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A Construction Method of Binary Orthogonal Complementary Sequences Sets in MC-CDMA System

Chen Hong[1] and Dai Qiang[2]

[1]Technique Department of Jiuquan Satellite Launching Center, Jiuquan, China
[2]Beijing NO.1 Satellite Earth Station, Beijing, China

Abstract—A construction method of binary orthogonal complementary sequences sets was presented in this paper. The numeration and simulation show that the sequences sets constructed by the means of this paper have low PAPR in the up-link of MC-CDMA. The method has strong practicality in spreading codes design and can be widely used in the system with large number of users.

Key words--MC-CDMA;PAPR; orthogonal complement sequences

1. Intruduction

The Multicarrier Code Division Multiple Access (MC-CDMA) which combing the advantages of OFDM and CDMA has high data rate, excellent frequency spectrum utilization rate and perfect ability of anti-multipath. But the high Peak to Average Power Ratio (PAPR) of the Orthogonal Frequency Division Multiplex (OFDM) becomes the main cause which restricts the application of OFDM. The traditional methods of PAPR reduction have high system complexity, low data rate and will result in signal distortion. In fact, there is a close relation to the PAPR of specified MC-CDMA user and its distributed spreading frequency sequence[1]. Now, the study of constructing the spreading frequency sequences with low PAPR has become one of the hotspots in MC-CDMA mobile communication. It is shown that the transmitting signal's PAPR of binary complementary sequences set is no more than 3dB [2]. Therefore, it is very important to search enough binary complementary sequences sets with satisfied spreading gain and low PAPR. A novel construction method of binary orthogonal complementary sequences based on the traditional extension methods is presented in this paper. In this way, we can easily construct this kind of binary orthogonal complementary sequence whose length equals to the number of the sequences. The correctness of the method in this paper is also verified by the result of numerical calculation and computer simulations.

2. Some Definitions About the Binary Complementary Sequences Sets

Above all, the aperiodic autocorrelations of the N-th spreading sequences $x=[x_1, x_2, ..., x_N]$ is defined by

$$\psi_{x,x}(l) = \sum_{i=1}^{N-|l|} x_i x_{i+|l|}, \qquad |l| \le N-1 \tag{1}$$

Definition 1:assuming that the sum of the aperiodic autocorrelations of two N-th spreading sequences $a=[a_1, a_2, ..., a_N]$ and $b=[b_1, b_2, ..., b_N]$ satisfy

$$\psi_{a,a}(l) + \psi_{b,b}(l) = 2N\delta(l) \tag{2}$$

E-mail address: Chenhong7409@163.com, Korla_daiqiang@sina.com

where $\delta(l)=\begin{cases}1 & l=0\\ 0 & otherwise\end{cases}$, a and b are a pair of binary complementary sequences and any of them is called a complementary sequence. $\psi_{a,a}$ and $\psi_{b,b}$ are the aperiodic autocorrelations of the sequence a and the sequence b respectively. The length of a binary complementary sequence is the sum of squares of two integers and it has been certificated that no 18-th binary complementary sequences exit[3]。

Definition 2：assuming that a pair of binary sequences matrixes A and B, $(A_i \in A, 1 \le i \le p)$, B $(B_i \in B, 1 \le i \le p)$, if they satisfy the following condition:

$$\psi_{A_i,A_i}(l)+\psi_{B_i,B_i}(l)=2N\delta(l) \tag{3}$$

where ψ_{A_i,A_i} and ψ_{B_i,B_i} are the aperiodic autocorrelation of the responding row vectors of the matrixes A and B, then A and B are called a pair of binary complementary sequence sets.

Definition 3: if the upper part row vectors and the corresponding bottom half row vectors of a binary sequences matrix A $(A_i \in A, 1 \le i \le p)$ constitute complementary pairs which conforms to (4) , then the binary matrix A is called a complementary pairs set.

$$\psi_{A_i,A_i}(l)+\psi_{A_{i+\frac{p}{2}},A_{i+\frac{p}{2}}}(l)=2N\delta(l) \qquad (1 \le i \le \frac{p}{2}) \tag{4}$$

Definition 4: if a binary sequences matrix A of order N satisfies $A \cdot A^T = NI_N$ (I_N is the identity matrix)，then matrix A is called a binary orthogonal sequences set.

