

A Medical Image Segmentation Based on Global Variational Level Set

WANG Yanwei^{a,b,*}, PAN Yanming^c, FENG Kejian^c, FENG Yukuan^c

^a HuaRui College of Northeast Petroleum University, Harbin Heilongjiang, PR China

^b Automation College of Harbin Engineering University, Nantong street No.145, Harbin, Heilongjiang, PR China

^c Mudanjiang Medical Collage, Mudanjiang, Heilongjiang, PR China

Abstract. A medical image segmentation based on global variables differential level set is proposed in this paper for medical images with complex topological structure, strong contrast and low noise characteristics. It make full use of the image area information, build a energy model, and using variation gradient information to establish a global energy model to get the minimization value, which is geodesic active contour (GAC) model. Experimental results show that the method set in the initial outline of the evolution without success to avoid the re-initialization and correction process, thus saving computing time. With traditional methods and TV and CV method, the method convergence stable segmentation accuracy is good, easy parameter adjustment and split speed, better medical treatment of low contrast, blurred image.

Keywords: Level Set, Medical Image Segmentation; GAC; Global Minimum

1. Introduction

Medical image segmentation is a key technology in medical image processing, its task is to extract interesting objects from the medical image in order to serve the clinician computer-aided diagnosis.

Variation level set method is an image segmentation method for no parameters based on geometric active contour model, which essence is a low-dimensional space to describe the problem embedded in the high dimensional space. And it has ability of handling topology changes to get a stable result in numerical calculation. Therefore, the level set method is widely used. Paper 1 Ye Xiu-Fen studied the level set method is applied in radar image processing, and literature 2 research the level set method on holographic images.

At present, many researchers study the variation level set method applied in medical image segmentation. Variation level set method widely used in tracking and modeling, and it flourishes in the past decade research both in theoretical and practical. Paper 3 Chen Yunjie studied the level set used in the MR image segmentation, and the references 4 Gao Hui and Oksam Chae studied the level set method applied in the application of CT images, references 5 refers to an automatic image segmentation level set, and the Bayesian model in level set is adopted in the references .In addition the paper 7 studied the literature Variation level set applied in image segmentation. From the references we can see that the variation level set methods become very popular in image processing methods, therefore, the variation level set is well suited for complex topologies strong noise and low contrast areas of medical image analysis.

Variation level set segmentation method, which is advantages to deal with the variety topology, is an important branch in medical applications .It is essential that transform the higher dimensional space into a low-dimensional space to describe the evolution to achieve medical image segmentation.

2. TV Level Set Principle

* Wang Yanwei et al.Tel.:+86-13694500371; fax: +86-451-87174018.
E-mail address: xianxinyue@163.com.

2.1. Variable Differential Principle

Set a two-dimensional energy functional of the form,

$$E(u) = \iint F(x, y, u, u_x, u_y) dx dy \quad (1)$$

In order to obtain the minimum energy solution u , Euler-Langrange equation is applied in variable distribution.

$$\frac{\partial F}{\partial u} - \frac{d}{dx} \left(\frac{\partial F}{\partial u_x} \right) - \frac{d}{dy} \left(\frac{\partial F}{\partial u_y} \right) = 0 \quad (2)$$

The minimum energy extremum value is acquired to solving Euler-Langrange equation. It is known that the Euler-Langrange equation is a nonlinear partial differential equation (PDE), and it discredited to get non-linear simultaneous algebraic equations. Therefore, the time needed to introduce auxiliary variables, at the same time, the static nonlinear PDE problem transform into a dynamic PDE problem. And the solution of Euler-Langrange equation is the evolution in a steady state.

2.2. Level Set Segmentation Principle

The level set implicit expression of the closed curve is the following.

$$C = \{(x, y) | u(x, y) = c\} \quad (3)$$

Curve C changes as previously mentioned, time parameters is introduction, therefore, a closed curve can be expressed as two-dimensional ,following the time function ,u (x, y) the level set.

$$C(t) = \{(x, y) | u(x, y, t) = c\} \quad (4)$$

Derivative the EQ.4 was perfection:

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \nabla u \cdot \left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t} \right) = 0 \quad (5)$$

If $\left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t} \right) = \frac{\partial C}{\partial t} = V$, then $\frac{\partial u}{\partial t} = -\nabla u \cdot V = -|\nabla u| \left(\frac{\nabla u}{|\nabla u|} \cdot V \right) = |\nabla u| N \cdot V = \beta(F) |\nabla u|$. In which $\beta(F) = N \cdot V$ is a energy

functional F as a function of a velocity function. The basic equation of the level set method is a development of PDE, it belongs to a class of Hamilton-Jacobi equation, Should go to the variation problem to be selected when the numerical viscosity solution of the wind program requirements.

