2011 International Conference on Computer Science and Information Technology (ICCSIT 2011)
IPCSIT vol. 51 (2012) © (2012) IACSIT Press, Singapore
DOI: 10.7763/IPCSIT.2012.V51. 116

Estimation of BPSK Carrier Frequency Based on the High-Order Cyclic Cumulants

Liu Liangkai[a,*], Cheng Jian, Li Yongtao, Liu Tao

[a] baixia, biaoying No.2, Nanjing, 210007, China

Abstract. Aiming at the application of BPSK signal demodulation on burst communication, an algorithm based on high-order cyclic cumulants is introduced to estimate the carrier frequency of BPSK signal. This algorithm does not need a training sequence. Due to the fact that the high-order cycle can suppress the stationary and non-stationary Gaussian noise, the algorithm is tolerant to the noise mentioned above. The simulations show that the algorithm based on high-order cyclic cumulants can estimate the carrier frequency effectively and accurately.

Keywords: BPSK; cyclic cumulants; frequency estimation; carrier estimation

1. Introduction

The classical ways of Carrier Frequency Estimation are the algorithm based on feedback loop and the algorithm based on the principle of Maximum Likelihood (ML). The frequency estimation algorithm based on feedback loop costs too much time, so it is not suitable for burst communication. The algorithm based on Maximum Likelihood is complex and costs a large number of operations.

Aiming at the blind estimation of multi-carrier CDMA sub-carrier frequencies under Gaussian noise background, [3, 4] introduced an algorithm based on high-order cyclic cumulants to estimate carrier frequency. This algorithm does not need a training sequence. Due to the fact that the high-order cyclic cumulants can suppress the stationary and non-stationary Gaussian noise, it is tolerant to the noise mentioned above [5, 6]. The algorithm is developed and applied to estimate carrier frequency of BPSK in this paper.

The paper is organized as follows. Section Ⅱ introduces both analog signal and digital signal model of BPSK. The frequency estimation algorithm based on cyclic cumulants is shown in Section Ⅲ. Simulation results and analysis are presented in Section Ⅳ and conclusions are drawn in Section Ⅴ.

2. Signal model

For a BPSK modulated signal, the RF waveform at the transmitter is:

$$s(t) = a(t)\exp(j2\pi ft + \theta_m(t)) \tag{1}$$

where $a(t)$ is the amplitude of the transmitted signal, f is the frequency and $\theta_m(t)$ is the phase.

We just define the impulse response of the communication channel $h(t)$:

$$h(t) = \beta\exp(j\gamma)\delta(t-\tau) \tag{2}$$

where β is the attenuation factor, γ is the phase factor and τ is the propagation delay from the transmitter to the receiver.

The receiver signal, whose parameter we want to estimate, is written as:

$$r_{rcv}(t) = s(t) * h(t) + n(t) \tag{3}$$

* Corresponding author.
E-mail address: liangkailiu@sina.com.

and $n(t)$ is the white Gaussian noise, which is a variety of possible mechanisms, such as additive thermal noise generated by electronic devices; man-made noise, e.g., automobile ignition noise; and atmospheric noise, e.g., electrical lightning discharges during thunderstorms.

After the down-conversion, the received signal can be expressed as:

$$r(t) = a_r(t)\exp(j2\pi f_e t + \theta_r(t)) \tag{4}$$

where $a_r(t)$ is the amplitude of the received signal, f_e is the frequency offset after down-conversion and $\theta_r(t)$ is the phase of received signal.

In digital communication system, the received signal must be sampled periodically, which is expressed as:

$$r(n) = a_r(n)\exp(j2\pi f_e n + \theta_r(n)) \tag{5}$$

where $a_r(n)$ is the signal amplitude after sampled and $\theta_r(n)$ is the carrier phase after sampled.

