

## An Adaptive Tracking Algorithm for Radar Target

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**Abstract.** A new adaptive tracking method for radar target is proposed in this paper, in this method, velocity and acceleration are estimated by limited memory of least square method, position is estimated by one-step filter. In the estimation course of velocity and acceleration, dynamic number of limited memory points is adjusted according to maneuver detecting, to ensure the smooth for straight flight and less lag for swerve. In the position filter, a roll angle predicted method is proposed, and the roll angle is used for adaptive adjusting of Kalman filter parameter. Simulation result shows that it tracks the target smooth in straight flight and rapid in swerve flight, owning a high engineering value.

**Keywords:** radar target, adaptive tracking, maneuver detecting, roll angle predicted, Kalman filter

### 1. Introduction

Radar target tracking algorithm is based on the premise of target motion model, it is necessary to ensure the objectives of non-stop smooth track, but also need to achieve fast tracking of maneuvering turn. But in the actual engineering, theoretical model and the actual movement usually has a big difference, meanwhile, there are some radar measurement error data. Therefore, how to estimate the unknown parameters from observed data, to correct filter gain, to achieve the purpose of fast and accurate maneuvering target tracking by adaptive Kalman filter, is very important[1-4]. In this paper, prediction based on dynamic limited memory for adaptive Kalman filter is proposed. Different from the traditional target tracking is that, velocity and acceleration is estimated using the dynamic limited memory prediction of measurement points, but position is estimated using one step filtering. Using this method, the estimated velocity and acceleration is easily for engineering realization. The impact of random measurement error on the velocity estimation is reduced. In terms of location filtering, Kalman filter input disturbance and measurement noise is amended based on the target state, as well as real-time radar performance parameters. Through the above methods, it can achieve adaptive maneuvering target tracking, both to maintain a good smooth track for straight fly, but also to achieve fast track when the target turns.

### 2. Adaptive Tracking Strategy

Position tracking for moving objects can be viewed as a multidimensional time-varying systems, the way of using the classical Kalman filter to track the target can be described as follows.

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} + \mathbf{w}_{k-1} = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}(\mathbf{u}_{k-1} + \mathbf{B}^{-1}\mathbf{w}_{k-1}) \quad (1)$$

Where  $\mathbf{x} \in R^n$  is the state variable, in the equation (1),  $n \times n$  gain matrix  $\mathbf{A}$  is a Linear map from previous  $k-1$  state to the current  $k$  state.  $n \times l$  Gain matrix  $\mathbf{B}$  represents the control input  $\mathbf{u} \in R^l$  gain.

Define the observed variable  $\mathbf{z} \in R^m$ , the measurement equation is by:

$$\mathbf{z}_k = \mathbf{H}\mathbf{x}_k + \mathbf{v}_k \quad (2)$$

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Where  $m \times n$  gain matrix  $\mathbf{H}$  represents the gain from state to observation. Random signals  $\mathbf{w}$  and  $\mathbf{v}$  represent process noise and observation noise respectively. Assumed them as independent, following a normal white noise

$$p(\mathbf{w}) \sim N(0, \mathbf{Q}) \quad (3)$$

$$p(\mathbf{v}) \sim N(0, \mathbf{R}) \quad (4)$$

Using Kalman filtering, the basic equations are as follows:

$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_{k-1} + \mathbf{B}\mathbf{u}_{k-1} \quad (5)$$

$$\mathbf{P}_k = \mathbf{A}\mathbf{P}_{k-1}\mathbf{A}^T + \mathbf{Q} \quad (6)$$

$$\mathbf{K}_k = \mathbf{P}_k\mathbf{H}^T(\mathbf{H}\mathbf{P}_k\mathbf{H}^T + \mathbf{R})^{-1} \quad (7)$$

$$\hat{\mathbf{x}}_k = \mathbf{x}_k + \mathbf{K}_k(\mathbf{z}_k - \mathbf{H}\mathbf{x}_k) \quad (8)$$

$$\mathbf{P}_k = \mathbf{P}_k\mathbf{H}^T(\mathbf{H}\mathbf{P}_k\mathbf{H}^T + \mathbf{R})^{-1} \quad (9)$$