Noted that the derivation of the level set method, embedded functions assigned constant value does not matter, so you can take $c=0$, so we are concerned with curves can be attributed to the zero level set embedding function. Thus, the curve evolution problem can be Equivalent function embedded into the initial conditions given by the evolution $u_0(x, y)$ of the level set equation $u(x, y)$.

$$C(0) = \{(x, y) | u(x, y, 0) = u_0(x, y)\} \quad (6)$$

Once the initial curve fit the equation $u(x, y, t) = 0$ at any time in evolution ,then the current level set curve C (t) is set, which the minimum energy function of the Euler-Langrange equation is obtained, and the curve evolution is achieved.

All in all, the numerical calculation in the level set method can cause the unstable or too complicated. Therefore, the most commonly used signal distance function $u(x, y)$, it is $-d((x, y), C)$ means curves outside, otherwise insides the curve. Among them, $d((x, y), C)$ denotes the Euclidean distance between the point (x, y) and the curve C. Its main advantage is $|\nabla u| = 1$, the function of the uniform rate of change, which is benefit for to the stability of numerical calculation.

2.3. Total Variation Level Set Algorithm

The motion equations derived from minimize energy of a closed curve functional in Curve evolution applied to image processing. Therefore, the energy functional, which is the level set image segmentation for the geodesic active contour model, can be expressed as following.

$$E(C) = \int_c g(|\nabla I(C(s))|) ds \quad (7)$$

When the curve parameters $C(0) = C(1)$, the original function can be written as that:

$$E(C(p)) = \int_0^1 g(C(p)) |C_p(p)| dp \quad (8)$$

From the level set principle, the corresponding function for the PDE like that:

$$\frac{\partial u}{\partial t} = [g(|\nabla I \times [C(s)]|)k - \nabla g \bullet N]|\nabla u| \quad (9)$$

If the Heaviside function is introduced in the PDE model, the energy functional can be rewritten as the EQ10 by the Green formula.

$$E(u) = \iint_{\Omega} g(x, y)|\delta(u)dx dy| \quad (10)$$

Which $\delta(u) = dH(u)/du$, $H(u)$ function is the Heaviside function, when $u > 0$, the $H(u)$ value is 1. Using the variation method, the level can be set on the type of evolution equation.

$$\frac{\partial u}{\partial t} = \partial(u)div(g \frac{\nabla u}{|\nabla u|}) \quad (11)$$

Actual the regularization $\delta_{\varepsilon}(u)$ is approximation for the EQ12.

$$\frac{\partial u}{\partial t} = \partial_{\varepsilon}(u)div(g \frac{\nabla u}{|\nabla u|}) \quad (12)$$

Minimum for the energy functional problem solving, through the embedded function $u(x, y)$, and use the Heaviside function and Green formula, then the functional $E(C)$ transformed into $E(u)$ by variation different, and using the gradient descent flow or level set method to get PDE value, the numerical steady-state solution is acquired, therefore, the total variation level set method is applied in image processing methods.

3. The TV Level Set Applied in Medical Image Segmentation

Level set segmentation, in the curve evolution process, needs to loop until convergence in every PDE iteration solution, and computation based only on the boundary than the active contour to a large number of image segmentation. In addition, the level set method for image segmentation based on the model with the same problem, namely, the energy function may be non-convex curve on the initial outline of the evolution is very sensitive, and lead to failure results in local minimal value.

