

# A Modified Phase Transmit Sequences Approach to Reduce PAPR in LTE System

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**Abstract.** A high peak-to-average power ratio (PAPR) is a major shortcoming of multi-carrier transmission such as OFDM (orthogonal frequency division multiplexing) in LTE (Long Term Evolution) system. Optimum partial transmit sequences (OPTS) is one of the best methods in reducing PAPR, but there is a corresponding exponential increase in complexity. Recently, two suboptimal techniques for improving the complexity of an OFDM signal have been proposed: Iterative PTS (IPTS) and Dual-Layered PTS (DLPTS). In this paper, a modified PTS algorithm called Two-Step PTS (TS-PTS) is presented for searching out the suboptimal phase factors. It combines IPTS and DLPTS that achieves similar performance but with lower computational complexity.

**Keywords:** long term evolution, orthogonal frequency division multiplexing, peak-to-average power ratio, partial transmit sequences

## 1. Introduction

The Long Term Evolution (LTE) standard, specified by the 3rd Generation Partnership Project (3GPP) in Release 9, defines the next evolutionary step in 3G technology. LTE adopts Orthogonal Frequency-Division Multiple Access (OFDMA) for the downlink. OFDMA extends the multi-carrier technology OFDM to provide a very flexible multiple access scheme. OFDM subdivides the bandwidth available for signal transmission into a multitude of narrowband sub-carriers, arranged to be mutually orthogonal, which either individually or in groups can carry independent information streams. One of the major drawbacks is the potentially large peak-to-average power ratio (PAPR) characteristic of a multi-carrier signal with a large number of sub-channels. The high PAPR brings on the OFDM signal distortion in the nonlinear region of high power amplifier (HPA) and the signal distortion induces the degradation of bit error rate (BER) [1].

Recently many approaches [2] have been proposed to reduce the PAPR. The simplest method for PAPR reduction might be amplitude clipping [3], in which signals exceeding a certain prescribed limit are clipped before transmission, but it causes both in-band distortion and out of band radiation in the signal spectrum. Coding [4], is another popular technique to reduce the PAPR. It aims to select one of those code-words with low PAPR to reduce PAPR, but it incurs the rate decrease. ACE (Active Constellation Extension) in [5] is proposed. Selected mapping (SLM) technique generates a set of sufficiently different candidate data blocks, and selects the most favourable for transmission [6]. Partial transmit sequence (PTS) [7]-[10] is proposed to provide improved PAPR statistics with little cost in efficiency, but optimum PTS has a corresponding exponential increase in complexity. Two suboptimal techniques for reduce the computational cost of IFFT (Inverse Fast Fourier Transform) operations have been proposed: Iterative PTS (IPTS) and Dual-Layered PTS (DLPTS). IPTS [7] is very simple, but its performance is not good. DLPS with different

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implementations is proposed in [8] to further reduce the PAPR. Based on these techniques, a modified algorithm for finding out the suboptimal phase factors for PTS called Two-Step PTS (TS-PTS) is proposed. Simulation result shows that TS-PTS not only performs better than IPTS in reducing PAPR, but reaches less computational complexity than DLPTS.

The rest of this paper is organized as follows. Section 2 introduces the definition of PAPR of LTE system. Then, the principles of the optimum PTS technique as well as two suboptimal PTS techniques with low complexity are explained in Section 3. In Section 4, the derivation of our proposed reduction complexity approach and the modified TS-PTS method is presented. The complexity analysis and simulation results are discussed in Section 5 and it is followed by conclusions in Section 6.

## 2. LTE system description

LTE system adopts OFDMA for the downlink. OFDM can be easily implemented by FFT (Fast Fourier Transform) and IFFT processors. In transmitter side, IFFT converts signal in frequency domain into signal in time domain. The time-continuous signal  $s_l^{(p)}(t)$  in [11] on antenna port  $P$  in OFDM symbol  $l$  in a downlink slot is defined by

$$s_l^{(p)}(t) = \sum_{k=-\lfloor N_{RB}^{DL} N_{sc}^{RB} / 2 \rfloor}^{-1} a_{k^{(-)},l}^{(p)} \cdot e^{j2\pi k \Delta f (t - N_{CP,l} T_s)} + \sum_{k=1}^{\lfloor N_{RB}^{DL} N_{sc}^{RB} / 2 \rfloor} a_{k^{(+)},l}^{(p)} \cdot e^{j2\pi k \Delta f (t - N_{CP,l} T_s)} \quad (1)$$

for  $0 \leq t < (N_{CP,l} + N) \times T_s$  where  $k^{(-)} = k + \lfloor N_{RB}^{DL} N_{sc}^{RB} / 2 \rfloor$  and  $k^{(+)} = k + \lfloor N_{RB}^{DL} N_{sc}^{RB} / 2 \rfloor - 1$ . The variable  $N$  equals 2048 for  $\Delta f = 15$  kHz sub-carrier spacing and 4096 for  $\Delta f = 7.5$  kHz sub-carrier spacing.  $N_{CP,l}$  is the cyclic prefix length of OFDM symbol  $l$ . Table 1 lists the value of  $N_{CP,l}$  that shall be used.

