

## Harmonic Distortion Modeling of Fully-Balanced Third-order Butterworth Gm-C Filter by a Block Diagram Approach

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**Abstract.** Accurate distortion modeling of fully-balanced Gm-C filter is the main subject of this paper. Based on the phasor method, a block diagram approach was developed in this paper. To demonstrate the flexibility of this approach, a third-order Butterworth Gm-C filter was analyzed by using the proposed approach. A good agreement is shown between theoretical analysis and transistor level simulation by Spectre. The approach is suitable to be used in early steps of the design and it can give the designer a thorough understanding of the distortion of fully-balanced Gm-C filters.

**Keywords:** harmonic distortion; nonlinear distortion; Gm-C filter; block-diagram approach; phasor method.

### 1. Introduction

Continue-time filter is extensively used in a large number of electronic systems such as audio, video and communication products. It is apparent to all that the continue time filters implemented with transconductance amplifiers and capacitors known as Gm-C are very popular for some reasons such as reducing power dissipation or obtaining tunability. It is usually that the linearity is sacrificed for satisfying speed or power constraints. As a result, the distorted output of the filter contains nonnegligible high-order harmonic components. It is hard to control the linearity without deep understanding of the distortion behaviors of the Gm-C filter. In recent years, several papers were engaged in distortion or nonlinear analysis of the Gm-C filters [1]-[5]. It worth noting that the method adopted in paper [5] is development of the analysis used to deal with distortion modeling of other circuit such as operational amplifier or integrators [6]-[8]. Following the pioneering works of [5]-[8] and exploring the phasor method, we obtained a block diagram approach. The proposed approach regards the full-balanced filter as multi-stage and feedback structures. Therefore, it is easy to understand and use. In addition, the proposed approach is flexible to deal with other filter structures. In this paper, the distortion modeling of a fully-balanced third-order Butterworth filter as shown in Fig.1 was given to demonstrate accuracy of the approach.

The paper is organized as follows. In section II, the third-order Butterworth Gm-C filter is divided into two stages and the nonlinear coefficients, the third-order distortion factor are calculated in different subsections. In section III, the theoretical results are compared with the transistor level simulation by Spectre. Simple discussions are given in this section. Section IV concludes this paper.

### 2. Distortion Modeling of The Third-Order Butterworth Gm-C Filter

The approach developed in this paper is suitable for distortion modeling of commonly used Gm-C filters. To simplify the derivation process, we divided the filter into two nonlinear stages. The first stage is a nonlinear transconductance stage and it is modeled in subsection A. The second stage is a nonlinear resistance

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stage and it is analyzed in subsection B. In subsection C, the nonlinear coefficients of the two stages are assembled to obtain the nonlinear coefficients of the Gm-C filter.

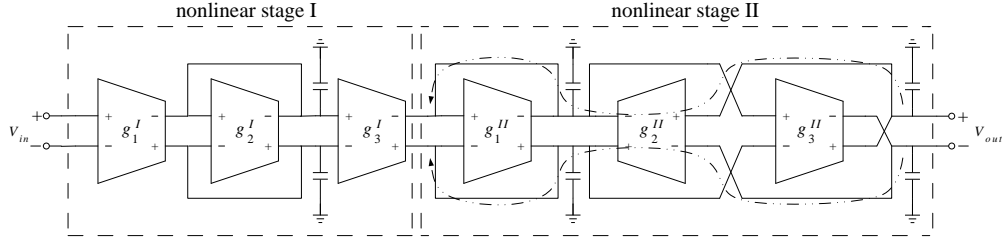


Fig. 1. Third-order Butterworth Gm-C filter

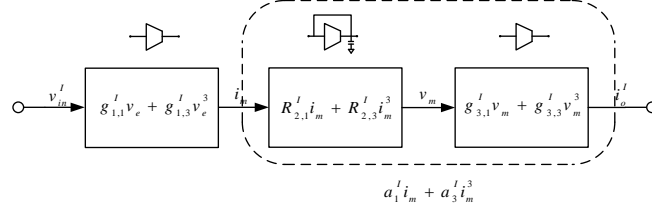


Fig. 2. Block diagram of 1<sup>st</sup> nonlinear stage

## 2.1. Coefficients of 1<sup>st</sup> Nonlinear Stage

The block diagram of the first nonlinear stage is shown in Fig.2. With the negative feedback and parallel connected capacitor at output, the second transconductance amplifier of the first nonlinear stage, i.e.  $g_{m2}^I$ , can serve as a nonlinear resistance  $R_2^I$ . Providing the input current of the nonlinear resistance  $i_m = x_s e^{j\omega t}$  and the output voltage  $v_m = R_{2,1}^I x_s e^{j\omega t} + R_{2,3}^I x_s^3 e^{j3\omega t}$ , we can insert them into the KCL equation

$$i_m - g_2^I v_m = j\omega C_L v_m \quad (1)$$

Omitting high order terms and equating the terms with same exponential, we get the expressions of nonlinear resistance coefficients as

$$R_{2,1}^I(j\omega) = \frac{1}{j\omega C_L + g_{2,1}^I} \quad (2)$$

$$R_{2,3}^I(j\omega) = \frac{-g_{2,3}^I}{(j3\omega C_L + g_{2,1}^I)(j\omega C_L + g_{2,1}^I)^3} \quad (3)$$

