

## ACO-based Joint Distribution: A Feasibility Study

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**Abstract.** Joint distribution not only can lower the cost and increase efficiency through resource sharing, but also can improve the heavy traffic and reduce pollution. In this paper, we consider a location routing problem (LRP) with time window and solve it by ant colony optimization (ACO). We combine the distribution distance and total cost before and after the joint distribution, and find that, the ACO-based plan is more effective and cost saving.

**Keywords:** LRP; ACO; Joint distribution

### 1. Introduction

In an ever more demanding society, having customers less and less willing to wait for the products they want to acquire, decisions concerning the location of distribution centers (DC) and tracing of distribution routes are a central problem, having implications on the complete supply chain<sup>[1]</sup>. The transportation cost and the service time should be considered in the location-distribution system.

This paper considers two-level distribution system: There are a set of customers and potential distribution centers (DCs) be represented by points on the plane. Each customer has a certain demand (units of load); the location cost of each DC is known, as well as the unitary cost of distribution. The vehicles have a certain capacity (units of load). A time interval or time window is also associated with each customer to constrain the time of service. This problem can be seen as a Location-Routing Problems with time windows. The purpose of this LRP is to choose the DCs that must be opened and to draw the routes from these DCs to the customers, having as an objective the minimization of the total cost (location and distribution costs).

We want to solve several problems as follows.

- A. *Minimize the transportation cost by using the ant colony algorithm.*
- B. *Choose the DCs from potential distribution centers*
- C. *Improve customer satisfaction by considering the time windows.*
- D. *Consider multi-product distribution by transforming the volume and weight into the same criterion.*
- E. *Solve the multi-objective planning problems by changing some setting of the algorithm.*

A LRP, generally speaking, can be assimilate to a Vehicle Routing Problem in which the optimal number and location of the facilities are simultaneously determined with the vehicles scheduling and the circuits (route) release so to minimize a particular function[2].

LRP is NP-hard since it is constituted by two NP-hard problems (the classical location problem and the vehicle routing problem). In fact, both of the latter problems can be viewed as special cases of the LRP. If we

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require all customers to be directly linked to a depot, the LRP becomes a standard location problem. If, on the other hand, we fix the depot locations, the LRP reduces to a VRP [3].

In the literature, the LRP is solved by two methods. One is the exact solution method, which can only tackle relatively small instances due to the complexity of location-routing. Another is the heuristic solution methods, which is usually classified to clustering-based, iterative and hierarchical heuristics.

In recent years metaheuristics are increasingly used to solve LRPs. These include tabu search, simulated annealing, genetic algorithm and so on. Ant colony optimization has been successfully applied to some classic compounding optimization problems, e.g. travelling salesman, telecommunication routing, etc. Recently, it has also been applied to solve VRP or VRPTW.

This paper aims to test the feasibility of ACO in LRP with Time Windows. The remainder of the paper is organized as follows. In Section 2 we describe LRP with time windows. In Section 3, ACO and some improvement strategies are presented. In Section 4, some computational results are discussed and lastly, the conclusions are provided in Section 5.

## 2. Problem Description

### 2.1 Problem Background

This problem is proposed based on the real world which a group company has five kinds of goods to deliver to the retails. Those retails are mostly same.

At present, the distribution of each good is outsourced to different third-party logistics companies. But the group company considers that the integration will be more effective and cost saving. Therefore, the group company wants to implement the joint distribution for all the goods and outsource the distribution to only one third-party logistics company. But it faced a problem that how to integrate all the distribution network and how to choose the best distribution center. In addition, in the real world, every retail has a time limit that the service at any customer starts within a given time interval, called a time window. Time windows are called soft when they can be considered non-biding for a penalty cost. They are hard when they cannot be violated, i.e., if a vehicle arrives too early at a customer, it must wait until the time window opens; and it is not allowed to arrive late [4]. This is the case we consider here. This situation is similar to the location-routing problem with hard time windows.

### 2.2 Problem Definition

A set of consumers and potential facility is given. If  $d_i$  is the demand of a consumer, each consumer with  $d_i > 0$  must be allocated to a facility so to completely satisfy  $d_i$ . The consignment is delivered through vehicles that depart from a facility and operate on circuits that include more customers. The set-up cost of a center and the unitary distribution cost have been fixed. The vehicles and the potential centers have limited capacity. Facilities location and vehicles routes have to be determined so to minimize the overall costs (location and distribution costs).