The initial value of  $\mathbf{P}$  is not important, will be converged in Iteration. In real-time radar target tracking, the difficulty of using Kalman filter lies in: 1) the estimation of target status. Usually radar can not provide velocity information, the velocity/ acceleration information of target should be calculated by position information, random measurement errors will lead to large fluctuations in the position information, which seriously affect the velocity/ acceleration estimation. 2) Determine the magnitude of noise. Observation noise  $\mathbf{v}$  is not only about the precision of radar detection, but also relevant with the distance from the radar station. Input uncertainty  $\mathbf{w}$  reflects the accuracy of input  $\mathbf{u}$ , it has a great relationship with target status. When target is Maneuvering to turn, there is obviously lagging of input estimation, at the same time, the uncertainty will greatly increase. In this paper, prediction based on dynamic limited memory for adaptive Kalman filter is proposed. Target status is estimated using the dynamic limited memory prediction of measurement points, but position is estimated using one step filtering. Observation noise  $\mathbf{R}$  is corrected according to the latest measured information and radar performance parameters. For process noise  $\mathbf{Q}$  correcting, a new maneuver detecting method is proposed, dynamic number of limited memory dot is adjusted according to maneuver detecting, then  $\mathbf{Q}$  is corrected according to the target maneuver information. Algorithm framework is shown in figure 1, adaptive management strategy mainly includes two points: 1) through the maneuver detecting, dynamic number of limited memory dot is adjusted according to maneuver detecting, to ensure the smooth for straight flight and less lag for swerve. 2) the target roll angle is estimated in maneuver detecting,  $\mathbf{Q}$  is real-time adjust by the roll angle feedback, achieving a fast turn track.

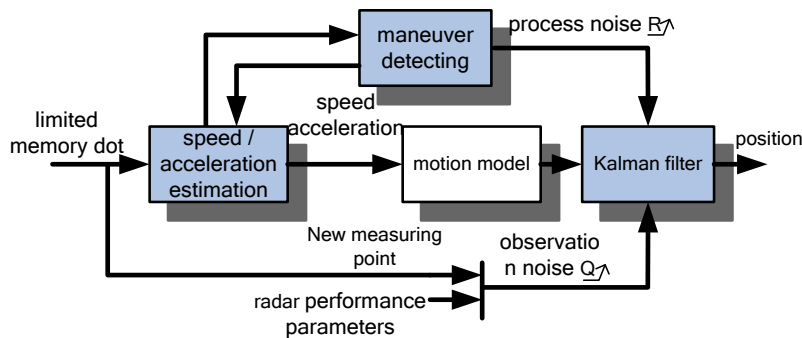


Figure 1. Adaptive radar target tracking framework diagram

### 3. Dynamic limited memory velocity and acceleration estimation

To avoid random measure errors causing target speed, heading changing too large, so in the calculation, limited memory points (5 to 7 points) is used to velocity/ acceleration smooth calculation, not just the recent two points. In addition, the number of points needs to consider the status of the target, if the target is in the maneuver turn, the number of points should be reduced accordingly in the velocity and heading estimation. So dynamic number of limited memory points used in velocity/acceleration smooth calculation can take into account both the characteristics of straight fly and maneuver turn. When target is in straight fly, more points can be calculated to make the estimation smooth, reducing the impact of measurement error. When target is

in maneuver turn, less points can be calculated to reduce the lag of velocity and acceleration estimation, close to the target current state.