Then we can use the similar steps to assemble the equivalent second resistance  $R_2^I$  and the third amplifier  $g_3^I$ . The current gain coefficients representing the two parts in dashed box in Fig.2 can be written as

$$a_1^I(j\omega) = g_{3,1}^I R_{2,1}^I(j\omega) \quad (4)$$

$$a_3^I(j\omega) = g_{3,1}^I R_{2,3}^I(j\omega) + g_{3,3}^I [R_{2,1}^I(j\omega)]^3 \quad (5)$$

After obtaining the current gain coefficients, we can assemble them with the transconductance coefficients of the first transconductance amplifier in the first nonlinear stage. Repeating the steps as above, immediately we get the coefficients of the first nonlinear stage as

$$h_1^I(j\omega) = a_1^I(j\omega) g_{1,1}^I \quad (6)$$

$$h_3^I(j\omega) = a_1^I(j\omega) g_{1,3}^I + a_3^I(j\omega) (g_{1,1}^I)^3 \quad (7)$$

## 2.2. Coefficients of 2<sup>nd</sup> Nonlinear Stage

Observing the circuit topology of the third-order Gm-C Butterworth filter in Fig.1, we can find easily the second transconductance amplifier provides a feedback path from output to input of this nonlinear stage which is marked with a dash dotted line. Therefore, the block diagram of the second nonlinear stage is shown in Fig.3.

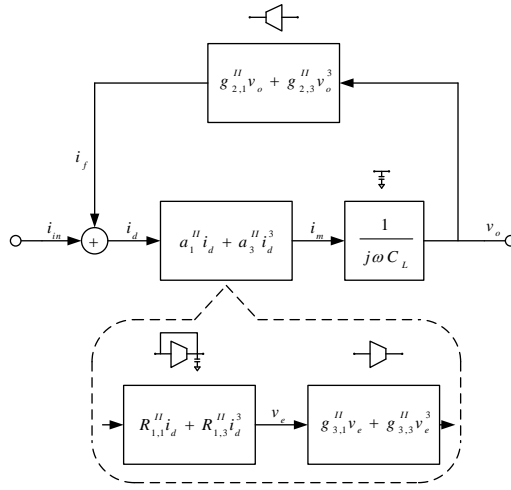


Fig..3. Block diagram of 2<sup>nd</sup> nonlinear stage

The connected two parts in dashed box in this stage is the same as the counterpart in first nonlinear stage. Hence, we can easily obtain the current gain coefficients by using the results in above subsection and gave the expressions as

$$a_1''(j\omega) = g_{3,1}'' R_{1,1}''(j\omega) \quad (8)$$

$$a_3''(j\omega) = g_{3,1}'' R_{1,3}''(j\omega) + g_{3,3}'' [R_{1,1}''(j\omega)]^3 \quad (9)$$

where

$$R_{1,1}''(j\omega) = \frac{1}{j\omega C_L + g_{1,1}''} \quad (10)$$

$$R_{1,3}''(j\omega) = \frac{-g_{1,3}''}{(j3\omega C_L + g_{1,1}'')(j\omega C_L + g_{1,1}'')^3} \quad (11)$$

Because the loading effect of capacitors at output of the filter, the feedforward path in Fig.3 is represented by a nonlinear resistance. The coefficients of this nonlinear resistance can be obtained by assembling the two blocks at lower part in Fig.3. The equivalent nonlinear resistance can be represented by coefficients

$$\begin{aligned} R_1''(j\omega) &= \frac{a_1''(j\omega)}{j\omega C_L} \\ &= \frac{g_{3,1}''}{j\omega C_L (j\omega C_L + g_{1,1}'')} \end{aligned} \quad (12)$$

$$\begin{aligned} R_3''(j\omega) &= \frac{a_3''(j\omega)}{j3\omega C_L} \\ &= \frac{-g_{3,1}'' g_{1,3}'' + g_{3,3}'' (j3\omega C_L + g_{1,1}'')} {j3\omega C_L (j3\omega C_L + g_{1,1}'')(j\omega C_L + g_{1,1}'')^3} \end{aligned} \quad (13)$$