3. Carrier frequency estimation algorithm

The prominent characteristic of digital communication signals is cyclic stationary characteristic, so cyclic cumulants has become an effective tool of signal analysis and processing. Theoretically, high-order cyclic cumulants could be completely inhibited any stationary Gaussian or non-Gaussian noise and non-stationary Gaussian noise [7, 8]. So we can obtain higher Signal-to-Noise Ratio, which is conducive to signal parameter estimate, in the high-order cyclic cumulants domain [9].

Generally, the time-varying k-order moment function (TMF) of cyclic stationary signal $r(t)$ is defined as the mathematical expectations of k-order hysteretic product [10]:

$$m_{kr}(t;\tau) = \hat{E}^{(\alpha)}\{\prod_{j=0}^{k-1} r(t+\tau_j)\} = \hat{E}^{(\alpha)}\{L_{kr}(t;\tau)\} \tag{6}$$

where $\hat{E}^{(\alpha)}\{\bullet\}$ is sinusoidal extraction operator and $L_{kr}(t;\tau)$ is k-order hysteretic product of signal $r(t)$.

$$\hat{E}^{(\alpha)}\{g(t)\} = \sum_{\alpha} < g(t)e^{-j2\pi\alpha t} >_t e^{j2\pi\alpha t} \tag{7}$$

where $< g(t)e^{-j2\pi\alpha t} >_t$ represent time average of $g(t)e^{-j2\pi\alpha t}$, and make $\tau_0 = 0$.

[11] indicates that TMF and time-varying cumulant function describe the time-varying characteristics of non-stationary signals, it can't be estimated by a single observation data. Because the cyclic matrix and cyclic cumulants are not the function of time, they can be calculated based on a single observation sample.

For fixed delay $\tau_1, \tau_2, \ldots, \tau_k$, the Fourier Series Expansion of $m_{kr}(t;\tau)$ is:

$$m_{kr}(t;\tau_1,\tau_2,\ldots,\tau_{k-1}) = \sum_{\alpha\in A_k} M_{kr}^{\alpha}(\tau_1,\tau_2,\ldots,\tau_{k-1})e^{j\alpha t} \tag{8}$$

$$\begin{aligned} M_{kr}^{\alpha}(\tau_1,\tau_2,\ldots,\tau_{k-1}) &= \lim_{T\to\infty}(1/T)m_{kr}(t;\tau_1,\tau_2,\ldots,\tau_{k-1})e^{j\alpha t} \\ &= < m_{kr}(t;\tau_1,\tau_2,\ldots,\tau_{k-1})\, e^{j\alpha t} >_t \end{aligned} \tag{9}$$

where α is cyclic frequency, k is the order of α and Fourier coefficient M_{kr}^{α} is the k-order cyclic moments in the cyclic frequency α.

For the BPSK signal, A_k is the cyclic frequency set of the k-order cyclic matrix.

$$A_k = \{\alpha : M_{kr}^{\alpha}(\tau_1,\tau_2,\ldots,\tau_{k-1}) \neq 0, 0 \le \alpha \le 2\pi\} \tag{10}$$

Just make some modifications on formula (9), we define the k-order sample cyclic moments of the received signal $r(n)$, which has been sampled, as follows:

$$\begin{aligned} M_{kr}^{\alpha}(\tau_1,\tau_2,\ldots,\tau_{k-1}) &= \lim_{T\to\infty}(1/T)\sum_{n=0}^{N-1} r(n+\tau_0)r(n+\tau_1)\cdots r(n+\tau_{k-1})e^{-jk2\pi\alpha n} \\ &= < r(n+\tau_0)r(n+\tau_1)\cdots r(n+\tau_{k-1})e^{-jk2\pi\alpha n} >_n \end{aligned} \tag{11}$$

where T is interval of time average and N is the number of sample point.