The problem is bound by the followings conditions:

- 1) *The demand of each customer must be satisfied;*
- 2) *Each customer must be served by a single vehicle;*
- 3) *Each route begins and ends to the same facility;*
- 4) *The time windows of the customers and capacity constraints of the vehicles are observed.*

### 2.3 Mathematical Formulation

If  $D$  is the set of potential facilities, and  $D = \{1, 2, \dots, m\}$ ;  $I$  is the customers set, and  $I = \{m+1, m+2, \dots, m+n\}$ ;  $V$  the vehicles set, and  $V = \{1, 2, \dots, k\}$ ;  $N$  is set of all nodes, and  $N = D \cup I = \{1, 2, \dots, m+n\}$

The mathematical formulation of the problem is the following:

$$\min \sum_{r \in D} y_r G_r + \sum_{k \in V} \sum_{i \in N} \sum_{j \in N} X_{ijk} C_{ij} + \sum_{K \in V} F_K \sum_{i \in D} \sum_{j \in I} X_{ijk}$$

s.t:

$$\sum_{k \in V} \sum_{i \in N} X_{ijk} = 1, \forall j \in N, i \neq j \quad (2.1)$$

$$\sum_{i \in N} \sum_{j \in D} X_{ijk} \leq Q_k, \forall k \in V, i \neq j \quad (2.2)$$

$$\sum_{k \in V} \sum_{j \in D} Q_k X_{rjk} \leq V_r, \forall r \in D \quad (2.3)$$

$$\sum_{i \in N} X_{ijk} = \sum_{i \in N} X_{ijk}, \quad \forall k \in V, i, j \in N, i \neq j \quad (2.4)$$

$$\sum_{i \in D} \sum_{j \in I} X_{ijk} \leq 1, \forall k \in V \quad (2.5)$$

$$\sum_{i \in I} (x_{rik} + x_{ijk}) \leq 1 + z_{rj}, \forall r \in D, j \in N, k \in V \quad (2.6)$$

$$\sum_{j \in I} X_{rjk} \leq y_r, \forall k \in K, r \in D \quad (2.7)$$

$$a_i \leq w_i \leq b_i, \forall i \in I \quad (2.8)$$

$$w_j = \max(w_i + t_{ij} + s_i, a_j), \forall i \in I, \forall j \in I, k \in K, i \neq j \quad (2.9)$$

$$x_{ijk} = (w_i + t_{ij} + s_i) - w_j \leq 0, \forall i \in I, \forall j \in I, k \in K, i \neq j \quad (2.10)$$

$$X_{ijk} \in \{0,1\}, \forall i, j \in N, k \in V \quad (2.11)$$

$$y_r \in \{0,1\}, \forall r \in D \quad (2.12)$$

$$z_{ij} \in \{0,1\}, \forall i \in D, j \in N \quad (2.13)$$

$$U_{ik} - U_{jk} + (m+n)X_{ijk} \quad (2.14)$$

$$U_{ik} \geq 0, \forall i \in I, k \in V \quad (2.15)$$

$$\sum_{i \in N} X_{ijk} = 0, \forall k \in V \quad (2.16)$$

$$\sum_{k \in V} X_{rik} + z_r + z_i \leq 2, \forall r \in D, \forall i \in D \quad (2.17)$$

where:

$c_{ij}$  is edge  $i - j$  unit cost,;  $i, j \in N$ ;

$d_{ij}$  is distance between  $i$  and  $j$ ;

$G_r$  is the set-up cost for facility,  $r \in D$

$F_k$  is the set-up cost for vehicles;

$V_r$  is the capacity of the facility I;

$q_j$  is the demand of the customer  $j$ ;

$Q_k$  is the vehicle  $k$  capacity;

$a_i$  is the earliest time that customer  $i$  is ready to be serviced;

$b_i$  is the latest time that customer  $i$  is ready to be serviced;

$w_i$  is the real starting time that customer  $i$  begin to be serviced,  $w_i \in [a_i, b_i]$ ;

$s_i$  is the service time of customer I;

$t_{ij}$  is the travelling time from customer I to customer  $j$ ;

$x_{ijk} = 1$  if vehicle  $k$  goes from  $i$  node to  $j$  node;

$y_r = 1$  if a facility is set-up at node  $r$ ;

$z_{ij} = 1$  if customer  $j$  is allocated to facility I;

$U_{ik}$  is the auxiliary variable.

The flow chart of LRPTW is presented at Fig 1, in which the potential facilities and customer demand decide the location of the distribution center, and the variety of goods, delivery time and other limits determine the delivery routing and the loading of goods.

