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Fast Algorithm of Parameters Estimation of High Speed Moving Target Based on Conjugate Gradients and Its Test

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Abstract—Wideband signal processing is always realized by wideband correlative processing which has a time scale and time delay 2-D peak value search calculation structure. Aiming at the relatively high parameters estimation precision, the structure should have a high search density, but it would cause a heavy operation which system cannot afford. Then a fast algorithm of parameters estimation based on conjugate gradients was advanced. The computer simulation of fast algorithm was accomplished and parameters estimation deviation variance curves were calculated by Monte Carlo method. The simulated reflected signal of high-speed moving target was collected in lake test and the fast algorithm were applied on the target detection and parameters estimation under the low SNR. The results of simulation and lake test showed that the fast algorithm based on conjugate gradients could reach the peak value by low operation and relatively high parameters estimation precision by iterative calculation.

Keywords-parameters estimation; conjugate gradients; fast algorithm; Monte Carlo; lake test

1. Introduction

Detection and parameters estimation on high-speed moving target is focused greatly by researchers, and its key point is how to choose appropriate transmitting signal and corresponding algorithm of signal process. The reflected signal of target has the rich information about target, weaker reverberation and improved performance in signal detection and parameters estimation when using a wideband signal as transmitting signal, which makes wideband signal process an active area of parameters estimation of underwater high speed moving target. Wideband signal processing is realized by wideband correlative processing which has a time scale and time delay 2-D peak value search calculation structure. Aiming at the relatively high parameters estimation precision, the structure must have a high search density, so it would cause a heavy operation which system cannot afford which limit its application. To solve the problem, many fast algorithms are brought forward. In [1] and [2], continuous wavelet transform (CWT) is advanced as a method of wideband signal process calculation. From [3] to [5], some fast algorithms which apply to CWT are advanced, such as FFT, Mellin transform, fast algorithm of “a trous”[1-5]. Although these algorithms can reduce the operation, the system can not afford it. Therefore, a fast algorithm of parameters estimation based on conjugate gradients is advanced in the paper. The results of simulation and lake test showed that the fast algorithm could obtain the peak value with low operation and relatively high parameters estimation precision by iterative calculation..

2. Calculation Structure of Parameters Estimation of Wideband Correlative Processing

Considering transmitting signal is $f(t)$, receiving signal is $r(t)$, then:

$$r(t) = g(t) + n(t) \quad (1)$$

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where $g(t)$ is reflected signal of target; $n(t)$ is White Gauss Noise (WGN). Assume the speed of target is v, the expression of target echo is showed as follow:

$$g(t) = \frac{1}{\sqrt{a_0}} \cdot f(\frac{t-\tau_0}{a_0}) \tag{2}$$

where a_0 and τ_0 is time scale and time delay related to target speed and range respectively. $a_0 = (c+v)/(c-v)$.The detection statistic expression is as below [6]:

$$\eta = \max_{a_m,\tau_n} \left| \int_0^T r(t) \cdot \frac{1}{\sqrt{a_m}} f^*(\frac{t-\tau_n}{a_m}) dt \right| \mathop{\gtrless} \gamma' \tag{3}$$

where a_m and τ_n is discrete time scale and time delay respectively. γ' is the detection threshold.

Wideband correlative processing can be realized by continuous wavelet transform (CWT). Let transmitting signal $f(t)$ be mother wavelet, the receiving signal $r(t)$ is transformed into function $W_f r(a,b)$ with $b = \tau$. In the engineering application of wideband correlative processing, time scale a and time delay b will be discrete, calculate the values of correlative processing point by point, then search out the position of peak value and its corresponding a and b are parameters estimation value. Let scale a and time delay b be discrete, gives a_i, b_j, $i = 1,2,\cdots,I$, $j = 1,2,\cdots,J$. The calculating expression of CWT is:

$$W_f r(a_i, b_j) = \frac{1}{\sqrt{a_i}} \int_{-\infty}^{+\infty} r(t) f^*(\frac{t-b_j}{a_i}) dt \tag{4}$$

The expression of parameters estimation is as below:

$$\{\hat{a}, \hat{b}\} = \arg\max_{a_i,b_j} \left| W_f r(a_i, b_j) \right| \tag{5}$$

The value of parameters estimation $\{\hat{a}, \hat{b}\}$ is the position corresponding to peak of $|W_f r(a_i, b_j)|$. To ensure to get the value of parameters estimation precisely, the intervals of time scale a_i and time delay b_j should be narrow enough. But it also brings a problem of large operation. Thus a fast algorithm is needed.

