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Modeling on Condition Mornitoring of Cigarette Manufacture Equipment

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Abstract. Vibration monitoring is an effective means of condition monitoring, which is importance to manufacture. But some times, unqualified cigarette is produced because of an undetected abnormal condition. So condition monitoring based on the cigarette quality is necessary for cigarette manufacture equipment. Traditional condition test models include CSTF, CSCF, TSTF, TSCF. In this paper, a new model for condition monitoring of cigarette manufacture equipment is proposed, which is more efficient. The distribution function, expectation of test length, and the probability of state are given by using the finite Markov chain imbedding approach. In order to improve the efficiency of condition monitoring, a process for obtaining the optimal parameters of model is proposed.

Keywords: condition mornitoring, cigarette manufacture equipment, markov chain.

1. Introduction

Vibration monitoring is an effective means of condition monitoring for cigarette manufacture equipment. But some times, unqualified cigarette is produced although vibration monitoring result is normal. The most obvious reason for this phenomenon is that there are some undetected abnormal conditions. These undetected abnormal conditions of cigarette manufacture equipment can be found by cigarette quality test. So condition monitoring based on the cigarette quality test is necessary for cigarette manufacture equipment.

Start-up demonstration test model can be used to monitor the condition of cigarette manufacture equipment. Each quality test can be seen as a start-up test. The first start-up test model was given by Hahn & Gage [1]. After that, The CSTF start-up test model was proposed and studied by Balakrishnan and Chan [2], Govindaraju and Lai [3], and Martin [4], in which the practitioner accepts the equipment if k consecutive successes are observed prior to a total of f failures, and rejects the equipment otherwise. Smith and Griffith [5-7] analyzed the CSTF start-up test model by using the finite Markov chain imbedding methodology. It is shown that the probability mass function, distribution function, mean and variance of the test length, the probability of acceptance and rejection can be obtained easily by this method. Additionally, research showed that the finite Markov chain imbedding approach could be readily adapted to the non-i.i.d. case. They also investigated procedures to determine the optimal parameters k and f for the test model and procedures to estimate the probability p of a start-up test being success.

Two new tests named the TSTF and CSCF tests were introduced by Smith and Griffith [8], in which the acceptance/rejection criteria are based on consecutive or total successes and failures. Same analysis was performed for these tests by using the finite Markov chain imbedding methodology. In addition, their researches show that each of TSTF, CSTF, and CSCF test models is beneficial with no overall best test.

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In this paper we will propose a new type of quality test model that are improved and more sophisticated to allow for more precise monitor the actual situation of the cigarette manufacture equipment. Additional, we will give analytical expressions for the acceptance and rejection probabilities, the probability mass function, the distribution function and the mean and the variance of the test length by using the finite Markov chain imbedding methodology. It makes the computation of all above indexes easy.

2. Modeling and Analysis

2.1. Modeling

The cigarette manufacture equipment under test is regarded as qualified, if either k_{cq} consecutive qualified cigarette quality tests or k_q qualified ones are observed before the occurrence of either k_u unqualified or k_{cu} consecutive unqualified cigarette quality tests. The procedure terminates if either it is judged qualified or unqualified. This new test model is named CQTQCUTU. Obviously, the total number of tests till termination is of interest. Let Z be the random variable presenting the total number of tests.

Given the probability of qualified of each test result (p), it has been observed that an intelligent choice of the k_q, k_{cq}, k_u, k_{cu} parameters may lead to a reduction in the expected number of tests without an undesired change of the probability of acceptance.

2.2. Analysis

Consider the Markov Chain $\{X_n\}$ with state space

$$\left\{\begin{matrix}(Q_C, Q_T, U_C, U_T): 0 \le Q_C < k_{cq}, \\ 0 \le Q_T < k_q, 0 \le U_C < k_{cu}, 0 \le U_T < k_u\end{matrix}\right\} \cup \{AS_Q\} \cup \{AS_U\}$$

where $X_n = (Q_C, Q_T, U_C, U_T)$ means that after the n^{th} cigarette test there are last consecutive Q_C cigarette quality test results are qualified, total Q_T cigarette quality test results are qualified, last consecutive U_C cigarette quality test results are unqualified, total U_T cigarette quality test results are unqualified. AS_Q means that after the n^{th} cigarette test there are either k_{cq} consecutive qualified cigarette quality tests or k_q qualified ones. AS_U means that after the n^{th} cigarette test there are either k_u unqualified or k_{cu} consecutive unqualified cigarette quality tests.