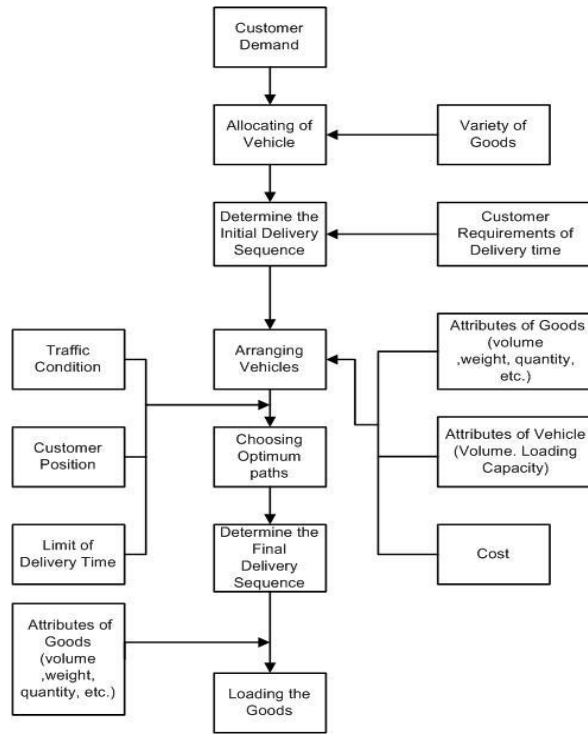


Fig. 1: Flow Chart of LRPTW

## 2.4 AN ACO-BASED HEURISTIC FOR THE LRPTW

### 1) Ant Colony Optimization (ACO)

The Ant algorithms are one of the most recent approximate optimization methods to be developed. These algorithms are inspired by the behavior of real ants in the wild [5], and more specifically, by the indirect communication between ants within the colony via the secretion of chemical pheromones.

Colomi, Dorigo and Maniezo proposed a metaheuristics technique: ants are procedures that build solutions to an optimization problem. According to how the solution space is explored some values are recorded in a similar way as pheromone acts, and objective values of solutions are associated with food sources. With time more pheromone is deposited on the more frequented trails. When constructing a VRP solution a move can be assigned a higher probability of being selected if it has previously led to a better solution in previous iterations.

At each iteration of the basic Ant Colony method, each ant builds a solution of the problem step by step. At each step the ant makes a move in order to complete the actual partial solution choosing between elements of a set  $A_k$  of expansion states, following a probability function. For each ant  $k$  probability  $P_k(i, j)$  of moving from present state  $i$  to another state  $j$  is calculated taking into account:

- Attractiveness  $\eta$  of the move according to the information of the problem.
- Level  $\tau$  of pheromone of the move that indicates how good the move was in the past.
- A  $Tabu_k$  list of forbidden moves.

In the ant algorithm original version formula for  $P_k(i, j)$  is:

$$P_k(i, j) = \begin{cases} \frac{[\tau(i, j)]^\alpha [\eta(i, j)]^\beta}{\sum_{(i,z) \notin Tabu_k} [\tau(i, z)]^\alpha [\eta(i, z)]^\beta} & \text{if } (i, j) \notin Tabu_k \\ 0, & \text{if } (i, j) \in Tabu_k \end{cases}$$

where  $\alpha$ ,  $\beta$  are parameters that are used to establish the relative influence of  $\eta$  versus  $\tau$ . After iteration  $t$  is complete, that is when all the ants have completed their solutions, the pheromone levels are updated to:

$$\tau(i, j) = \psi \tau(i, j) + \Delta \tau(i, j)$$

where  $\psi$  is a coefficient representing the level of pheromone persistence and  $\Delta \tau(i, j)$  represents the contributions of all ants that chose move  $(i, j)$  in their solution.

## 2) The frame of ACO for the LRPTW

The frame of ACO for the LRPTW is presented as follows.

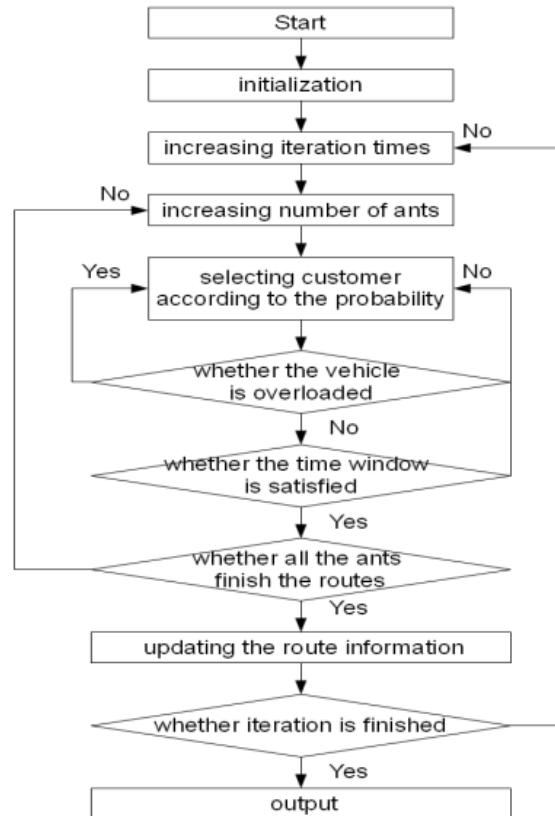


Fig. 2: The frame of Ant Colony Optimization

## 3. EXPERIMENT AND COMPUTATIONAL RESULTS

### 3.1 The Solomon instance

Because there is no suitable set of benchmark problems in the literature, we consider tested our algorithm on some benchmark instances. Solomon (1987)[6] proposed a set of 164 instances that have remained the leading test set ever since. We will test our algorithm by these instances, and then the performances will also comparable to our LRPTW.