3. Fast Algorithm based on Conjugate Gradients

3.1. Calculation of Conjugate Direction

Defining parameters estimation vector be $\hat{w} = [\hat{a}, \hat{b}]^T$, a is time scale, b is time delay. The result of correlative processing is $R(w)$. Take Taylor expansion of $R(w)$ at the initial value point $w_1 = (a_1, b_1)^T$, omit the high order terms, the result is:

$$R(\hat{w}) = R(w_1) + R'(w)|_{w_1} (\hat{w} - w_1) + \frac{1}{2} R''(w)|_{w_1} (\hat{w} - w_1)^2 \tag{6}$$

The expression of correlative processing peak value is:

$$R(\hat{w}) = R(w_1) + B^T (\hat{w} - w_1) + \frac{1}{2} (\hat{w} - w_1)^T H (\hat{w} - w_1) \tag{7}$$

where $B = R'(w)\big|_{w_1} = \left[\partial R/\partial a, \partial R/\partial b\right]\Big|_{a_1,b_1}^T$ is constant vector, $H = \begin{bmatrix} \partial^2 R/\partial a^2 & \partial^2 R/\partial a \partial b \\ \partial^2 R/\partial b \partial a & \partial^2 R/\partial b^2 \end{bmatrix}\Bigg|_{a_1,b_1}$ is constant matrix, and the correlative processing result $R(w)$'s partial gradients is $\nabla R(w) = B + Hw$. At the peak point of $\hat{w}$, it gives:

$$B + H\hat{w} = 0 \tag{8}$$

Suppose a group of W-Dimension vectors d_i conjugate with H:

$$d_j^T H d_i = 0 \qquad \forall i \neq j \tag{9}$$

When H is a positively definite matrix, the vectors above are linear independent, and form a complete non-orthogonal radix in weight space. The difference value from initial points w_1 to $\hat{w}$ can be expressed as a linear combination of conjugate direction vectors.

$$\hat{w} - w_1 = \sum_{i=1} \lambda_i d_i \tag{10}$$

Define $$w_j = w_1 + \sum_{i=1}^{j-1} \lambda_i d_i \tag{11}$$

Then (11) can be transformed into its iterative form:

$$w_{j+1} = w_j + \lambda_j d_j \tag{12}$$

The iterative approach is parallel with conjugate directions, λ_i controls the iterative step length. Let $d_j^T H$ multiply (10) and with expression of $B + H\hat{w} = 0$, gives :

$$-d_j^T (B + Hw_1) = \sum_{i=1} \lambda_i d_j^T Hd_i \tag{13}$$

Accordingly gives:

$$\lambda_j = -d_j^T \left(B + Hw_1\right) \Big/ \left(d_j^T Hd_j\right) \tag{14}$$

λ_j also can be expressed as[7]:

$$\lambda_j = -d_j^T g_j \Big/ \left(d_j^T Hd_j\right) \tag{15}$$

where $g_j = g(w_j) = B + Hw_j$

$$g_{j+1} - g_j = H(w_{j+1} - w_j) = \lambda_j Hd_j \tag{16}$$

With scalar product of d_j and (16), and the λ_j in (15), gives:

$$d_j^T \cdot g_{j+1} = 0 \tag{17}$$

As the same, according to (16), gives:

$$d_k^T \cdot (g_{j+1} - g_j) = \lambda_j d_k^T Hd_j = 0 \quad \forall k < j \le W \tag{18}$$

Take induction on (17) and (18), gives:

$$d_k^T \cdot g_j = 0 \quad \forall k < j \le W \tag{19}$$

The expression above shows that g_j is orthogonal with all conjugate directions. The conjugate direction can be calculated by the method below:

Use the minus gradients direction as the first search direction, that is $d_1 = -g_1$, the follow-up direction is the linear combination of current gradients and former search direction:

$$d_{j+1} = g_{j+1} + \beta_j d_j \tag{20}$$

According to the conjugate condition $d_j^T Hd_i = 0$, $j = i$. β_j should be:

$$\beta_j = g_{j+1}^T Hd_j \Big/ \left(d_j^T Hd_j\right) \tag{21}$$

3.2. Algorithm Based on Conjugate Gradients

The essential of parameters estimation is to search for the position of peak value of correlative processing. The whole course can be expressed as below:

$$\eta = \max_{a_i, b_j} \left| W_f r(a_i, b_j) \right| \underset{<}{\overset{>}{}} \gamma' \tag{22}$$

$$\{\hat{a}, \hat{b}\} = \arg\max_{a,b} \left| W_f r(a_i, b_j) \right| \tag{23}$$

When a high parameter estimation precision is wanted, the operation is large because of the high density of net search. Therefore, adopting the conjugate gradients method, to get the result of step control parameter and to form iterative conjugate direction vectors for iterative calculation, the results of parameters estimation can be calculated with low operation.