To simplify the velocity estimation, assuming that the target is in the constant linear motion, the track use one order polynomial denotation, that is:

$$x(V_x, t) = x_0 + V_x t \quad (10)$$

Where  $x$  is the amount of position,  $x_0$  is Initial position,  $V_x$  is velocity,  $t$  is time, use the least square estimation method[5], the coefficient equation is:

$$\bar{x}_0 n + \bar{V}_x \sum_{i=1}^n t_i = \sum_{i=1}^n x_i \quad (11)$$

$$\bar{x}_0 \sum_{i=1}^n t_i + \bar{V}_x \sum_{i=1}^n t_i^2 = \sum_{i=1}^n t_i x_i \quad (12)$$

Coefficient is:

$$\bar{x}_0 = \frac{\Delta x}{\Delta} = \frac{\sum_{i=1}^n x_i \sum_{i=1}^n t_i^2 - \sum_{i=1}^n x_i t_i \sum_{i=1}^n t_i}{n \sum_{i=1}^n t_i^2 - (\sum_{i=1}^n t_i)^2} \quad (13)$$

$$\bar{V}_x = \frac{\Delta V}{\Delta} = \frac{n \sum_{i=1}^n x_i t_i - \sum_{i=1}^n x_i \sum_{i=1}^n t_i}{n \sum_{i=1}^n t_i^2 - (\sum_{i=1}^n t_i)^2} \quad (14)$$

In the velocity estimation, least square estimation of the speed is the target current speed, the acceleration can be calculated by the rate of change rate of the two recent points.

$$\bar{a}_x = \frac{\Delta \bar{V}_x}{\Delta t} \quad (15)$$

In the process of target detection, information access has the same time interval  $T$ , assuming that the first time information is 0, that is  $t_1 = 0$ , so  $t_2 = T$ ,  $t_3 = 2T$ ,  $t_i = (i-1)T$ ,  $t_n = (n-1)T$ , substituting  $t_i = (i-1)T$  into the expression (13,14) and transformed, finally get:

$$\bar{x}_0 = \sum_{i=1}^n \frac{4(n-1) - 6i}{n(n+1)} x_i = \sum_{i=1}^n \eta_0(i) x_i \quad (16)$$

$$\bar{V}_x = \sum_{i=1}^n \frac{6(2i - n - 1)}{T(n^2 - 1)n} x_i = \sum_{i=1}^n \eta_V(i) x_i \quad (17)$$

Where:

$$\eta_0(i) = \frac{4(n-1) - 6i}{n(n+1)} \quad (18)$$

$$\eta_V(i) = \frac{6(2i - n - 1)}{T(n^2 - 1)n} \quad (19)$$

Variance of velocity is:

$$\sigma_V^2 = \sum_{i=1}^n \eta_V^2(i) \sigma_{x_i}^2 \quad (20)$$

Variance of acceleration is:

$$\sigma_{a_x}^2 = \sum_{i=1}^n \frac{\eta_V^2(i) \sigma_{x_i}^2}{2T} \quad (21)$$

As can be seen from the variance, the accuracy of velocity and heading estimation is proportional to coordinate measurement precision, and it is relevant to measuring numbers  $n$ , the larger  $n$  is, the smaller  $\eta_V$  is. That is to say the smaller estimated variance of velocity and acceleration is, the higher the precision is, but

when  $n$  is large, historical measurement information should be used more, and estimation is more lag, if the target is in maneuver turn, the estimation error will lead to greater. Therefore, it is necessary to add the maneuvering detection part, if the target maneuver is detected, the limited memory number of points decreased.

#### 4. Maneuver detecting

In maneuvering target tracking, to determine whether the target maneuvering is or not is very important[6,7]. It is usually determined by the state threshold method in engineering. In this paper, a roll angle predicted method is proposed, the vector relations of speed and acceleration is considered, combined the coordinated turn characteristics of aircraft to approximately solve the roll angle. Stress analysis of coordinated turn can be seen in figure 2.