The test sets reflect several structural factors in vehicle routing and scheduling such as geographical data, number of customers serviced by a single vehicle and the characteristics of the time windows. Customers are distributed within a  $[0,100]^2$  square instances. Customers are distributed within a  $[0,100]^2$  square. The instances are divided into 6 groups (test-sets) denoted R1, R2, C1, C2, RC1 and RC2. Each of the test sets contain between 8 and 12 instances. But here, we just choose the RC2 for our instance since the characteristics is more similar to our problem background which is described above. RC2 test-sets some customers are placed in clusters while others are placed randomly. In addition, RC2 has a long scheduling horizon allowing routes with more than 30 customers to be feasible which is also fix to our background.

### 3.2 Comparative analysis before and after optimization

The comparison of present way of distribution and the ACO-based joint distribution are shown in Fig 3 and Fig 4, in which the total distance of resent way is 1386 and the total distance of the ACO-based joint distribution is 830.0013. While the present way needs five vehicles and the ACO-based joint distribution needs three vehicles.

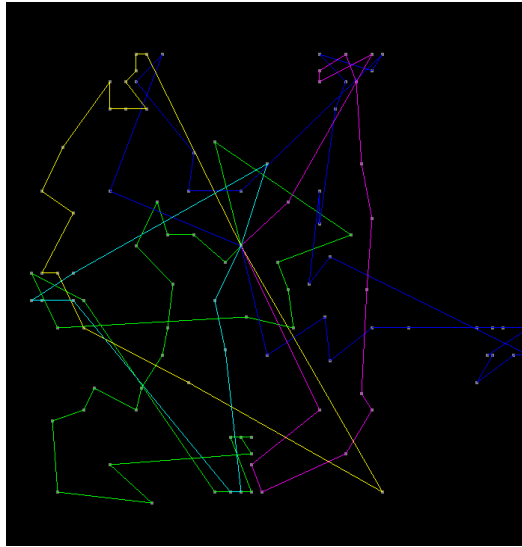


Fig. 3: The present way of distribution

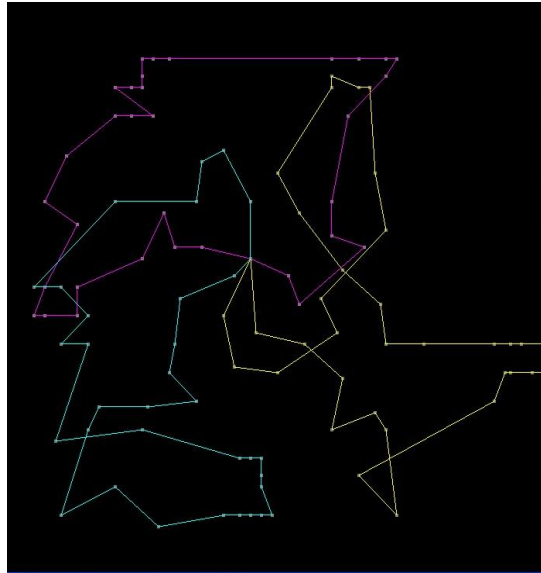


Fig. 4: The ACO-based joint distribution

By comparison, we can observe that the distribution distance is shorter if we implement the joint distribution. In addition, there are other benefits as follows.

- 1) *If we transform the specifications of multi-product into standard one, we can solve the more real problem with multi-product.*
- 2) *When the customer's demands change, the ACO-based heuristic can compute the problems in an acceptable computational time.*
- 3) *If we change the objective function, we can also transfer the problem to an objective of finding the shortest travelling time and so on. Therefore, it can solve a multi-objective problem.*

#### 4. Conclusions

This paper is originated from the real case that a multi-product group company wants to integrate the distribution system. We use the ant colony optimization to solve the distribution location problem and vehicle routing problem of the joint distribution of different goods. The result shows that the joint distribution can reduce the total cost and increase efficiency.

However, there are some more complicated problems we are not involved right now, such as the nonlinear path, traffic jam and so on which will also affect the optimization effect.

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