The details of fast algorithm is as follows: firstly, do correlative processing with wide scanning interval, get rough value of parameters estimation $\{a_1, b_1\}$; Then, take this $\{a_1, b_1\}$ as iterative initial value, do the iterative calculation based on conjugate gradients to get the new value of the parameter estimation until reaching the scheduled high precision which the system needed, the iterative process can be expressed as below:

$$\left[\hat{a}_{j+1}, \hat{b}_{j+1}\right]^T = \left[\hat{a}_j, \hat{b}_j\right]^T + \lambda_j d_j \tag{24}$$

where $\hat{a}_j$ and $\hat{b}_j$ are the results of the step *j* iterative search, λ_j is step length coefficient, d_j is conjugate direction at the point of $(\hat{a}_j, \hat{b}_j)$. The flow chart of fast algorithm is shown in Fig.1:

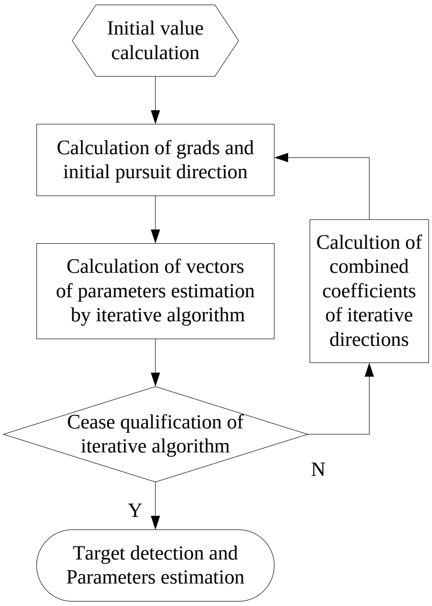

Figure 1. Flow chart of Fast algorithm based on conjugate gradients

The flow chart of fast algorithm can be described as below[8]:

1) Adopting wide scanning intervals to do correlative processing can get the initial value $w_1=(\tau_1,a_1)^T$ *of iterative calculation in fast algorithm;*

2) Calculating the gradients value $g_1=\nabla R\,|\,w_1$ *at the initial value of* $w_1(\tau_1,a_1)$, *let* $d_1=-g_1$ *be the initial direction value of iterative calculation.*

3) Get the parameter estimation point w_{j+1}, *and corresponding gradients* g_{j+1} *and iterative direction linear combination coefficient* β_j, *that is :* $g_{j+1}=g_j+\lambda_j Hd_j$, $\beta_j=g_{j+1}^T Hd_j \big/ d_j^T Hd_j$

4) Calculate iterative coefficient λ_j *and conjugate direction* d_{j+1}, $\lambda_j=-d_j^T g_j \big/ d_j^T Hd_j$, $d_{j+1}=-g_{j+1}+\beta_j d_j$

5) Get the next point w_{j+1} *by iterative calculation:* $w_{j+1}=w_j+\lambda_j d_j$

If $B+Hw_{j+1}=0$, then it is considered that peak point $\hat{w}$ is reached, iterative process stops, get the corresponding result $R(\hat{w})$ at point $\hat{w}$ then comparing it with threshold, and $\hat{w}$ is the parameter estimation value. Or, switch to step (2), repeat the iterative calculating to get new parameter estimation value.

3.3. Simulation

To test the validity of fast algorithm above, and to analyze its performance of parameter estimation, simulation results of fast algorithms are given. The transmitting signal is wideband LFM signal, noise is WGN. Let SNR be from -20dB to 10dB, interval is 2dB. Parameters estimation variances of time scale and time delay which calculated by Monte Carlo method is shown in Fig.2 and Fig.3.