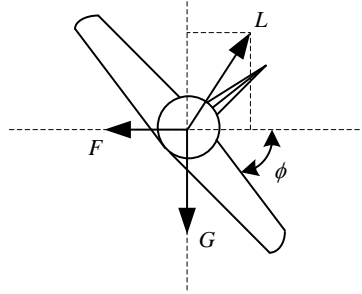


Figure 2. Stress analysis of coordinated turn

Where  $L$  is aerodynamic lift,  $G$  is gravity,  $F$  is centrifugal force,  $\phi$  is roll angle. When coordinated turning, in the vertical direction, the vertical component of lift and gravity make the balance, keeping the aircraft flying in the horizontal plane. Horizontal component of lift and centrifugal force make the balance[8], aircraft turns at a constant angular velocity  $\dot{\psi}$  in circular motion in the horizontal plane.

$$L \cos \phi = G = mg \quad (22)$$

$$L \sin \phi = F = m \dot{\psi} V \quad (23)$$

Where  $m$  is mass of the target,  $g$  is acceleration of gravity,  $V$  is fly velocity, by equation (22,23) simultaneous calculated, can get:

$$\dot{\psi} = \frac{g}{V} \tan \phi \quad (24)$$

Define vector  $\mathbf{a} = [a_x, a_y]$ ,  $\mathbf{V} = [V_x, V_y]$ , according to vector calculations, acceleration and speed can get the angle cosine, as follows:

$$\cos \langle \mathbf{a}, \mathbf{V} \rangle = \frac{(\mathbf{a}, \mathbf{V})}{|\mathbf{a}| |\mathbf{V}|} \quad (25)$$

Decompose the acceleration to the vertical direction of velocity, Centrifugal force  $F$  can be approximately obtained:

$$F = m \sqrt{a_x^2 + a_y^2} (\sqrt{1 - \cos^2 \langle \mathbf{a}, \mathbf{V} \rangle}) \quad (26)$$

Substituted equation (23,24) into the above equation:

$$m \sqrt{a_x^2 + a_y^2} (\sqrt{1 - \cos^2 \langle \mathbf{a}, \mathbf{V} \rangle}) = m \frac{g}{V} \tan \phi V \quad (27)$$

Roll angle can solve for:

$$\phi = a \tan \left( \frac{\sqrt{a_x^2 + a_y^2} (\sqrt{1 - \cos^2 \langle \mathbf{a}, \mathbf{V} \rangle})}{g} \right) \quad (28)$$

Can be seen from the above equation, Roll angle of the aircraft is only associated with velocity and acceleration. The larger roll angle is, the more quickly the target turns. Determine whether the target is maneuvering or not, we can set a threshold of roll angle, if the roll angle exceeds the threshold, the target is in maneuvering state.

## 5. Design of filter parameters

Define the target state vector  $\mathbf{x}=[x, \dot{x}, y, \dot{y}]'$ , observation vector  $\mathbf{z}$  is  $[x, y]'$ , input vector  $\mathbf{u}$  is  $[a_x, a_y]'$ , Sampling period for the radar is  $T$ , parameters in equation of state can be described as follows:

$$\mathbf{A} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (29)$$

The design of the filter parameters is directly related to the effect of position filtering. Observation noise  $\mathbf{R}$  is mainly due to the random error caused by radar observations, which can be determined by the distance and radar measurement accuracy. The process noise  $\mathbf{Q}$  is usually difficult to determine, the value is entered as a reflection of the level of uncertainty.

### 5.1 R Determination

In the target tracking problem, the obvious advantage of Cartesian coordinate system is that Filtering and the extrapolation process can be implemented on the linear dynamic model. But the radar measurements are done in the spherical coordinate system, therefore, using a Cartesian coordinate system measurement errors inevitably exist in the coupling[9]. Consider the two-dimensional coordinates, measuring distance is  $\rho$ , azimuth angle is  $\theta$ , Tracking the X-Y Cartesian coordinate system completed, Measurement of the form is:

$$x = \rho \cos \theta \quad (30)$$

$$y = \rho \sin \theta \quad (31)$$

Measurement covariance matrix is:

$$\mathbf{R} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{xy}^2 & \sigma_y^2 \end{bmatrix} \quad (32)$$

where:

$$\sigma_x^2 = \sigma_\rho^2 \cos^2 \theta + \rho^2 \sigma_\theta^2 \sin^2 \theta \quad (33)$$

$$\sigma_y^2 = \sigma_\rho^2 \sin^2 \theta + \rho^2 \sigma_\theta^2 \cos^2 \theta \quad (34)$$

$$\sigma_{xy}^2 = 0.5 \sin 2\theta (\sigma_\rho^2 - \rho^2 \sigma_\theta^2) \quad (35)$$