3. Trditional Extension Methods of Complementary Sequence Pairs

The PAPR of a binary complementary sequence is no more than 3dB. It is pointed out that only four pairs of kernel sequences were found and the sequence lengths of them are 2,10 and 26 respectively. The four pairs of kernel sequences are listed below[4]:

{K2a}=11,{K2b}=10;
{K10a}=10010,10001;{K10b}=00001,00110;
{K26a}=01001,10111,10101,11100,11101,0;
{K26b}=10110,01000,01111,11100,11101,0.

In a MC-CDMA mobile system, there are many users and different spreading sequences should be distributed to different users. Therefore, the number of the complementary sequence pairs must be extended based on the kernel sequence pairs. On the other hand, the length of the sequences must be extended to obtain enough spreading gains. The number and the length can be commonly extended according to the characters of the binary complementary sequences [3].

Character 1: if $\{a_n\}$ and $\{b_n\}$ are a pair of binary complementary sequences, then

a) The data-converted sequences of $\{a_n\}$ and $\{b_n\}$ are a pair of complementary sequences.

b) The sequence in reverse order of $\{a_n\}$ and the sequence $\{b_n\}$ are a pair of complementary sequences.

c) Assuming that the sequence $\{c_n\}$ and $\{d_n\}$ are respectively derived from $\{a_n\}$ and $\{b_n\}$ with reversed bit in even order, so $\{c_n\}$ and $\{d_n\}$ are a pair of complementary sequences.

Character 2: if $\{a_n\}$ ($\{a_n\}=a_1 a_2,...,a_n$)and $\{b_n\}$ ($\{b_n\}=b_1 b_2,...,b_n$)a are a pair of binary complementary sequences, we may have the conclusions as follow:

a)assuming that $s_1 = ab = a_1a_2...a_Nb_1b_2..b_N$, $s_2 = a\bar{b} = a_1a_2...a_N\bar{b}_1\bar{b}_2..\bar{b}_N$ （where $\bar{b}$ means to reverse the responding bit of sequence b）, the sequence s_1 and the sequence s_2 are a pair of complementary sequences.

b) assuming that $s_1 = a_1b_1a_2b_2...a_Nb_N$, $s_2 = a_1\bar{b}_1a_2\bar{b}_2...a_N\bar{b}_N$, the sequence s_1 and the sequence s_2 are a pair of complementary sequences.

In addition, Shapiro and Rudin have found another S-R binary complementary sequence[6]. S-R sequence is the subclass of complementary sequence and has all of the characters of binary complementary sequences. It is easy to extent the number and length of the binary complementary sequences based on S-R sequences. For example, we assume that sequences $\{\alpha_i\}$ and $\{\beta_i\}$ are two 2-th binary complementary sequences and we can obtain 4-th binary complementary sequences through connecting the front part and the reversed end of the sequence $\{\alpha_i\}$ or $\{\beta_i\}$. In this way, we can extent the length of a complementary sequence in MC-CDMA system.

But some questions exit in the method above:

a) We can only compound new S-R sequences one by one based on the existed binary sequences. When there are many users in a MC-CDMA mobile system, the workload is heavy.

b) We can only extent the length of a pair of complementary sequences according to character 2 and the constitution method of S-R sequence.

c) The constituted complementary sequences may not be orthogonal. For example, the sequence T_1=[1 1 1 1 1 -1 -1 1] and the sequence T_2=[1 -1 1 -1 1 1 -1 -1] are a pair of complementary sequences, the sequence S_1=[1 1 1 -1 1 1 -1 1] and the sequence S_2=[1 1 1 -1 -1 -1 1 -1] are another pair of complementary sequences, but these two pairs of sequences are not orthogonal. If non-orthogonal spreading sequences are used in MC-CDMA systems, the multiple access interference(MAI) will be serious when there are many active users in the mobile system.

