

Performance Analysis of System Outage Based on Network Coding in Mobile Satellite Communication System

Sun Wei^{a,*}, Zhang Gengxin^a, Jing Hu^a

^aInstitute of Communications Engineering, PLA University of Science and Technology, Nanjing 210007, China

*Institute of Communications Engineering, PLA University of Science and Technology, Pox Box 14, No 2 Biao Ying Road, Yu Dao Street, Nanjing, Jiang Su, 210007, China

Abstract. This paper proposes a novel network-coded cooperation scheme in mobile satellite communication system. The scenario under consideration is one which two “partner”—Terminal A and Terminal B— cooperate in transmitting information to a single destination; Each partner transmits both locally generated information and adaptive information that is formed based on the observed information from the other partner. A key observation is that when one partner knows the other partner’s local information, it can exploit that knowledge in the next transmission to improve the BER of the System. This leads to an encoding scheme in which each partner transmits the algebraic superposition of its local and the other information, and the superimposed codeword is interpreted differently at the two partners based on their different a priori knowledge. Decoding at the destination is then carried on by iterating between the codewords from the two partners. It is shown via simulation that the proposed scheme provides substantial coding gain over other techniques, including those based on time multiplexing and signal superposition.

Keywords: network coding, cooperative diversity, system outage

1. Introduction

The wireless channel in mobile satellite communication is a typical fading channel. To obtain reliable communications, there is a significant need for method of combating detrimental effects in this wireless fading. Most of the current existed mobile satellite systems use the convolution coding and interleave techniques to overcome the effects of the fading. Some of them also use diversity reception techniques, such as Globalstar system [1]. Because of its significant capacity and performance, spatial diversity has been widely accepted as one of the most effective ways to combat fading [2] [3] over wireless channels. However, a large amount of relay nodes needs to be disposed and more frequency resources have to be used. Multiple-input-multiple-output (MIMO) [4][5] is another option, which requires the transmitter to have a number of transmit antennas. In mobile satellite communications, because of constraints from the equipment’s size, hardware and power consumption, the advantages of transmit diversity techniques are not practical. As a result, alternate approaches, such as distributed antenna system (DAS) [6] and user cooperative transmission, have been proposed to provide spatial diversity.

So far, various protocols in cooperative communication, which allow the relay to simply process and forward what it has heard from a certain source node, have been proposed, Where “process” includes amplifying, repetition coding and some more complicated channel coding, etc. for example, Laneman et al. develop cooperative protocols for a pair of terminals based upon relays amplifying their received signals or fully decoding and repeating information [7]. In [8], the authors propose an opportunistic relaying

* Corresponding author. Tel.: +0-86-15996301567.
E-mail address: sunwei_lijuan@163.com.

cooperation scheme which achieves the same diversity-multiplexing tradeoff as achieved by the space-time coded cooperation scheme. Hunter et al. [8] [9] introduce coded cooperation which combines cooperation with channel coding. To differentiate from the network-coded cooperation (NCC) schemes in the following, we call the above mentioned protocols conventional cooperation (CC) schemes.

The idea of network coding was first proposed by Yeung et al. [10] to enhance the capacity of noiseless wired network. In this paper, we investigate the additional diversity gain facilitated by this type of network coding in wireless networks. As an initial study, we do not incorporate other distributed channel coding in this paper, however, it is possible to apply existing channel coding techniques on top of the network coding scheme studied here for further performance improvement.

The probability of system outage is adopted as the criterion for our analysis of small network (containing two or three nodes). System outage occurs when the destination (e.g., the base station) is unable to correctly receive data from any one of its users. Through theoretical analysis and numerical evaluation between CC schemes and NCC schemes, we show that network coding offers improved diversity and more design flexibility in wireless network.

The rest of this paper is organized as follows. Section II describes how network coding can be applied in two-user network and system description. The performance of the system is analyzed in section III. We also provide the numerical evaluation of system performance in Section IV and conclude our work in Section V.

