

Robust Image Segmentation Using FCM Based on New Kernel-Induced Distance Measure with Membership Constraints

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Abstract. The accurate and effective algorithm for segmenting image is very useful in many fields, especially in medical image. In this paper, we present a novel algorithm for fuzzy segmentation of magnetic resonance (MR) images. The algorithm is realized by modifying the objective function in the conventional fuzzy C-means (FCM) algorithm using a kernel-induced distance metric and a membership constraints on the membership functions. With synthetic image and the clinical MR images, the experiments show that our proposed algorithm is effective.

Keywords: FCM Cluster; brain MR image; image segmentation; kernel method

1. Introduction

The accurate and effective algorithm for segmenting image is very useful in many fields, especially in medical image that is important for neural diseases. Because of the advantages of magnetic resonance imaging (MRI) over other diagnostic imaging [1], the majority of researches in medical image segmentation pertains to its use for MR images. There are a lot of methods available for MR image segmentation [1-5]. However, because of the spatial intensity inhomogeneity induced by the radio-frequency coil in MR images, those intensity-based algorithms have proven to be problematic, even when advanced techniques such as non-parametric, multi-channel methods are used [1]. In fact, intensity inhomogeneity is impact on every image and we have to solve the problem with new method. Wells [1] developed a new statistical approach based on the expectation-maximization (EM) algorithm, but the results are too dependent on the initial values, extremely consuming the time and just looking for local maximum point. Ahmed [6] formulated by modifying the objective function of the standard FCM algorithm to compensate for such inhomogeneities. The method is outperformed, but because the average of immediate neighborhood is influenced localized measurements, and it is no good for accuracy segmentation of image. A recent approach proposed by Frank and Zhu [7-8] is to modify the FCM objective function by rewarding the crisp membership degrees. The corresponding fuzzy membership functions fulfill boundedness of support and unimodality. However, the methods lack enough robustness to noise and outliers.

The kernel methods are one of the most researched subjects within machine-learning community in recent years and have been widely applied to pattern recognition and function approximation. In [9], the novel kernel method based on the kernel-induced distance measure was proposed to increase the robustness to noise and outliers. The proposed approach did not adopt so-called dual representation for each centroid, but directly transform all the centroids in the original space, together with given data samples, into high-dimensional feature space. Such transformation led to the robust non-Euclidean distance measures, the computational simplicity and intuitive interpretation.

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In this paper, we used for reference Frank [7] and modified the objective function using the kernel-induced distance measure to enhance robustness and compensate for the intensity inhomogeneities. The synthetic image and real MR images help us to verify the algorithm and results were good consistent and our methods were verified.

2. Fcm with improved fuzzy partition

Frank et al. [7] proposed a modification to the standard FCM by introducing a term that reward crisp membership degrees. The modification can be interpreted as standard FCM using distances to the Voronoi cell of the cluster rather than using distances to the cluster prototypes. The resulting partitions of the modified algorithm are much closer to those of the crisp original methods. The modified objective function of FCM with improved fuzzy partition(FCM-IFP) is defined as follows:

$$J = \sum_{j=1}^n \sum_{i=1}^c u_{ij}^2 d^2(x_j, p_i) - \sum_{j=1}^n a_j \sum_{i=1}^c (u_{ij} - \frac{1}{2})^2 \quad (1)$$

subject to

$$\sum_{i=1}^c u_{ij} = 1, \forall j = 1, \dots, n \quad (2)$$

where d_{ij} is Euclidean distance from sample x_j to cluster center p_i defined as:

$$d_{ij} = \sqrt{\sum_{k=1}^s (p_{ik} - x_{jk})^2} \quad (3)$$

For the j th sample $x_j (1 \leq j \leq n)$ and the i th cluster center $p_i (1 \leq i \leq c)$, there is a membership degree $u_{ij} \in [0,1]$ indicating with what degree the sample x_j belongs to the cluster center vector p_i . The value of the so-called fuzzifier m is set to 2. Applying derivative to Eqs. (1) and (2), one can derive the computational formulae of u_{ij} and p_i as:

$$v_i = \frac{\sum_{j=1}^n u_{ij}^2 x_j}{\sum_{j=1}^n u_{ij}^2} \quad (4)$$

$$u_{ij} = \frac{1}{\sum_{k=1}^c \left(\frac{d^2(x_j, p_i) - a_j}{d^2(x_j, p_k) - a_j} \right)} \quad (5)$$

where x_j represents the j th sample and

$$a_j = \min_{1 \leq i \leq c} \{d_{ij}^2\} - \eta (0.01 \leq \eta \leq 0.2) \quad (6)$$

3. Fcm with improved fuzzy partition based on kernel-induced distance

In the last years, a number of powerful kernel-based learning machines[10], e.g. Support Vector Machines (SVM), Kernel Fisher Discriminant (KFD) and Kernel Principal Component Analysis (KPCA) were proposed and have found successful applications such as in pattern recognition and function approximation. A common philosophy behind these algorithms is based on the following kernel (substitution) trick, that is, a (implicit) nonlinear map from the data space X to the mapped feature space F , $\Phi: X \rightarrow F$. An input data space X with low dimension is mapped into a potentially much higher dimensional feature space F , which aims at turning the original nonlinear problem in the input space into potentially a linear one in rather high dimensional feature space. Therefore, every linear algorithm that only uses inner products can be easily extended to a nonlinear version only through the kernels satisfying the Mercer's conditions[11].

In the following are given typical radialbasis function (RBF):

$$K(x, y) = \exp\left(-\frac{\sum_{i=1}^d (x_i - y_i)^2}{\sigma^2}\right) \quad (7)$$

where d is the dimension of vector x . Obviously, $K(x, x) = 1$ for all x and the above RBF kernels.