4. Construction Method of Binary Orthogonal Complementary Sequences in This Paper

Aiming at the above problems, a fast construction method of binary orthogonal complementary sequences is presented in this paper. With this method, we can easily construct the orthogonal complementary sequences whose number equals to the sequence length. The step of this method can be expressed as follow.

a) Construct $N \times N$ Walsh-Hadamard matrix

Considering that a 2×2 Walsh-Hadamard matrix $H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$,the N×N Walsh-Hadamard matrix can be generated by the recursion

$$H_N = H_{\frac{N}{2}} \otimes H_2 = \begin{bmatrix} H_{\frac{N}{2}} & H_{\frac{N}{2}} \\ H_{\frac{N}{2}} & -H_{\frac{N}{2}} \end{bmatrix} \tag{5}$$

where $\otimes$ denotes Kronecker product operation of a matrix，$N=2^p$, p=1,2,…

b）Construct a pair of N-th S-R complementary sequences

The N-th S sequence and R sequence of the S-R sequences can be generated by the means of recursion [7] . Considering that the sequence s_2 is[1 1] and the sequence c_2 is [1 -1]，the sequences s_N and c_N can be generated as following:

$$s_N = s_{\frac{N}{2}} c_{\frac{N}{2}} \tag{6}$$

$$c_N = s_{\frac{N}{2}} \bar{c}_{\frac{N}{2}} \tag{7}$$

c）Multiplying the sequence s_N or c_N with the corresponding row vectors of matrix H_N, a pair of binary orthogonal complementary matrix H_N' and H_N'' can be generated. For example, the 8-th S-R complementary sequence s_8 is[1 1 1 -1 1 1 -1 1] and c_8 is [1 1 1 -1 -1 -1 1 -1],the matrix H_8' and the matrix H_8'' can be generated through multiplying the sequence s_8 or the sequence c_8 with the corresponding row vectors of 8×8 Walsh-Hadamard matrix. It can be tested that H_8' and H_8'' is a pair of complementary sequence set and both matrixes are orthogonal complementary matrixes whose upper row vectors and line vectors constructing complementary sequence sets.

$$H_8' = \begin{bmatrix} +1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 \\ +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 \\ +1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 \\ +1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 \\ +1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 \\ +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 \\ +1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 \end{bmatrix} \qquad H_8'' = \begin{bmatrix} +1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 \\ +1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 \\ +1 & +1 & -1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & -1 & -1 & -1 & +1 & -1 & -1 \\ +1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 \\ +1 & -1 & +1 & +1 & +1 & -1 & -1 & -1 \\ +1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 \\ +1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 \end{bmatrix}$$

It can be validated that the sequences sets H_N' and H_N'' constructed by the method presented in this paper have the characters as following:

a) H_N' is a binary complementary sequences set.

Proof: considering two S-R sequences of length 2 s_2 is [1 1] and c_2 is [1 -1], the complementary sequences s_N and c_N can be expressed as following:

$$s_N = s_{\frac{N}{2}} c_{\frac{N}{2}} = s_{\frac{N}{4}} c_{\frac{N}{4}} s_{\frac{N}{4}} \bar{c}_{\frac{N}{4}} \tag{8}$$

$$c_N = s_{\frac{N}{2}} \bar{c}_{\frac{N}{2}} = s_{\frac{N}{4}} c_{\frac{N}{4}} \bar{s}_{\frac{N}{4}} c_{\frac{N}{4}} \tag{9}$$

Assuming that $H_2^s = H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ and $H_2^C = H_2 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$, so the matrix H_N' can be generated as following:

$$H_N' = s_N .* H_N = s_N .* \begin{bmatrix} H_{\frac{N}{2}} & H_{\frac{N}{2}} \\ H_{\frac{N}{2}} & -H_{\frac{N}{2}} \end{bmatrix} = \begin{bmatrix} A \\ B \end{bmatrix} \tag{10}$$