2. System Description

The scenarios addressed in this paper are depicted in Fig.1. Two Terminal A and B work in cooperation to deliver their packets X_A and X_B to a common destination Satellite D. we refer to user A and B as each other's partner. In CC scheme, as shown in Fig. 1(a), user A transmits at first section, and then Node B transmits. Each source node receives the packet sent by its partner user and attempts to decode its partner's information. If decoding is successful, then some of this information — the part that originated at the transmitting partner — will be relayed in a future transmission to provide the satellite D with spatial diversity. If a source node fails to decode its partner's information, therefore each source send its own packet again in the next time slot.

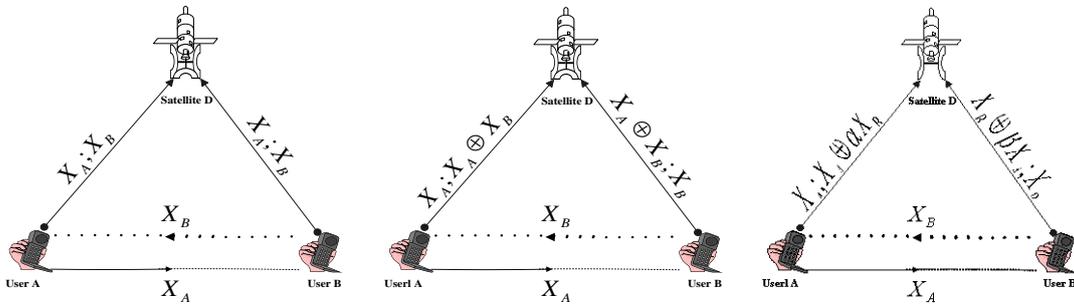


Fig. 1. (a)Conventional cooperation scheme; (b) NCC-1 scheme; (c)NCC-2 scheme

In NCC scheme, when user A (B) successfully decode the packet of B (A), in the next time slot, we classify the NCC scheme into two kinds, NCC-1 and NCC-2, by the what information to be transmitted. If user A (B) relays $X_A \oplus X_B$, it is called NCC-1 as shown in Fig. 1(b). If node A (B) relays $X_A \oplus \alpha X_B$ ($X_B \oplus \beta X_A$), where α and β are elements of a finite field F_q of $q = 2^n$ and $\alpha \neq \beta$, it is called NCC-2, as shown in Fig. 1(c).

The following analysis and evaluations are based on these simple scenarios.

3. Performance Analysis

Suppose that the probability of outage of the link from node X to node Y is p_{xy} . Considering all the scenarios in Fig. 1, the probabilities of system outage, which the Satellite D can not successfully decode X_A and X_B , will be expressed as follow.

Theorem 1: In Fig.1 (a), the probability of system outage can be expressed as follow:

$$P_{out}^1 = p_{ad}(1-p_{ba}(1-p_{ad}))(p_{ab}+(1-p_{ab})p_{bd})+p_{bd}(1-p_{ab}(1-p_{bd}))(p_{ba}+(1-p_{ba})p_{ad})-p_{ad}^2p_{bd}^2 \quad (1)$$

Proof: the probabilities of outage of the user A and B are P^a and P^b , respectively. And the joint probability of outage of the user A and B is P^{ab} .

$$P^a = p_{ad}(p_{ba}p_{ad}(p_{ab}+(1-p_{ab})p_{bd})+(1-p_{ba})(p_{ab}+(1-p_{ab})p_{bd})) \quad (2)$$

$$P^b = p_{bd}(p_{ab}p_{bd}(p_{ba}+(1-p_{ba})p_{ad})+(1-p_{ab})(p_{ba}+(1-p_{ba})p_{ad})) \quad (3)$$

$$P^{ab} = p_{ad}^2p_{bd}^2 \quad (4)$$

Because

$$P_{out}^1 = P^a + P^b - P^{ab} \quad (5)$$

Thus we can get the formula (1) by formula (2), (3) and (4) into formula (5).