The transition probabilities are of the form

$$P\left(X_n = (Q_C + 1, Q_T + 1, 0, U_T) \middle| X_{n-1} = (Q_C, Q_T, U_C, U_T)\right) = p, \text{ if } Q_C < k_{cq} - 1 \text{ and } Q_T < k_q - 1;$$

$$P\left(X_n = (0, Q_T, U_C + 1, U_T + 1) \middle| X_{n-1} = (Q_C, Q_T, U_C, U_T)\right) = 1 - p, \text{ if } U_C < k_{cu} - 1 \text{ and } U_T < k_u - 1;$$

$$P\left(X_n = AS_Q \middle| X_{n-1} = (Q_C, Q_T, U_C, U_T)\right) = p, \text{ if } U_C = k_{cu} - 1 \text{ or } U_T = k_u - 1;$$

$$P\left(X_n = AS_U \middle| X_{n-1} = (Q_C, Q_T, U_C, U_T)\right) = 1 - p, \text{ if } U_C = k_{cu} - 1 \text{ or } U_T = k_u - 1;$$

$$P\left(X_n = AS_Q \middle| X_{n-1} = AS_Q\right) = 1;$$

$$P\left(X_n = AS_U \middle| X_{n-1} = AS_U\right) = 1;$$

For other situation, the transition probabilities are zeros. Based on above transition rules, we can easily obtain the one-step probability transition matrix **P**.

We now make a more general approach using the Markov chain methodology. This will produce a general approach to finding acceptance/rejection probabilities and also to finding the probability mass function and distribution function of the test length, and the mean and variance of the test length. This works for each of the Markov chains described in the previous section. For each Markov chain there are absorbing states, which correspond to the termination of the quality test. Let A denote the set of absorbing states and $|A|$ denote the number of absorbing states. In fact, two sets consisting of each of these absorbing states are recurrent classes. The remaining states are transient which we will denote by T and likewise the number of transient states by

$|T|$. Written in canonical form, the one-step transition probability matrix P for the Markov chain is $\begin{bmatrix} \mathbf{P}_1 & \mathbf{0} \\ \mathbf{R} & \mathbf{Q} \end{bmatrix}$, where $\mathbf{P}_1$ is the $|A| \times |A|$ identity matrix for the absorbing states, $\mathbf{R}$ is a $|T| \times |A|$ matrix containing the one-step probabilities of the transient states to the recurrent (absorbing) states, $\mathbf{Q}$ is a $|T| \times |T|$ matrix containing the one-step probabilities among the transient states, and $\mathbf{0}$ is the $|A| \times |T|$ zero matrix. The one-step probabilities of $\mathbf{R}$ and $\mathbf{Q}$ are determined by the transition probabilities given for each test. The first row of $\mathbf{Q}$ contains the one step transition probabilities from state (0,0,0,0).

To compute the test length, we will define the following notations, each of i and j refer to an ordered pair. Let,

$\mathbf{I}_{|T|\times|T|}$ = identity matrix of dimension $|T| \times |T|$,

$\mathbf{M}_{|T|\times|T|} = (\mathbf{I}_{|T|\times|T|} - \mathbf{Q}_{|T|\times|T|})^{-1}$- the fundamental matrix of dimension $|T| \times |T|$,

$\mathbf{e}_m$ = column vector of length t where the m^{th} element is one and the remaining elements are zero.

$\mathbf{e}_m^{'}$ is defined to be the transpose of $\mathbf{e}_m$,

$\mathbf{u}_{\{RS\}}$ = column vector where all the elements corresponding to the rejection states are one, and the remainder of the elements are zero.

$\mathbf{1}_z$ = column vector of ones of length z

N_{ij} = random variable that represents the number of times the process visits state j before it eventually enters a recurrent state, having initially started from state i $(i, j \in T)$.

$\mu_{ij} = E(N_{ij})$ for $i, j \in T$.

$\mathbf{M}_\rho = \left[\sum_{j\in T} \mu_{ij} \right] = \mathbf{M}\mathbf{1}_{|T|}$ = column vector such that the m^{th} element is the sum of the m^{th} row of $\mathbf{M}$.

$\mathbf{M}_{\rho^2} = \left[\left(\sum_{j\in T} \mu_{ij} \right)^2 \right]$ = diag $(\mathbf{M}_\rho)\mathbf{M}_\rho$ - column vector such that the m^{th} element is the square of the sum of the m^{th} row of $\mathbf{M}$. Note: diag $(\mathbf{M}_\rho)$ is a diagonal matrix whose entries are the corresponding entries of $\mathbf{M}_\rho$.

Consider a quality test for i.i.d. Bernoulli tests with constant probability of success p and with the first transient state being the initial state.

The expected value of the test length Y is

$$E(Y) = \mathbf{e}_1^{'}\mathbf{M}\mathbf{1}_t. \tag{1}$$

The variance of Y is

$$\text{Var}(Y) = \mathbf{e}_1'\left[(2\mathbf{M} - \mathbf{I})\underline{\mathbf{M}}_\rho - \underline{\mathbf{M}}_{\rho^2}\right]. \tag{2}$$

The probability mass function of Y is

$$P(Y = m) = \mathbf{e}_1^{'}\mathbf{Q}^{m-1}\mathbf{R}\mathbf{1}_a. \tag{3}$$

The distribution function of Y is

$$P(Y \le m) = \mathbf{e}_1'\sum_{i=0}^{m-1}\mathbf{Q}^i\,\mathbf{R}\,\mathbf{1}_a \text{ where } \mathbf{Q}^0 = \mathbf{I}. \tag{4}$$

The probability of rejection and acceptance are given by

P(test result is qualified) $= \mathbf{e}_1^{'}\mathbf{M}\mathbf{R}\mathbf{u}_{\{RS\}}$

$$= 1 - \text{P(test result is unqualified)}. \tag{5}$$

The proofs to (1)-(5) are given in Bhat [9].