Recently, many scholars studied in the accuracy segmentation. Li proposed a need to reinitialize the model to construct a signed distance to keep unvaried (penalty function) in 2005[8], it improve the level set segmentation speed. Gao will be split equation multiphase disaster and get graded level set segmentation method [9], successfully solving Mumford-Shah model sensitive to the initial outline problem. Subsequently, papers 10 mention that penalty function is applied in CV model for biomedical target extraction. Recently, Law constructed a set of random level hybrid model, combining gradient information based on the level set method and the Basin-Hopping global optimization algorithms for a class integrated optimization method to solve the Murnford-Shah of non-convex variation problem, to achieve a global optimization division [11].

To solve the speed problem in the traditional level set method in the past, especially in local minimum energy function can not be achieved segmentation result, we propose a global variation segmentation method. In this method, the use of the image area information, build a posterior probability-based energy model, then a very small variation in the overall framework, the combination of geodesic active contour (GAC) model uses gradient information to establish a global variation energy model, and finally construct a new integrated image segmentation model.

Assuming that the image area is consist of Ω_1 and Ω_2 , that is, and any two $\Omega_1 \cup \Omega_2 \dots \cup \Omega_n = \Omega$ $\Omega_i \cap \Omega_j = 0$, $i \neq j$. Based on the maximum a posteriori probability criterion, image segmentation task is to look for pattern classification based on the data D from M , maximum a posteriori is the following.

$$P(M/D) = \frac{P(D/M)P(M)}{P(D)} \quad (13)$$

Since $P(D)$ is a fixed value, so to maximize $P(M/D)$ that the $E(M) = P(D/M)P(M)$ will get minimized. Applied to the image, is the model M is the current point Ω_i estimate of test problems. D is the input image data I , to maximize the posterior probability problem can be written as

$$P(x \in \Omega | I(x) = s) = \frac{P(I(x) = s | x \in \Omega_i)P(x \in \Omega_i)}{P(I(x) = s)} \quad (14)$$

That is $P_i(s) = P(I(x) = s | x \in \Omega_i)$. According to the contour length of the curve the current point estimate probability $P(x \in \Omega_i)$, Assuming that each point is independent in the image, and the shortest line is the split curve. At this point, there

$$E(\Gamma) = -\sum_{i=1}^n \int_{\Omega_i} \log P_i(x) dx + v \int_{\Gamma} ds \quad (15)$$

From the EQ15, we see that Curve Γ is split result, and posterior probability model is got from the variation level set. It constructed the model based on region information. Thus the gradient information is not dependent on the boundary, as the CV model is based on the area of active contour models. The probabilistic model is different from CV model, which dealing with the outside by the Constance value. It probability model deals with the region information by the logarithm of Gaussian function to broaden the scope in theory.

Image segmentation information given gradient active contour model, as opposed to region-based model when the level set method speeded long time to computing the iterative results of numerical PDE. However, the evolution model based on gradient information in the segmentation is oriented, which has the stable solutions, and widely used, that is the geodesic active contour model (Geodesic active contour, GAC).

$$E_{GAC} = \int_0^{L(C)} g(|\nabla I(C(s))|) ds \quad (16)$$

Using the level set, take C to the level set function value, then the model can be written as the EQ17 by the Heaviside function.

$$E_{GAC}(\phi) = \int_{\Omega} g(x) |\nabla H(\phi)| dx \quad (17)$$

The function $g(\cdot)$ is expressed as EQ.18.

$$g(x) = \frac{1}{1 + \beta |\nabla(G_a(x) * I(x))|^p} \quad (18)$$

The parameter β is a normal positive number. Parameter p value is 1 or 2. And parameter G_a is the standard deviation σ of the Gaussian function. The input image convolution is smoothing of Gaussian model. Then the global variation level set model as the EQ19.

$$E(\phi, \lambda) = \int_{\Omega} g(x) |\nabla H(\phi)| dx - \lambda \sum_{i=1}^n \int_{\Omega} H(\phi_i) \log P_i(x) dx \quad (19)$$

Where the Heaviside function is regularized, such as Li setting, Then the global optimization algorithm steps for obtained steady-state solution is the following:

$$H_{\varepsilon}(z) = \begin{cases} 1, z > \varepsilon \\ 0, z < -\varepsilon \\ \frac{1}{2} [1 + \frac{z}{\varepsilon} + \frac{1}{\pi} \sin(\frac{\pi z}{\varepsilon})], |z| \leq \varepsilon \end{cases} \quad (20)$$