After these preparations, we can derive the coefficients of the second nonlinear stage by using the phasor method again. Providing that input  $i_m = x_s e^{j\omega t}$  and output  $v_o = h_1'' x_s e^{j\omega t} + h_3'' x_s^3 e^{j3\omega t}$ , inserting them into KCL equation  $i_d = i_m - i_f$ , we get

$$i_d = (1 - g_{2,1}'' h_1'') x_s e^{j\omega t} - [g_{2,1}'' h_3'' + g_{2,3}'' (h_1'')^3] x_s^3 e^{j3\omega t} \quad (14)$$

Inserting the expressions of  $v_o$  and  $i_d$  into the equation  $v_o = R_1'' i_d + R_3'' i_d^3$ , we get an equation as

$$\begin{aligned}
& h_1'' x_s e^{j\omega_0 t} + h_3'' x_s^3 e^{j3\omega_0 t} \\
& = R_1''(j\omega_0)(1 - g_{2,1}'' h_1'') x_s e^{j\omega_0 t} - R_1''(j3\omega_0)[g_{2,1}'' h_3'' + g_{2,3}'' (h_1'')^3] x_s^3 e^{j3\omega_0 t} \\
& + R_3''(j\omega) \times \left\{ (1 - g_{2,1}'' h_1'') x_s e^{j\omega_0 t} - [g_{2,1}'' h_3'' + g_{2,3}'' (h_1'')^3] x_s^3 e^{j3\omega_0 t} \right\}^3
\end{aligned} \tag{15}$$

Omitting high-order terms, we easily obtain the first-order coefficient of this stage as

$$h_1''(j\omega) = \frac{R_1''(j\omega)}{1 + R_1''(j\omega)g_{2,1}''} \tag{16}$$

Equating the terms with third-order exponential in equation (15), we get the third-order coefficient

$$h_3''(j\omega) = \frac{-g_{2,3}'' R_1''(j3\omega)[R_1''(j\omega)]^3 + R_3''(j\omega)}{[1 + R_1''(j3\omega)g_{2,1}''][1 + R_1''(j\omega)g_{2,1}'']^3} \tag{17}$$

### 2.3. 3<sup>rd</sup> Order Harmonic Distortion Factor of the Filter

After obtaining coefficients of the two nonlinear stages in above subsections, we can derive the nonlinear coefficients of the filter by assembling the coefficients of first and second stages. The task can be completed with the help of phasor method as above analyses and it is neglected in this subsection. Then the coefficients of the filter can be written as

$$\begin{aligned}
h_1^{I,II}(j\omega) & = h_1''(j\omega)h_1^I(j\omega) \\
& = \frac{R_1''(j\omega)a_1^I(j\omega)g_{1,1}^I}{1 + R_1''(j\omega)g_{2,1}''}
\end{aligned} \tag{18}$$

$$\begin{aligned}
& h_3^{I,II}(j\omega) \\
& = h_1''(j3\omega)h_3^I(j\omega) + h_3''(j\omega)[h_1^I(j\omega)]^3 \\
& = \frac{R_1''(j3\omega)[a_1^I(j3\omega)g_{1,3}^I + a_3^I(j\omega)(g_{1,1}^I)^3]}{1 + R_1''(j3\omega)g_{2,1}''} + \frac{[a_1^I(j\omega)g_{1,1}^I]^3 \{-g_{2,3}'' R_1''(j3\omega)[R_1''(j\omega)]^3 + R_3''(j\omega)\}}{[1 + R_1''(j3\omega)g_{2,1}''][1 + R_1''(j\omega)g_{2,1}'']^3}
\end{aligned} \tag{19}$$

Finally, the third-order harmonic distortion factor is obtained conveniently and it is written at the bottom of this page.

In many cases, the same design is used for every transconductance amplifier. Therefore, we can use a transconductance  $g$  in place of  $g_x'$  and  $g_x''$  ( $x=1,2,3$ ) in above equations. In many practical cases, the third-order nonlinear transconductance coefficient  $g_3$  is proportional to first-order one  $g_1$ . As a result, we use the product of ratio  $\varepsilon$  and linear transconductance  $g_1$  to present the third-order nonlinear coefficient  $g_3$ . Hence, the equation (20) can be simplified as

$$\begin{aligned}
HD3 & = \frac{1}{4} \left| \frac{h_3^{I,II}(j\omega)}{h_1^{I,II}(j\omega)} \right| v_{in}^2 \\
& = \frac{v_{in}^2}{4} \left| \frac{R_1''(j3\omega)[a_1^I(j3\omega)g_{1,3}^I + a_3^I(j\omega)(g_{1,1}^I)^3][1 + R_1''(j\omega)g_{2,1}'']}{R_1''(j\omega)a_1^I(j\omega)g_{1,1}^I[1 + R_1''(j3\omega)g_{2,1}'']} + \frac{[a_1^I(j\omega)g_{1,1}^I]^2 \{-g_{2,3}'' R_1''(j3\omega)[R_1''(j\omega)]^3 + R_3''(j\omega)\}}{R_1''(j\omega)[1 + R_1''(j3\omega)g_{2,1}''][1 + R_1''(j\omega)g_{2,1}'']^2} \right|
\end{aligned} \tag{20}$$