3. Optimization for Model

The test length depends on the probability of success of each test and on the choice of the various parameters k_{cq} , k_q , k_u , k_{cu} . The cigarette equipment should be regarded as qualified if the probability of cigarette equipment producing qualified cigarette (p) will be higher than some specific value p_U and it should be regarded as unqualified if the probability of cigarette equipment producing qualified cigarette (p) is lower than some initially set value p_L . Let α , β be the commonly known confidence limits. Using any of the procedures, it is thus required that

$$P\left(\text{equipment is regarded as qualified} \middle| p = p_U\right) > 1 - \beta$$

$$P\left(\text{equipment is regarded as qualified} \middle| p = p_L\right) < \alpha$$

Thus, a constrained optimization problem is set up: Find the values of k_{cq} , k_q , k_u , k_{cu} that will minimize the first moment E{Z} for p= p_U subject to the constraints on confidence level. A numerical example will be presented further.

4. Numerical Examples

In this section, an example will be given to show that our proposed analytical expression for related index and procedure for optimizing model is effective.

A producer wishes to establish a CQTQCUTU test to provide an aid of the decision-making in station monitoring for cigarette manufacture equipment. The test model should satisfy the following two goals:

a) If the probability of cigarette equipment producing qualified cigarette is greater or equal to 0.9, it should be judged qualified in probability at least 0.9, that is to say $P\left(\text{equipment is regarded as qualified} \middle| p > 0.95\right) > 0.9$;

b) If the probability of cigarette equipment producing qualified cigarette is less than or equal to 0.65, it should be rejected in probability at least 0.6, that is to say $P\left(\text{equipment is regarded as qualified} \middle| p < 0.65\right) < 0.4$.

By using proposed optimization procedure for model in section Ⅲ, it can be found that $k_{cq} = 5$, $k_q = 10$, $k_u = 2$, $k_{cu} = 3$ is a optimal parameter combination. Because

$$P\left(\text{equipment is regarded as qualified} \middle| p > 0.95\right) > 0.9804$$

$$P\left(\text{equipment is regarded as qualified} \middle| p < 0.65\right) < 0.2782$$

Otherwise, by using the formulas which introduced in Section Ⅱ, the distribution of the test length with $k_{cq} = 5$, $k_q = 10$, $k_u = 2$, $k_{cu} = 3$, $p = 0.95$ is as show in Fig 1.

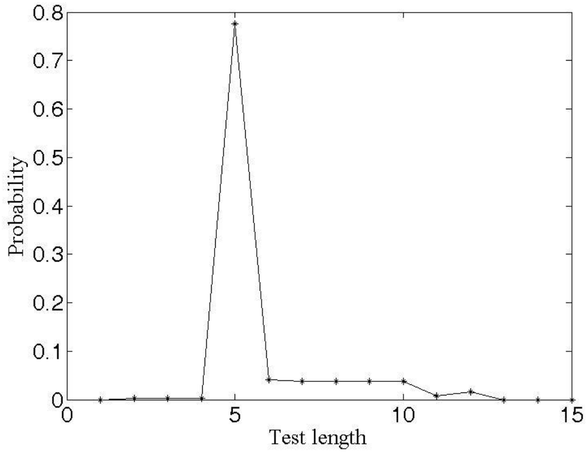

Fig. 1: Distribution of the test length with $k_{cq} = 5$, $k_q = 10$, $k_u = 2$, $k_{cu} = 3$, $p = 0.95$

The distribution of the test length with $k_{cq} = 5$, $k_q = 10$, $k_u = 2$, $k_{cu} = 3$, $p = 0.65$ is as show in Fig 2.

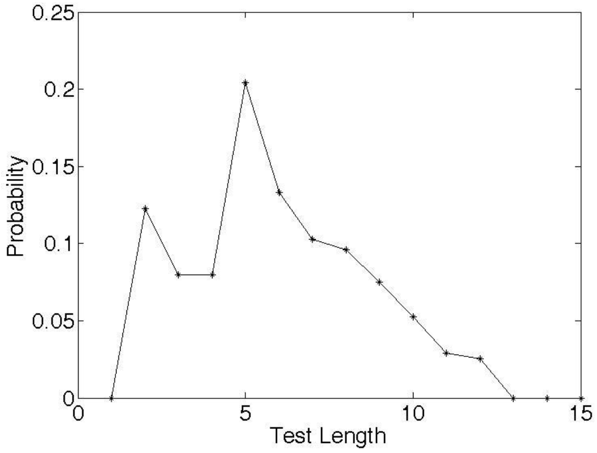

Fig. 2: Distribution of the test length with $k_{cq} = 5$, $k_q = 10$, $k_u = 2$, $k_{cu} = 3$, $p = 0.65$

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