Theorem 2: In Fig.1 (b), the probability of system outage can be expressed as follow:

$$P_{out}^2 = -3p_{ad}^2p_{bd}^2 + p_{ad}^2p_{ab}p_{bd}^2 - 2p_{ba}p_{bd}^2p_{ad} + 2p_{ad}^2p_{bd} + 2p_{bd}^2p_{ad} + p_{bd}^2p_{ba} + p_{ad}^2p_{ab} + p_{ba}p_{bd}^2p_{ad}^2 - 2p_{ad}^2p_{ab}p_{bd} \quad (6)$$

Proof: the probabilities of outage of the user A and B are P^a and P^b , respectively. And the joint probability of outage of the user A and B is P^{ab} .

$$P^a = p_{ad}(1-p_{ba})(1-p_{ad})(p_{ab}p_{bd}^2 + p_{bd}(1-p_{ab})) + p_{ad}^2(1-(1-p_{bd}^2)^2(1-p_{ab})) \quad (7)$$

$$P^b = p_{bd}(1-p_{ab})(1-p_{bd})(p_{ba}p_{ad}^2 + p_{ad}(1-p_{ba})) + p_{bd}^2(1-p_{ad}^2)^2(1-p_{ba}) \quad (8)$$

$$P^{ab} = p_{ad}p_{bd}(p_{ba}p_{ad}p_{ab}p_{bd} + p_{ba}p_{ad}(1-p_{ab}) + (1-p_{ba})p_{ab}p_{bd} + (1-p_{ab})(1-p_{ba})) \quad (9)$$

Because the probability of system outage is $P_{out}^2 = P^a + P^b - P^{ab}$, so the formula (6) can be gotten by using formula (7), (8) and (9).

Theorem 3: In Fig.1 (c), the probability of system outage can be expressed as follow:

$$P_{out}^3 = -p_{ad}p_{ab}p_{bd}^2 + 3p_{ad}^2p_{bd} + 2p_{ad}^2p_{ab}p_{bd}^2 - 3p_{ab}p_{ad}^2p_{bd} + 2p_{ba}p_{bd}^2p_{ad}^2 + 3p_{bd}^2p_{ad} + p_{ad}p_{ab}p_{bd}^2p_{ba} - p_{ad}^2p_{ab}p_{ba}p_{bd}^2 + p_{ad}^2p_{ab} + p_{ad}p_{ab}p_{bd}^2p_{ba} - p_{ad}^2p_{ab}p_{ba}p_{bd}^2 + p_{ad}^2p_{ab} \quad (10)$$

Proof: the probabilities of outage of the user A and B are P^a and P^b , respectively. And the joint probability of outage of the user A and B is P^{ab} . According to [12], if the size of finite field F_q is large enough, the destination can decode the source information by high probability. In the article, suppose that the size of finite field F_q is large enough, so that the node D can successfully recover X_A and X_B from the received messages $X_A \oplus \alpha X_B$ and $X_B \oplus \beta X_A$.

$$P^a = p_{ad}(p_{ad}p_{ab}p_{bd}^2 + p_{ad}p_{ab} + 2p_{ad}p_{bd} + p_{bd}^2 - 2p_{bd}p_{ad} - p_{bd}^2p_{ba} + p_{ba}p_{bd}^2p_{ad} - 2p_{ad}p_{ab}p_{bd}) \quad (11)$$

$$P^b = p_{bd}(p_{bd}p_{ba}p_{ad}^2 + p_{bd}p_{ba} + 2p_{bd}p_{ad} + p_{ad}^2 - 2p_{ad}p_{bd} - p_{ad}^2p_{ab} + p_{ab}p_{ad}^2p_{bd} - 2p_{bd}p_{ba}p_{ad}) \quad (12)$$

$$P^{ab} = p_{ad}p_{bd}(p_{ad} + p_{bd} - p_{ad}p_{bd} - p_{bd}p_{ba} + p_{ba}p_{bd}p_{ad} - p_{ad}p_{ab} + p_{ab}p_{ad}p_{bd}) \quad (13)$$

Because the probability of system outage is $P_{out}^3 = P^a + P^b - P^{ab}$, so the formula (10) can be gotten by using formula (11), (12) and (13).