Table 1. OFDM parameters

Configuration		Cyclic prefix length $N_{CP,l}$
Normal cyclic prefix	$\Delta f = 15$ kHz	160 for $l = 0$ 144 for $l = 1, 2, \dots, 6$
	$\Delta f = 7.5$ kHz	512 for $l = 0, 1, \dots, 5$ 1024 for $l = 0, 1, 2$

Where,  $t = nT_s$ ,  $T_s$  is a basic time unit,  $T_s = 1/(N \cdot \Delta f)$ ,  $N \cdot \Delta f$  is the sampling rate. Therefore,  $\Delta f t = \Delta f n T_s = \frac{n}{N}$ . Then, we remove the  $N_{CP,l}$  in (1) ( After IFFT, we can add cyclic prefix for each OFDM symbol ). Hence,

$$s_l(n) = \sum_{k=0}^{N_{RB}^{DL} N_{sc}^{RB}} a_{k,l} e^{j2\pi \frac{n}{N} (k - \lfloor N_{RB}^{DL} N_{sc}^{RB} / 2 \rfloor)}, \quad n=1, 2, \dots, N. \quad (2)$$

Then,

$$s_l(n) = e^{\frac{j2\pi (-\lfloor N_{RB}^{DL} N_{sc}^{RB} / 2 \rfloor)(n-1)}{N}} * N * IFFT(a, N), \quad n=1, 2, \dots, N. \quad (3)$$

PAPR of transmitted OFDM signals in one symbol period in (3) can be defined as the ratio of the maximum divided by the average power of the signal, is expressed as

$$PAPR = 10 \log_{10} \frac{\max\{|s_l(n)|^2\}}{E\{|s_l(n)|^2\}} \quad (dB) \quad (4)$$

## 3. The Partial Transmit Sequences Algorithm

The original sequence is divided into several sub-sequences and each sub-sequence is separately multiplied by different weights and then merged, which is the basic idea of optimum PTS algorithm. The combined received signal PAPR value is as small as possible by choosing a different weight vector.

### 3.1. Optimum PTS

The basic principles block diagram of OPTS approach is shown in Fig. 1.(a). Input data block  $X$  with the length  $N$  are partitioned into  $M$  disjoint sub-blocks  $X_m = [X_{m,0}, X_{m,1}, \dots, X_{m,N-1}]$ ,  $m = 1, 2, \dots, M$ . Where,

$\sum_{m=1}^M X_m = X$  and these sub-blocks are called the partial transmit sequences. The objective is to find sets of phase rotation factors  $b_m = \exp\{j\phi_m\}$ , combining  $X_m$  with them so that the resulting signals will have the lowest PAPR. This operation in time domain can be represented by

$$\hat{x} = \sum_{m=1}^M b_m \text{IFFT}\{X_m\} = \sum_{m=1}^M b_m x_m. \quad (5)$$

Where, the phase rotation factors  $b_m$  can be selected from an infinite number of phases  $\phi$ . By employing optimal binary phase sequences (OBPS), in which the phases are quantized to  $[\pm 1]$  and show a satisfactory PAPR reduction. The vectors  $b_m$  and  $\hat{x}$  becomes the optimized phase rotation factors and the resulting time-domain signals respectively, after an exhaustive search.

### 3.2. Iterative PTS

In [7], a simple method called Iterative PTS (IPTS) is introduced in order to reduce the computational complexity, instead of the optimum method to find the optimum weighting coefficients, and PAPR performance declines slightly. The phase rotation factors  $b_m = \pm 1, m = 1, 2, \dots, M$ .

This method can be expressed as:

- Set  $b_m = 1, m = 1, 2, \dots, M$ , using (3) and (4), we can calculate PAPR of OFDM signals with the value of  $PAPR1$ , and set  $index = 1$ ;
- Set  $b_{index} = 1$ , PAPR at this time is calculated by the same method with the value of  $PAPR2$ ;
- If  $PAPR1 > PAPR2$ ,  $b_{index} = 1$ ; otherwise  $PAPR1 = PAPR2$ ,  $b_{index} = -1$ ;
- $index = index + 1$ ;
- Repeat from step 2-4 if  $index < M + 1$ .

The amount of computation can be effectively reduced by this iterative PTS algorithm. Compared to  $2^{M-1}$  IFFT operations of optimum PTS, the computational cost of IPTS is only  $M$  IFFT operations.