There are two ways to kernelize FCM: one is to view every centroid as a mapped point in the feature space and use their dual forms, i.e., a linear combination of all data samples, to replace original centroids to get clustering results in the feature space but not in the original space as in [11]. In this way, the resulting clustering are not easily interpreted intuitively in the original space. The other [12] is to still view every centroid as a data point in the original space and directly transform them, together with the data samples, into the feature spaces and then carry out clustering. The advantages of doing so are that clustering is performed still in the original space and thus the results can easier be interpreted intuitively and computational simplicity. In this paper, we adopt the latter kernelizing way.

In this subsection, we will construct the corresponding kernelized version of the FCM-IFP algorithm based on kernel- induced distance (KFCM-IFP). We modify the objective function with the mapping Φ as follows:

$$KJ = \sum_{j=1}^n \sum_{i=1}^c u_{ij}^2 \|\Phi(x_j) - \Phi(p_i)\|^2 - \sum_{j=1}^n a_j \sum_{i=1}^c (u_{ij} - \frac{1}{2})^2 \quad (8)$$

Now through the RBFkernel substitution, we have:

$$\|\Phi(x_j) - \Phi(p_i)\|^2 = K(x_j, x_j)K(p_i, p_i) - 2K(x_j, p_i) = 1 - 2K(x_j, p_i) \quad (9)$$

Then (8) can be rewritten as

$$KJ = \sum_{j=1}^n \sum_{i=1}^c u_{ij}^2 (1 - 2K(x_j, p_i)) - \sum_{j=1}^n a_j \sum_{i=1}^c (u_{ij} - \frac{1}{2})^2 \quad (10)$$

We can prove that the measures based on the RBF kernel are robust but those based on polynomials are not by means of the Huber's robust statistics [13].

The parameter α_j in the second term controls the effect of the membership reward. By an optimization way similar to the standard FCM algorithm, the objective function KJ can be minimized under the constraint of as stated in (2). We then can obtain u_{ij} and p_i as follows:

$$u_{ij} = \frac{1}{\sum_{k=1}^c \frac{(1 - K(x_j, p_k)) - \alpha_j}{(1 - K(x_j, p_k)) - \alpha_j}} \quad (11)$$

$$p_i = \frac{\sum_{j=1}^n u_{ij}^2 K(x_j, p_i) x_j}{\sum_{j=1}^n u_{ij}^2 K(x_j, p_i)} \quad (12)$$

4. Experimental demonstration

In this section, we describe the application of the KFCM-IFP segmentation on synthetic and real brain MR images corrupted with noises.

4.1. The synthetic images

The first experiment applies our algorithms to a synthetic test image. The image with 64×64 pixels includes two classes with two intensity values taken as 0 and 90. We test our algorithms performance when corrupted by "Gaussian" and "salt and pepper" noises, respectively, and the results are shown in Fig. 1.

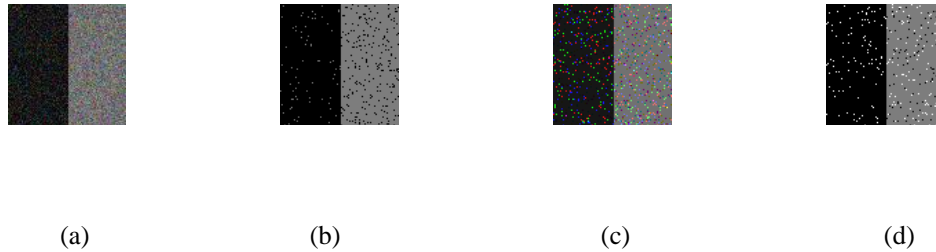


Fig. 1. Segmentation results on synthetic images: (a) original image with gaussian noises; (b) KFCM-IFP results; (c) original image with salt and pepper noise; (d) KFCM-IFP results

4.2. The real MR images

To test the performances of our algorithms under different kinds of noises on the real brain MR images, we do the following experiments. Fig. 2 show the segmentation results on real T1-weighted MR images with artificially added “Gaussian” and “salt and pepper” noises, respectively. Note that MR images typically do not suffer from “salt and pepper” noise, and we add such type of noise just for the show of robustness to noises of our algorithms.

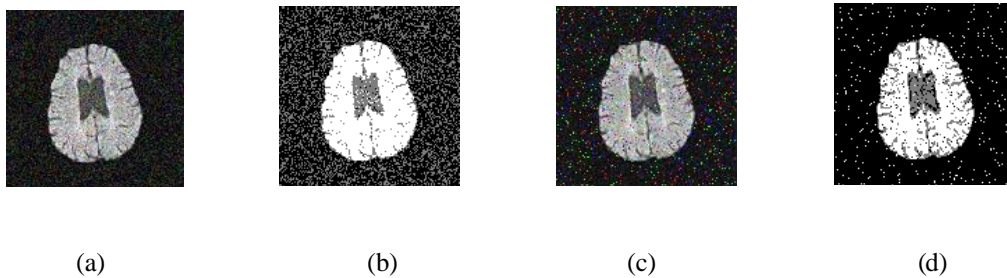


Fig. 2. Segmentation results on real images: (a) original image with gaussian noises; (b) KFCM-IFP results; (c) original image with salt and pepper noise; (d) KFCM-IFP results

On the whole, KFCM-IFP algorithm achieves better segmentation results under both noises and intensity inhomogeneities from the above figures.

5. Conclusions

FCM is one of the most well-known clustering algorithms. But its performance has been limited by Euclidean distance. In this paper, we propose the kernelized version of the FCM-IFP algorithm based on kernel-induced distance. The non-Euclidean distance measures show good robustness to noises and intensity inhomogeneities. Experiments on some synthetic and real images illustrate the advantages.

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7. References

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