To simplify the calculation without loss of generality, we simple assume the outage probability of the links from A to D and B to D are all p , besides the outage probability of the links between A and B are all q . The probabilities of system outage can be expressed as:

$$P_{out}^1 = 2p(1-q(1-p))(q+(1-q)p) - p^4 \quad (14)$$

$$P_{out}^2 = p^2(-2p^2 + 2p + 1 - 2q^2p + q^2 + q^2p^2) \quad (15)$$

$$P_{out}^3 = -p^2(-pq^2 + 7pq - 4p^2q + p^2q^2 - 2q - 6p + 4p^2) \quad (16)$$

As $p \ll q \ll 1$, three schemes achieve a diversity order of 2. Next, a numerical performance evaluation will be provided in Section IV.

4. Performance Evaluation

Suppose that the distance from the node A to the satellite D and that from the node B to the satellite D are comparable, and the channel gain is a randomized variable obeying the complex Gaussian distribution. That is $h \sim CN(0,1)$. As the channel gain is exponentially related to the distance according to free space

transmission model and distance from the node A and B to the satellite D changes a little during the transmission. To simplify the calculation, we assume that the BERs of the links between node A and B are q , and BERs are used as probability of outage. When BPSK modulation is adopted, the uplink BER can be calculated through the formula below:

$$p_e = E \left[Q \left(\sqrt{2|h|^2 SNR} \right) \right] = \frac{1}{2} \left(1 - \sqrt{\frac{SNR}{1+SNR}} \right) \quad (13)$$

Where $SNR=E/N_o$, E represents transmission power, and N_o is unilateral Gaussian noise power.

Fig. 2, Fig. 3, Fig. 4 and Fig. 5 show the probabilities of system outage with changed SNR, when q is equal to 0, 0.1, 0.5 and 1, respectively. We can see that with the increasing of q , the probabilities of system outage are close to the no cooperation scheme, which is CC scheme with q equal to 1. That is because, with the increasing of q , the assisting user can not recover the original information, and therefore the assisting user is gradually out of action, while the two kinds of NCC schemes perform better than the CC scheme, and the NCC-2 scheme does better than the NCC-1 scheme. The performance of the NCC-2 scheme is better than the NCC-1 scheme, which is because that when node D can not correctly receiver both information X_A and X_B at the first time slot, and user A and B successfully decode each other 's information, at the second time slot, for NCC-1 scheme, A and B will all send $X_A \oplus X_B$, even though satellite D correctly receiver assisting information $X_A \oplus X_B$ from A and B, satellite D can not decode X_A and X_B ; However, for NCC-2 scheme, at the second time slot, A and B will send $X_A \oplus \alpha X_B$ and $X_B \oplus \beta X_A$, if satellite D correctly receiver assisting information $X_A \oplus \alpha X_B$ and $X_B \oplus \beta X_A$, satellite D can decode X_A and X_B . When q is equal to 1, which means that the assisting user can not recover the original information from the partner, hence the performance of the three schemes is equal.