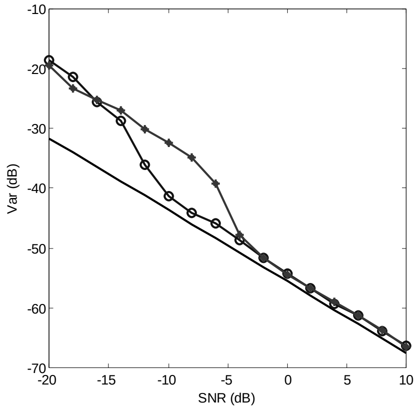

Figure 2. Scale estimation variance curve of algorithms

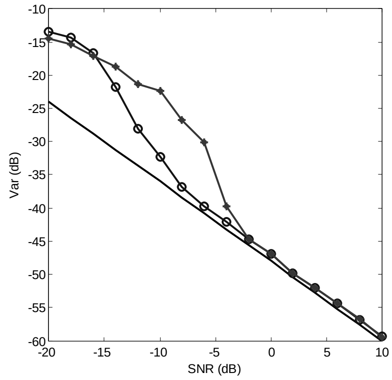

Figure 3. Time delay estimation variance curve of algorithms

The circle points in Fig.2 and Fig.3 are parameter estimation error variance of fast algorithm based on conjugate gradients, asterisk points are corresponding to correlative processing. When SNR>-3dB, parameter estimation variance of two algorithms are very approximate to CRLB. As SNR reduces, parameter estimation variance will departure the CRLB bit by bit, the parameter estimation variances of fast algorithm based on conjugate gradients is less than the those of correlative processing. Under the low SNR, two algorithms' parameter estimation error variance curves intercross which is brought by the fake peak under low SNR. In the Monte-Carlo simulation, the results of parameters estimation adopting fast algorithm based on conjugate gradients are approximate enough to real value with lower operation than that of correlative processing.

4. lake test

Due to the difficulty to collect the reflecting echo of high speed moving target, the test adopts a method which simulates the echo of high speed target to accomplish the dynamic target experiment to testify the performance of the fast algorithm. The lake test system is shown in Fig.4:.

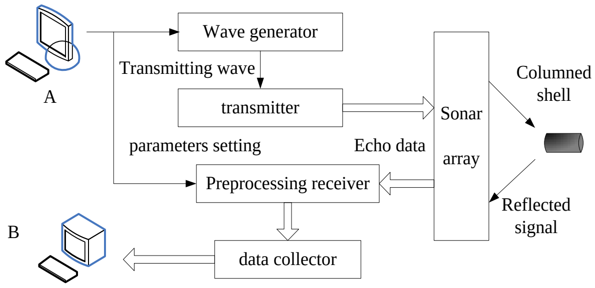

Figure 4. Flow chart of lake test

In lake test, wave generator to add an appropriate time scale on the transmitting signal to simulate the reflected signal of moving target, and then load it into the sonic array. After reflected by the target, the echo is received by sonic array, transferring into matching network through cable, then passes through the preprocessing channel, inputting into the computer and DSP system to be processed after being collected by mass memorizer.

Letting transmitting signal be wideband LFM signal, the reflected signal was processed with algorithms respectively. The time scale of reflected signal is set to be a=1.0278 and the corresponding simulated speed of target is v= 40kn. The range of target is 141.2m. Fig.5 shows the results of parameters estimation of the two algorithms in lake test

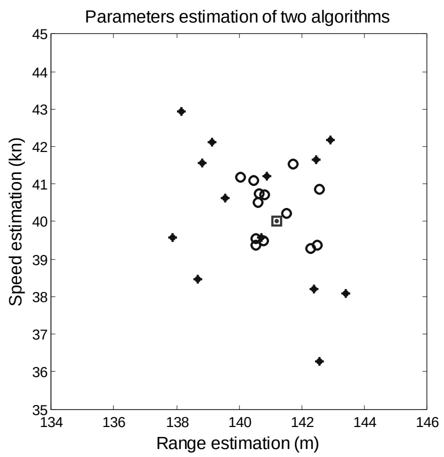

Figure 5. Result of parameters estimation of the two algorithms

In Fig.5, square point is the real value, asterisk points is the results of correlative processing and the circle points is results of fast algorithm based on conjugate gradients. Obviously, the results of fast algorithm are closer to real value. The fast algorithm has higher parameters estimation precision.

The results of lake test indicate that both of two algorithms can detect the reflected signal effectively in low SNR, but the fast algorithm based on conjugate gradients has a better precision on parameters estimation and lower operation.

5. Conclusion

Wideband correlative processing based on wavelet transform is an important tool in wideband active signal processing, thus more and more studies begin to focus on it and its corresponding fast algorithm. However, most studies are done just pointing to the research on fast algorithm for wavelet transform which brings an operation that the system cannot afford. Then fast algorithm based on conjugate gradients is advanced. This algorithm takes Taylor expansion at the parameter estimation initial point, forming a quadratic approaching function. Based on conjugate gradients method, calculating the approaching step length of parameter estimation and conjugate iterative direction, a fast algorithm is brought forward to obtain the values of parameters estimation. The validity and performance of fast algorithm is testified by computer simulation and lake test which makes the algorithm theoretically better and more applicable. Comparing with correlative processing, the fast algorithm can avoid heavy operation, lower the processing complexity and is able to search out the peak value of wideband correlative processing quickly and precisely, so as to realizing the parameters estimation perfectly with low operation.

6. References

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