no-identical distributed components, reliability approximations with different iteration count are as show in TABLE Ⅰ, where k=3, r=2, m=3, $p_{ij} = 1 - 0.0015[10(i-1) + j]$. It can be seen obviously that our proposed algorithm is effective.

4.3. Example 3: k -within-consecutive-Φ -out-of-(m,n): F system

For a k -within-consecutive-Φ -out-of-(m,n): F system with k=22, m=10 n=10 p=0.8, where Φ is a 6×6 matrix with a gap (2×2 matrix) in the top left corner, to obtain the approximation reliability computing time is less than 10 seconds. So our proposed algorithm is efficient.

Tab. 1: Comparison of real value and approximation under the situtaion of independent but no-identical distributed

		s=2	s=3	s=4	s=5
n=10	Approximation	0.9990	0.9960	0.9930	0.9895
	Real value	0.9989	0.9962	0.9928	0.9891
n=50	Approximation	0.5680	0.2060	0.0745	0.0355
	Real value	0.5689	0.2078	0.0767	0.0314

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