Where,  $\sigma_\rho^2$  and  $\sigma_\theta^2$  represent variance of distance and azimuth angle respectively. When  $\theta$  is not zero, or not equal to  $n\pi/2$ ,  $\sigma_{xy}^2$  is not zero, this means that the coupling measurement exists. As can be seen from the Kalman filter equations, when variance  $\mathbf{R}$  becomes small, measured variable  $\mathbf{z}_k$  increases its weight, and the Predict variable  $\mathbf{Hx}_k$  decreases its weight.

### 5.2 Q Determination

Variance  $\mathbf{Q}$  represents the uncertainty of the input disturbance. Considering the input is acceleration, its expression is the acceleration of the estimation variance, equation (21) is shown. When target is in turn maneuver, the acceleration is estimated lagging, meanwhile, accuracy will also be significantly increased, but equation (21) does not reflect this situation, so the roll angle feedback is introduced into the  $\mathbf{Q}$  calculation, seen in Figure 3. Estimated variance of the new acceleration consists of two parts: Variance of the original acceleration  $\sigma_{a_x}^2$  and roll angle correction  $K_\phi |\phi|$ . When the target is in turn maneuver, using the introduction of roll angle feedback to increase the target's input uncertainty, the size of feedback gain  $K_\phi$  represents the degree of input uncertainty, its value can be selected by trying to carry out simulation.

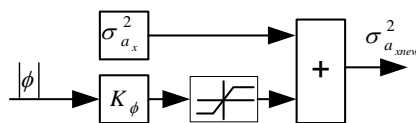


Figure 3. Roll angle feedback figure

Increase in  $\mathbf{Q}$  is actually increasing the priori estimation error covariance  $\mathbf{P}_k$ . Measured variable  $\mathbf{z}_k$  increases its weight, and the Predict variable  $\mathbf{H}\mathbf{x}_k$  decreases its weight, it helps turn fast.

## 6. The simulation results

Taking a simulation of radar data, straight fly and turning show in Figure 4,5 respectively. Figure 4 shows the track of straight fly simulation, two regions marked in dashed box clearly demonstrated that the track restrains disturbance observed effectively, target maintains a smooth track of its own, according to predicted flight direction. In Figure 5, the region marked in dashed box clearly demonstrated fast turn track, target can effectively detect the turn and track fast, the largest deviation is about 1km.

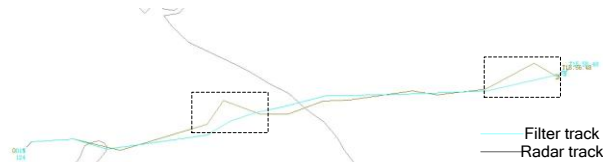


Figure 4. Straight fly track

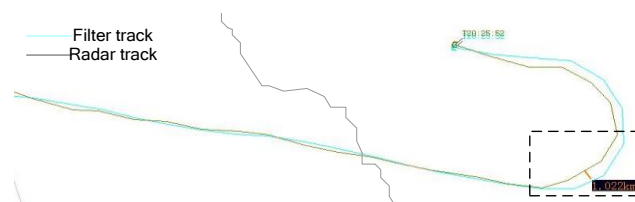


Figure 5. Turn track

## 7. Conclusion

Through the maneuver detecting method, Dynamic number of limited memory points is adjusted for velocity/acceleration estimation, ensuring the smooth for straight flight and less lag for swerve. In the position filter, a roll angle predicted method is proposed, and the roll angle is used for adaptive adjusting of Kalman filter parameter, it is a new attempt, The specific expression of feedback gain is subject to further study.

## 8. References

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