$$\begin{aligned} A = H_N^s &= s_N .* \begin{bmatrix} H_{\frac{N}{2}} & H_{\frac{N}{2}} \end{bmatrix} \\ &= s_{\frac{N}{4}} c_{\frac{N}{4}} s_{\frac{N}{4}} \bar{c}_{\frac{N}{4}} \begin{bmatrix} H_{\frac{N}{2}} & H_{\frac{N}{2}} \end{bmatrix} = \begin{bmatrix} H_{\frac{N}{2}}^s & \tilde{H}_{\frac{N}{2}}^s \end{bmatrix} \end{aligned} \tag{11}$$

$$\begin{aligned} B = H_N^c &= s_N .* \begin{bmatrix} H_{\frac{N}{2}} & -H_{\frac{N}{2}} \end{bmatrix} \\ &= s_{\frac{N}{4}} c_{\frac{N}{4}} s_{\frac{N}{4}} \bar{c}_{\frac{N}{4}} \begin{bmatrix} H_{\frac{N}{2}} & -H_{\frac{N}{2}} \end{bmatrix} = \begin{bmatrix} H_{\frac{N}{2}}^c & \tilde{H}_{\frac{N}{2}}^c \end{bmatrix} \end{aligned} \tag{12}$$

where $.*$ means to multiply sequence s_N with every row vector of a matrix. The front part of $\tilde{H}_N$ and H_N is identical and the rear part of $\tilde{H}_N$ and H_N is reverse and can be described as following:

$$\tilde{H}_N = P_N H_N Q_N \tag{13}$$

where, $Q_N = I_N$ is the identity matrix of order N and $P_N = \begin{bmatrix} 0 & I_{\frac{N}{2}} \\ I_{\frac{N}{2}} & 0 \end{bmatrix}$.

Because both matrix H_2^s and H_2^c are complementary matrix of order 2 , the (9) and (10) satisfy the length extension regulation of S-R sequence. So the row vectors of matrix A and B are all S-R sequences. We have known that S-R sequence is the sub-class of binary complementary sequence, so the row vectors of matrix A and B are all binary complementary sequences.

On the other hand, because $B = s_N .* \begin{bmatrix} H_{\frac{N}{2}} & -H_{\frac{N}{2}} \end{bmatrix} = c_N .* \begin{bmatrix} H_{\frac{N}{2}} & H_{\frac{N}{2}} \end{bmatrix}$, so every row vector of A and the corresponding row vector of B are a pair of complementary sequences and the matrix $H_N^{'}$ is a binary complementary matrix.

b) The matrix H_N' and H_N'' is a pair of complementary sequences set.

Proof: the matrix H_N'' can be constructed though multiplying the S-R sequence c_N of length N with the row vectors of N×N Walsh-Hadamard matrix. Therefore, H_N'' can be rewritten as

$$\begin{aligned} H_N^{''} = c_N .* H_N &= c_N .* \begin{bmatrix} H_{\frac{N}{2}} & H_{\frac{N}{2}} \\ H_{\frac{N}{2}} & -H_{\frac{N}{2}} \end{bmatrix} \\ &= s_N .* \begin{bmatrix} H_{\frac{N}{2}} & -H_{\frac{N}{2}} \\ H_{\frac{N}{2}} & H_{\frac{N}{2}} \end{bmatrix} = \begin{bmatrix} B \\ A \end{bmatrix} \end{aligned} \tag{14}$$

From (8) we have known that H_N' equals $\begin{bmatrix} A \\ B \end{bmatrix}$ and we have proofed that the row vectors of A and the row vectors of B constructed a pair of complementary sequences, so matrix H_N' and H_N'' is a pair of complementary sets.

c) Both H_N' and H_N'' are binary orthogonal sequence set.

Proof：because

$H_N' = s_N.*H_N = c_N.*\begin{bmatrix} H_{\frac{N}{2}} & H_{\frac{N}{2}} \\ H_{\frac{N}{2}} & -H_{\frac{N}{2}} \end{bmatrix}$, it is easy to know that $H_N' \cdot H_N'^{\,T} = NI_N$, so H_N' is binary orthogonal sequence set. We can also proof that H_N'' is a binary orthogonal sequence set for the same reason.