### 3.3. Dual-Layered PTS (DLPTS)

In [8], a suboptimal Dual-Layered PTS is proposed to reduce the computational complexity of OPTS with better performance than IPTS. The layers are classified as micro- and macro-optimization layers. In micro-optimization, the  $M$  sub-blocks are grouped into  $D$  divisions. With the un-optimized divisions assuming 1, optimization starts at the last division using OPTS. The optimized sequence for each division is retained to derive the optimal sequence for the next division. Therefore within each division all possible combinations of phase rotation factors are experimented to obtain a combination with minimum PAPR. On the second layer, a macro-optimization occurs, in which each individually optimized division is considered as a block for further PAPR reduction using OPTS. The computational complexity is  $2^{M/D} \cdot D + 2^{D-1}$  IFFT operations, which is less than OPTS when  $M$  is larger.

## 4. Two-Step PTS algorithm

Based on IPTS and DLPTS, a modified algorithm for finding out the suboptimal phase factors for PTS named Two-Step PTS (TS-PTS) is proposed. TS-PTS is not only better than IPTS on reducing PAPR performance, but reaches less computational complexity than DLPTS.

Fig. 1.(b) shows the modified algorithm, which can be expressed as:

First Step: we use IPTS as mentioned in section III. In this case, we will find the suboptimal set of phase rotation factors  $b'_m (m = 1, 2, \dots, M)$ . Then, PAPR is calculated with the value of  $PAPR_{first}$ .

Second Step: we use optimum PTS as mentioned in section III. The  $M$  sub-blocks are grouped into  $D$  divisions, so each division of the input data block is partitioned into  $R = M/D$  parts. Each division controls  $R$  bits of phase rotation factors called macro-factors  $c_d (d = 1, 2, \dots, D)$ .

Each individually optimized division is considered as a block for further PAPR reduction. In order to avoid performance loss, we assume macro-factor  $c_1 = 1$ , the other macro-factors  $c_d = \pm 1 (d = 2, 3, \dots, D)$  at the

beginning. Then,  $c_d (d = 1, 2, \dots, D)$  of each block multiples with the suboptimal phase factors  $b'_m (m = 1, 2, \dots, M)$  is modified by exhaustive search for appropriate phase factors to achieve the minimum PAPR comparing with PAPRfirst.

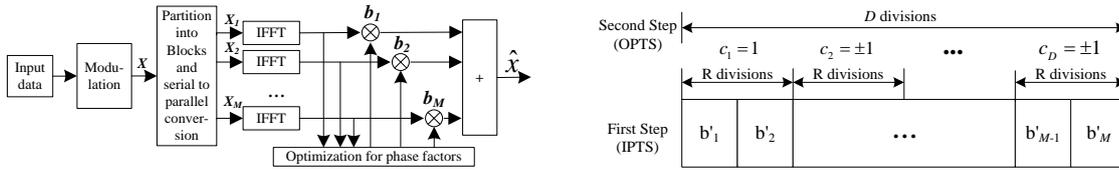


Fig. 1. (a) block diagram of PTS; (b) proposed TS-PTS

## 5. Complexity analysis and simulation results

### 5.1. Complexity analysis

Four PTS algorithms have been introduced above and their computational complexity analysis with different partitions  $M, D$  are showed in Table 2. As the value of partitions  $M, D$  increases, there is a corresponding increase in computational complexity. The IPTS algorithm requires the least IFFT operations, but with poor performance. The main contribution of our proposed TS-PTS method is to reduce the complexity of the OFDM system. The computational complexity of the modified TS-PTS declines largely compared with optimum PTS and DLPTS algorithm, particularly when  $M \geq 8$ .

Table 2. Computational complexity analysis

Partitions	OPTS	IPTS	DLPTS	TS-PTS
$M, D$	$2^{M-1}$	$M$	$2^{M/D} \cdot D + 2^{D-1}$	$M + 2^{D-1}$
M4D2	8	4	10	6
M8D2	128	8	34	10
M8D4	128	8	24	16
M16D2	32768	16	514	18
M16D4	32768	16	72	24
M16D8	32768	16	160	144

### 5.2. Simulation results

This section presents simulation results obtained with a standard compliant LTE physical layer downlink matlab simulation. As discussed in the above sections, the PAPR is associated with the time domain OFDM transmit signal (3). Because PAPR is a random variable, the complementary cumulative distribution function (CCDF) is used to evaluate the statistical properties of PAPR we consider the probability of PAPR exceeding a certain level  $PAPR_0$ . In Fig. 2.(a) and Fig. 2.(b), 7 OFDM symbols and 10000 blocks were generated to obtain the CCDFs. The simulation model is assumed to have 72 sub-carriers throughout and employs QPSK data symbols. The main simulation parameters are summarized in Table 3.