According to the property that cyclic matrix and cyclic cumulants can transfer to each other, we can make high-order matrix instead of high-order cyclic cumulants. This makes estimation of high-order cyclic cumulants simply [11]. Refer to this conclusion, we can get k-order cyclic cumulants of cyclic stationary signal $r(n)$ as:

$$C_{kr}^{\alpha}(\tau_1, \tau_2, ..., \tau_{k-1}) = \sum_{\cup_{p=1}^{q} I_p = I} [(-1)^{q-1}(q-1)! \prod_{p=1}^{q} M_{kr}^{\alpha}(\tau_{I_p})] \quad (12)$$

According to the formula (11) and (12), the cyclic cumulants of respective order can be expressed as follows:

$$C_{2r}^{\alpha}(\tau) = M_{2r}^{\alpha}(\tau) = R_r^{\alpha}(\tau) \quad (13)$$

$$C_{3r}^{\alpha}(\tau) = M_{3r}^{\alpha}(\tau_1, \tau_2) \quad (14)$$

$$\begin{aligned} C_{4r}^{\alpha}(\tau_1, \tau_2, \tau_3) &= M_{4r}^{\alpha}(\tau_1, \tau_2, \tau_3) - M_{2r}^{\alpha}(\tau_1) M_{2r}^{\alpha}(\tau_3 - \tau_2) \\ &- M_{2r}^{\alpha}(\tau_2) M_{2r}^{\alpha}(\tau_1 - \tau_3) - M_{2r}^{\alpha}(\tau_3) M_{2r}^{\alpha}(\tau_2 - \tau_1) \end{aligned} \quad (15)$$

when $\tau_1 = \tau_2 = \tau_3 = 0$,

$$\begin{aligned} C_{4r}^{\alpha}(0,0,0) &= M_{4r}^{\alpha}(0,0,0) - 3M_{2r}^{\alpha}(0) \times M_{2r}^{\alpha}(0) \\ &= < r^4(n) e^{-j8\pi\alpha n} >_n - 3 < r^2(n) e^{-j4\pi\alpha n} >_n^2 \end{aligned} \quad (16)$$

According all the formulas above, the 4-order cyclic cumulants of $r(n)$ is:

$$\begin{aligned} C_{4r}^{\alpha}(0,0,0) &= < r^4(n) e^{-j8\pi\alpha n} >_n - 3 < r^2(n) e^{-j4\pi\alpha n} >_n^2 \\ &= < a_r^{\,4}(n) e^{-j8\pi(f_e - \alpha)n} e^{j4\theta_r(n)} >_n - 3 < a_r^{\,2}(n) e^{-j4\pi(f_e - \alpha)n} e^{j2\theta_r(n)} >_n^2 \end{aligned} \quad (17)$$

Easily, we can see that formula (17) is not equal to 0 only at $\alpha = f_e$. It means that there exists a non-zero value of its 4-order cyclic cumulants only when the value of cyclic frequency α is equal to the carrier frequency of BPSK signal. So we can estimate the carrier frequency of BPSK by detecting the peak position.

4. Simulation and analysis

In order to verify the correctness of the mentioned algorithm, this paper carried out the following simulations. Simulation conditions are as follows:

The symbol rate is 50Hz. The carrier frequency of BPSK signal is 5000Hz and the search range of carrier frequency is $(5000 \pm 250)Hz$. The sampling frequency is 20 kHz.It means that the range of cyclic frequency α is $[4750\,Hz, 5250\,Hz]$. The cyclic frequency space is 1Hz. There are 400 sampling points in a data cycle.

In accordance with the above principles, we get the simulation results as follows:

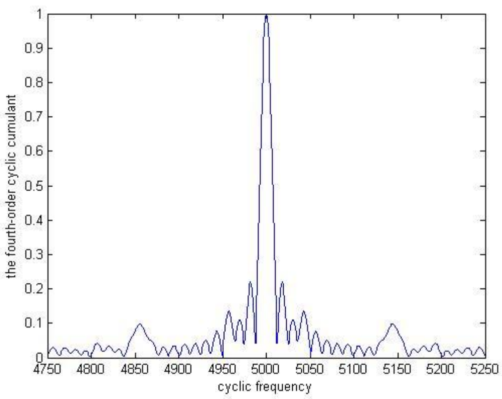

Figure 1. Eb/n0 = 5dB, T = 20ms

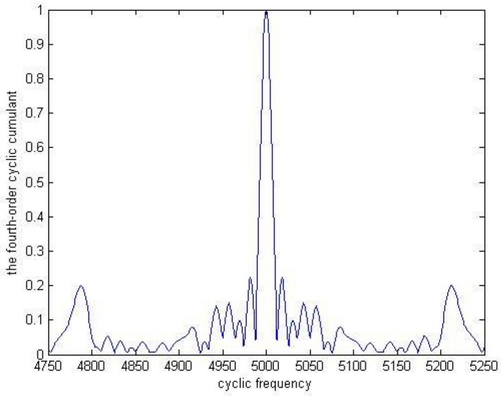

Figure 2. Eb/n0 = 2dB, T = 20ms

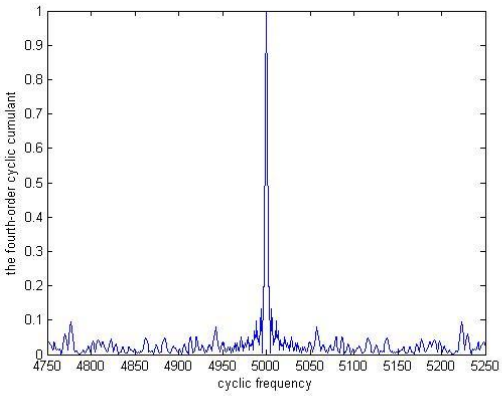

Figure 3. Eb/n0 = 5dB, T = 100ms

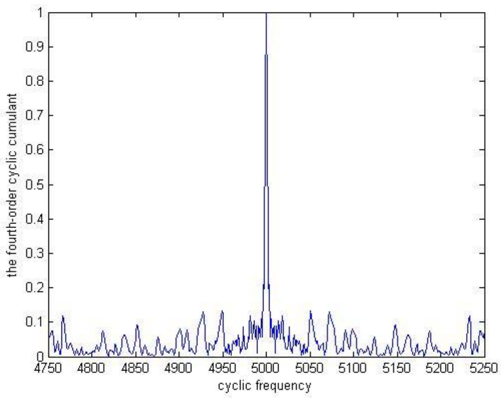

Figure 4. Eb/n0 = 2dB, T = 100ms

Figure 1 and 2 show the spectrum of cyclic cumulants when SNR=5dB and SNR=2dB with T=20ms respectively. And Figure 3 and 4 show the spectrum of cyclic cumulants when SNR=5dB and SNR=2dB with T=100ms respectively.

According to the simulations, we know the fact that where the cyclic frequency α is equal to the carrier frequency, there is a non-zero peak value of its cyclic cumulants. So the algorithm based on high-order cyclic cumulants can estimate the carrier frequency effectively and accurately. Because the high-order cycle can reduce the noise [5, 6], the carrier frequency estimate algorithm based on high-order cyclic cumulants can work well in low-SNR conditions. And the compare between Figure 1 and 2 or Figure 3 and 4 supports this conclusion.

Compare figure 1, 2 with figure 3, 4, we can obtain that if we extend the time interval T, we will get sharper spectrum of cyclic cumulants. This will be useful if high performance of carrier frequency estimate is required in low-SNR conditions. But the extendibility of time interval T will increase the computation.

Obviously, if we reduce the space of cyclic frequency, we will get frequency resolution improved. But the reduced cyclic frequency space will increase the number of cyclic frequency. This will increase the computation.

In order to reduce the computation, we can adopt two methods as follows:

1) First, apply DFT algorithm [15] to estimate the coarse frequency of the signal. Then set the coarse frequency as the range of cyclic frequency α. Finally, use the carrier frequency estimate algorithm based on high-order cyclic cumulants introduced to estimate the accurate frequency.
2) Refer to the idea of GPS signal frequency acquisition [16], we get the second method to reduce the computation. First, use the algorithm introduced in this paper to estimate the coarse frequency at large cyclic frequency space. Then set the coarse frequency as the range of cyclic frequency α and estimate the accurate frequency by applying the carrier frequency estimate algorithm based on high-order cyclic cumulants again.