$$\begin{aligned}
& HD_3(j\omega) \\
& = \frac{v_{in}^2}{4} \left| \frac{\varepsilon}{(j\omega C + g)^2} \frac{j\omega C(j\omega C + g) + g^2}{[j3\omega C(j3\omega C + g) + g^2]} \times \left\{ \frac{(j\omega C + g)^3 + j3\omega Cg^2}{j3\omega C + g} + \frac{g^4[3(j\omega C)^4 - g^4]}{[j\omega C(j\omega C + g) + g^2]^3} \right\} \right|
\end{aligned} \tag{21}$$

From above simplified expression, we find that the third-order harmonic distortion is directly proportional to ratio of  $g_{x,3}^{sg}$  to  $g_{x,1}^{sg}$  ( $sg=I,II$ ,  $x=1,2,3$ ). Although the third-order harmonic is mainly contributed by third-order nonlinearity, the frequency-dependent behavior is strongly affected by the linear transconductance of the amplifier and the load capacitance.

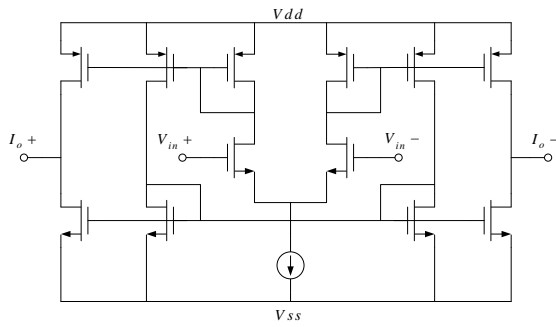


Fig.4. Fully-differential OTA with inherent CMF

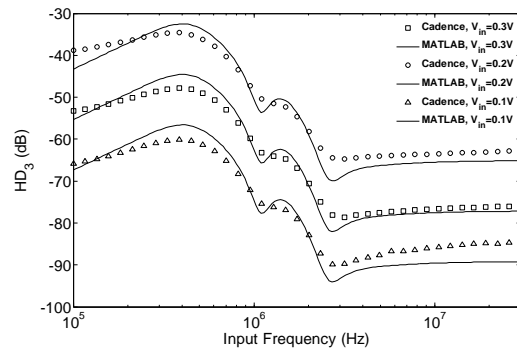


Fig. 5. Third-order distortion factor curve

### 3. Simulation

The accuracy of the proposed approach is verified in this section by comparing results between theoretical analysis and transistor level simulation. The transconductance amplifier used in this paper is a fully-differential OTA with inherent common mode feedback [9] and it is illustrated in Fig.4. The third-order Gm-C Butterworth filter shown in Fig.1 used a  $0.35\mu\text{m}$  CMOS process and was simulated in Spectre with sinusoid input signals with different amplitude. The tolerances of the simulator were set to relative small value to avoid excess numerical error. The output data of the transient simulation was post processed by Fourier analysis to get the harmonic components and the simulated third-order distortion factors. The curves of the simulated distortion factor are drawn in Fig.5. The nonlinear coefficients of the transconductance amplifier were extracted by using Fourier analysis when a sinusoid excitation signal is used as input of the amplifier. With these steps, we obtain the nonlinear coefficients  $g_{x,1}^{stg} = 83\mu\text{A}/\text{V}$  and  $g_{x,3}^{stg} = 125\mu\text{A}/\text{V}^3$  ( $stg=I,II$ ,  $x=1,2,3$ ). The ratio of third-order nonlinear coefficient to linear transconductance is calculated by the expression  $\varepsilon = g_3/g_1$ . All the load capacitors were sized equal to 10 pF, which yielding a cut-off frequency of 13.2 MHz. The equations derived in previous section were plotted in MATLAB and drawn in Fig.5. Excellent agreement has been observed between results of theoretical analysis and circuit simulation with EDA tool in the frequency range of interest.

### 4. Conclusion

In this paper, a block diagram approach based on phasor method was developed to model the distortion of a fully-balanced third-order Gm-C Butterworth filter. The theoretical results are in accordance with the circuit simulation at transistor level. In addition, the approach used in the paper shows flexibility for that it can be used to evaluate distortion of integrator, biquad and other high-order Gm-C filter. More important, the proposed approach gives more insights and understanding of the distortion behaviors of fully-balanced filters. It can serve as a helpful tool to predict the distortion in early steps of the continue time filters design and avoid time-consuming transient simulation.

### 5. References

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