5. Simulation Results

The PAPR of the uplink in the MC-CDMA system which using different spreading code such as Walsh sequence, OGold sequence[8]and the binary orthogonal complementary sequence generated in this paper is shown in Figure 1 and Figure 2. From Figure 1 , it is easy to know that Walsh sequence has the largest PAPR and the orthogonal complementary sequence has the lowest PAPR. Figure 2 shows that the PAPRs of complementary sequences with different length are no more than 3dB and conforms to the conclusion in reference 2 .

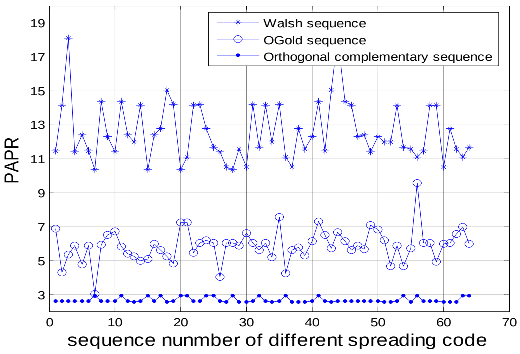

Figure 1 the PAPR of different spreading codes set （N_carrier=64）

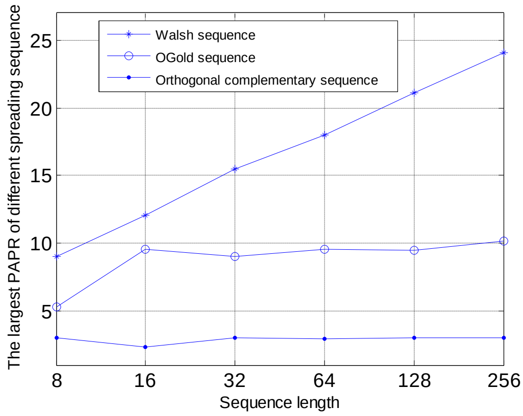

Figure 2. The largest PAPR of different sequence with different length

6. Conclusion

The method of constructing the binary orthogonal complementary sequences with low PAPR is presented in this paper. This method has low calculating works and strong practicality in spreading sequence design in MC-CDMA system. The method in this paper has more superiority when many users exit and the spreading sequence length is long in a MC-CDMA mobile system.

7. References

[1] Runseok Kang, Kweonhue Choi,Sooyoung Kim Shin, Reduced Search for optimum code sets to reduce papr in MC-CDMA System [J] 2002. The 5th International Symposium on Wireless Personal Multimedia Communications vol.1 135_139

[2] James A. Davis,Jonathan Jedwab, Peak-to-mean power control in OFDM, Golay complementary sequences,and Reed-Muller codes。[J] rIEEE Transactions on Information Theory Nov. 1999, 2397_2417.

[3] M.J.E. Golay, Complementary series,[J] IEEE Transactions on. Information Theory, vol.IT-7,pp.82-87,Apr.1961.

[4] Karkkainen Kari H.A, Leppanen Pentti A. Linear complexity of binary golay complementary sequences[J]. IEICE trans fund. 1996. E79-A(4):609-613.

[5] W.Rudin, Some theorems on Fourier coefficients, Proc. Amer. Math.Soc,1959,No.10,855-858.

[6] Byoung-Jo Choi,Chintha Tellambura, Lajos Hanzo, Crest factors of Shapiro-Rudin sequence based multi-code MC-CDMA signals[J] 2002. VTC Spring 2002. IEEE 55th Vehicular Technology Conference. vol.3,1472_1476

[7] Xiaojing Huang, Yunxin Li, Scalable complete complementary sets of sequences [J] 2002. GLOBECOM '02. IEEE Global Telecommunications Conference vol.2 1056_1060.

[8] Popovic Branistlav M. Efficient despreaders for multicode CDMA systems [J]. Proc IEEE ICUPC'97 [C].SanDiego,1997,516-520

BIOGRAPHIES

Chen Hong was born in Chendu, Si chuan province , China in 1974. He received M.S. and Ph.D. degrees both in National University Defense Technology in 2003 and 2007. He is currently the Senior Engineer of Jiuquan Satellite Launching Center of China. His main research interests are in the field of wireless communication and Radar signal processing.