5. Conclusion

An algorithm, which is suitable for estimating the carrier frequency of BPSK, is introduced in this paper. The deduction is based on the cyclic stationary characteristic of the received signal. The algorithm is of simple structure and doesn’t need a training sequence. High-order cycle can reduce any stationary Gaussian or non-Gaussian noise, as well as non-stationary Gaussian noise, so the algorithm introduced in this paper can estimate carrier frequency efficiently in low-SNR conditions. Two methods have been proposed to reduce the computation.

6. Acknowledgment

This work is supported by National High Technology Research and Development Program of China (“863” Program) (2008AA12A204, 2008AA12Z307).

7. References

[1] LI Jing, ZHU Jiang, ZHANG Eryang and SHEN Rongjun, “Second-Order Cyclic Statistics-Based Frequency Estimation Method,” Signal Processing, Vol.21. No.3, Jun. 2005 pp. 267-270.

[2] O. A. Dobre, Y. Bar-Ness and W. Su, “Higher-order cyclic cumulants for RF signal classification,” in Proc. MILCOM, 2003

[3] YU Zhi-ming, GUO Li-li and ZHAO Bing, “Blind estimation algorithm of multi-carder CDMA sub-carrier frequencies and its performance analysis,” Application Research of Computers. Vol.26 No.12, Dec.2009.

[4] YU Zhi-ming, GUO LI-li and QI Lin, “Blind Estimation of Multicarrier CDMA Sub-carrier Frequencies Based on the High-Order Cyclic Cumulants,” Wireless Communications, Networking and Mobile Computing, 2009, pp. 1-5.

[5] ZHENG Wen-xiu, ZHAO Guo-qing and LUO Ming, “Blind estimation of star-QAM carrier frequency based on the cyclostationary cumulants,” Systems Engineering and Electronics. Vol. 30 No. 2, Feb. 2008.

[6] ZHENG Wen-xiu, ZHAO Guo-qing and LUO Ming, “Blind Estimation of OFDM Sub-carrier Frequencies Based on the High-Order Cyclic Cumulants,” Journal of Electronics & Information Technology. Vol.30 No.2, Feb. 2008.

[7] ZHANG Xian-da and Bao Zheng, “Communication Signal Processing,” First version, Beijing: National Defence Industry Press, 2000: 11-37.

[8] ZHANG Xian-da, “Modern Signal Processing,” Beijing: TsingHua University Press, 2000: 373-377.

[9] A. Swami and B. M. Sadler, “Hierarchical digital modulation classification using cumulants,” IEEE Trans on Commun, 2000,48(3):416-429

[10] Fazel K and Kaiser S, “Multi carrier and spread spectrum systems,” Hoboken, N J: Wiley, 2003

[11] ZHANG Xian-da and Bao Zheng, “Nonstationary Signal Analysis and Processing,” Beijing: National Defense Industry Press, 2001: 346-349.

[12] Gardner W A and Spooner C M, "The Cumulant Theory of Cyclostationary Time-series, Part Ⅰ: Foundation," IEEE Transactions on Signal Processing, 1994, 42(12):3387-3408.

[13] WANG Jia-feng, YAO Jun, YUE Yang and QUAN You-bo, "Carrier Frequency Estimation Based on High-Order Cyclic Cumulant," Communications Technology. Vol.43.No.08, 2010.

[14] LUO Ming, "Research on Automatic Recognition and Parameter Estimation of Digital Communication Signals," Xidian University. Xi'an, China, Nov. 2005.

[15] L. C. Palmer, "Coarse frequency estimation using the discrete Fourier transform," IEEE Trans. Inform. Theory, pp. 104-109, Jan. 1974.

[16] Elliott D. Kaplan and Christopher J. Hegarty, "Understanding GPS Principles and Applications, Second Edition," Beijing: Publishing House of Electronics Industry, 2010, pp. 16