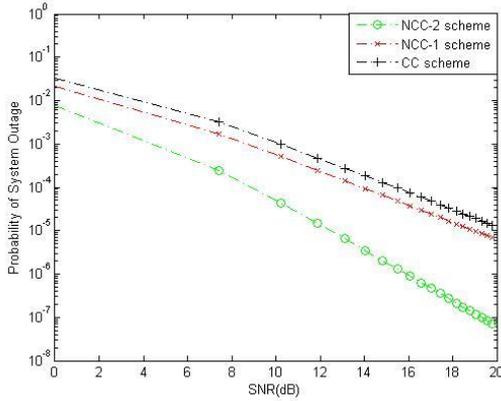


Fig. 2 probabilities of system outage with $q=0$

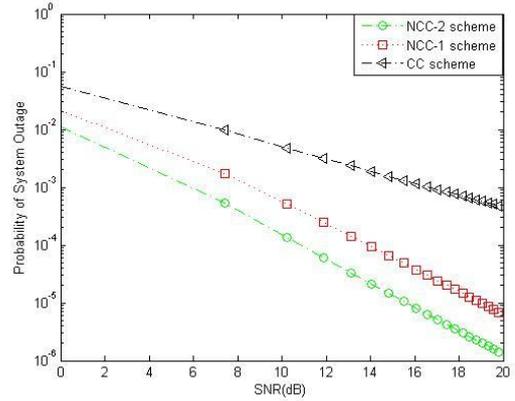


Fig. 3 probabilities of system outage with $q=0.1$

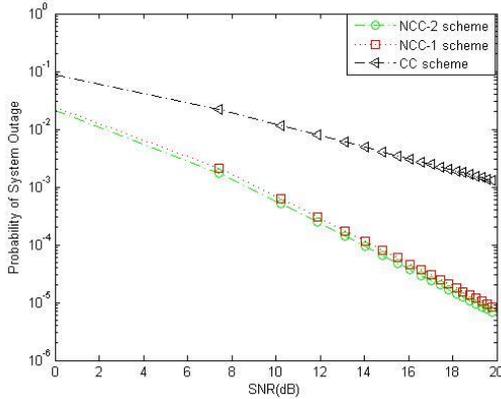


Fig. 4 probabilities of system outage with $q=0.5$

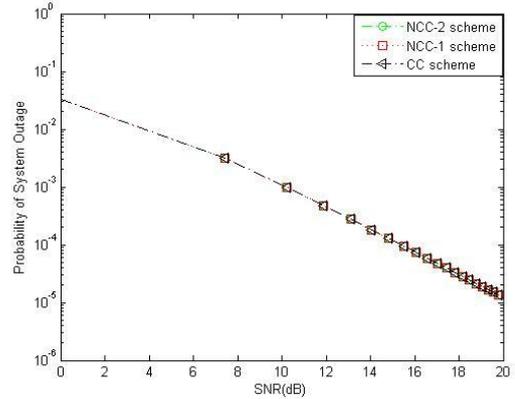


Fig. 5 probabilities of system outage with $q=1$

5. Conclusion

In this paper, we analyzed the application of network coding in mobile satellite communication that supports user cooperation between two terminals. Theoretical analysis and simulation show that network coding can improve the system performance. The research in this paper is based on decoding and forwarding

in the cooperative node. However, the cooperative node can also combine received information to transmit without decoding. And the destination node needs to decode with the help of physical technology. Analysis of amplifying and forwarding in the cooperative node will be concentrated in our future work. In addition, channel coding can be used on top of network coding, and future improvement would be possible.

6. Acknowledgements

This work is supported by National Natural Science Foundation of China (60972062 and 61032004), National High Technology Research and Development Program of China (“863” Program) (2008AA12A204).

7. References

- [1] GS-TR-94-0001, Description of the Globalstar System [S].Louie H , Burns M, Lima C. An introduction and user's guide to the IEEE Smart Grid Web Portal. *Innovative Smart Grid Technologies Conf. Europe (ISGT Europe)* 2010;1–5, .
- [2] A. Nosratinia, T.E. Hunter, A. Hedayat. Cooperative communication in wireless networks. *IEEE Communication Magazine* 2004; 42:74-80.
- [3] W. Roh., A. Paulraj. MIMO channel capacity for distributed antenna systems. *Proc. IEEE VTC 2002*: 2:207-209.
- [4] Saleh, A. A. Rustako. A. Roman, R.. Distributed antennas for indoor radio communications. *IEEE Transaction on Communications* 1987; 35:1245-1251.
- [5] Laneman, J.N. Wornell, G.W.. Distributed space-time coded protocols. *IEEE GLOBE-COM'02* 2002;1:77-81.
- [6] A. Nosratinia, T.E. hunter, A. Hedayat. Cooperative communication in wireless networks. *IEEE on Communications Magazine* 2004; 42: 74-80.
- [7] J. Laneman, D. Tse, and G. Wornell. Cooperative diversity in wireless networks: efficient protocols and outage behavior. *IEEE Trans. Inform. Theory* 2004; 50: 3062–3080.
- [8] A. Bletsas, A. Khisti, D. Reed, and A. Lippman. A simple cooperative diversity method based on network path selection. *IEEE J. Select. Areas Commun.* 2006; 24:659– 672.
- [9] T. Hunter and A. Nostratinia. Diversity through coded cooperation. *IEEE Trans. Wireless Commun.* 2006; 5: 283–289.
- [10] R. Ahlswede, N. Cai, S.-Y. Li, and R. Yeung. Network information flow. *IEEE Trans. Inform. Theory* 2000; 46: 1204–1216.