Table 3. Simulation parameters

Parameter	Value
System bandwidth	1.4 MHz
100Cyclic prefix	Normal
Resource Blocks	6
Sub-carriers pacing	15 KHz
Sub-carriers	72
Cell ID	0
Slots	1
Modulation	QPSK
Partitioned scheme	adjacent
Numbers of OFDM	7
IFFT Points	2048

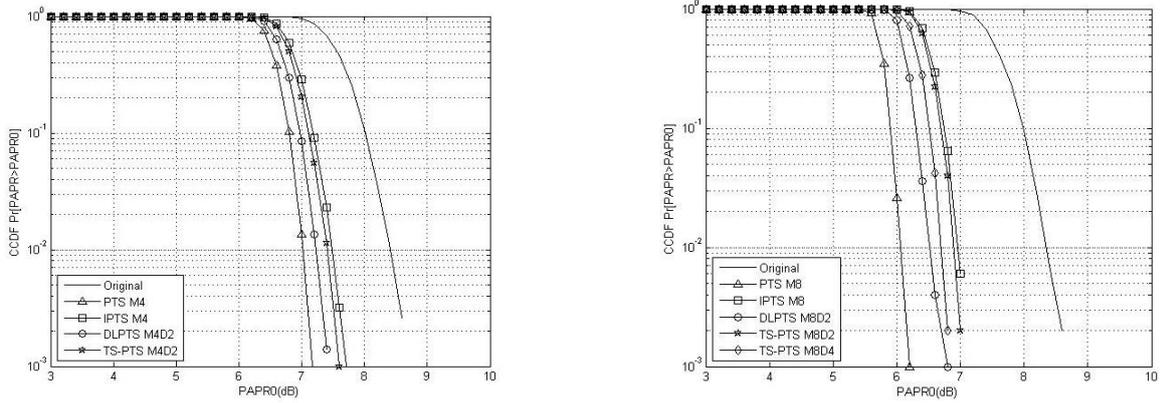


Fig. 2. (a) comparison of four PTS algorithms for  $M=4, D=2$ ; (b) comparison of four PTS algorithms for  $M=8, D=2$  or  $4$

The PAPR reduction curves of original signal, OPTS, IPTS, DLPTS and modified TS-PTS algorithms are shown in Fig. 2.(a) and Fig. 2.(b). It can be observed that the PAPR reduction efficiency of them is similar. The original OFDM signal has a PAPR which exceeds 8.7 dB for less than 0.1% of the blocks.

In Fig. 2.(a), when sub-blocks  $M = 4$  and divisions  $D = 2$ , by using the OPTS approach with the optimum binary phase sequence for combining, at 0.1% PAPR reduces to 7.2 dB. PAPR reduction for our modified TS-PTS algorithm can be reduced about 1.1 dB at 0.1% from 8.7 dB to 7.6 dB. The modified algorithm performs more than 0.2 dB better than IPTS, less than 0.3 dB worse than DLPTS. However, the modified algorithm only requires 60% IFFT operations of DLPTS, 75% IFFT operations of optimum PTS. Although IPTS algorithm requires less IFFT operations, its PAPR performance is worse than other PTS algorithm.

In Fig. 2.(b), when sub-blocks  $M = 8$  and divisions  $D = 2$  or  $4$ , comparing the performance of  $M = 4$ , the performance gain is about 1 dB. The CCDFs comparison between the modified PTS algorithms and other PTS algorithms is the same with  $M = 4$ . For  $D = 2$ , the computational complexity of the modified algorithm is only 29.4% of DLPTS, 7.8% of optimum PTS. For  $D = 4$ , the computational complexity of the modified algorithm is only 66.7% of DLPTS, 12.5% of optimum PTS.

Finally, we look at the effect of the divisions  $D$ . If  $D$  is increased, improved performance can be expected for the modified PTS algorithm. Of course, this occurs at an increasing level of computational complexity. When sub-blocks  $M$  is a constant, we can weigh PAPR performance benefit against computational complexity by adjusting the value of  $D$ .

## 6. Conclusion

Partial transmit sequence (PTS) technique has been proved efficient to improve the PAPR performance of OFDM signals in LTE system. However, optimum PTS technique requires an exhaustive search best combination of phase factors with a disadvantage of high search complexity. In this paper, by combining two suboptimal PTS algorithms (IPTS, DLPTS), a modified PTS scheme called TS-PTS has been proposed. The computational complexity analysis and simulation results show that the TS-PTS scheme with 72 sub-carriers reduces the computational complexity by 66.7% of DLPTS, 12.5% of OPTS with the performance more than 0.4 dB better than IPTS when sub-blocks  $M=8$ , divisions  $D = 4$ . Also, we can adjust the value of divisions  $D$  through a trade-off between computational complexity and PAPR performance. Especially, when the original sequence is partitioned into more sub-blocks and these sub-blocks are grouped into more divisions, the complexity of our proposed method is increased slightly with only neglected PAPR performance decrease.

## 7. Acknowledgements

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