Applying the variation methods, level set method and gradient descent method, the minimize energy function was satisfied by the PDE model solution.

$$\frac{\partial \phi}{\partial t} = H_{\varepsilon}'(\phi) [div(g \frac{\nabla \phi}{|\nabla \phi|}) - \lambda (\log P_i(x) - \log P_j(x))_{i \neq j}] \quad (21)$$

Then the global optimization algorithm steps for obtained steady-state solution is the following, First, Find an arbitrary function of a minimal solution, denoted by $\mu(x)$; Then take a value $\mu \in (0, 1)$, construction set $\Sigma = \{x \in R^2 : \mu(x) > \mu\}$; Finally, set the $\mu(x) = 1_{\Sigma(x)}$ is the global minimum solution.

4. Experiment

The first image is geometric tools which pixel is 200 * 200dpi. There are three methods used in the geometric tools image. and the initial conditions were set as part of the small round profile, the external contours of whole circle and square outline, the processing time in three methods were 5.219702 , 36.087686 and 22.422463 seconds.



(a) small circle initial (b) external circle initial (c) square outline initial

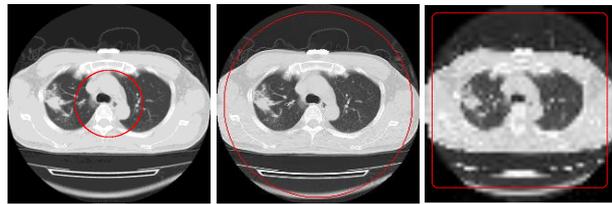


(d) result of small circle (e)result of external circle (f) result of square outline

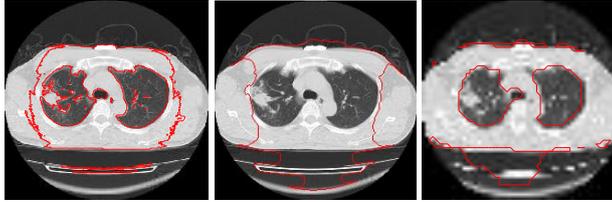
Fig.1. The three processing methods of geometric tools image

From the Fig.1, the results from the three groups can be seen that the first time a method and results are superior to the second and third methods, the second of three methods can only identify the tools of the outer contour, the internal geometry is not recognized.

In addition, the chest CT image is used to processing, the results of treatment services for medical diagnostics, to make an objective analysis and condition to determine. In order to achieve accurate segmentation, it is essential to set the proper initial conditions.



(a)small circle initial (b) external circle initial (c) square outline initial



(d) result of small circle (e)result of external circle (f) result of square outline

Fig2. The three processing methods of medical image

Three methods used for medical image segmentation, from the Fig.2, the first result is effectively to segmented chest CT image, and the middle image only identify outline, still the last image is to identify the inner contour partly. The proposed method (left) is better than the other two methods.

Table 1 depicts the processing time of three different initial conditions. Small round second test set of initial conditions was better than the other two times, so our method is better in time and effect.

TABLE 1 THE TIME OF DIFFERENT INITIALS

method time	Small circle	Outline circle	Outline square
Image I	5.219702	36.087686	22.422463
Image II	96.266205	97.668428	290.122986

The regular Heative function is used to optimize, the experimental results show that this method can be better displayed in the current curve and the piecewise constant function, this division processed image as shown below.



Fig3. The processed image of our method
Fig. 1. (a) first picture; (b) second picture

5. Conclusion

In this paper an medical image segmentation based on global variables differential level set is proposed for medical images. First, image area information is used to build a posterior probability-based energy model, and then a very small variation is available partly, and combination the geodesic active contour (GAC) model with gradient information to build a global variation energy model. Finally, new integrated image segmentation model is acquired. It is essential for setting the initial conditions. Experimental methods show our method is convergence and stability, which is better than the CV model. It has ability of accuracy and speed. And it parameter adjustment simple and fast division, to better medical treatment of low contrast, blurred image capabilities.

6. Acknowledgements

This work was financially supported by the Mudanjiang scientific and technological project (G2011s0020).

7. References

- [1] Louie H , Burns M, Lima C. An introduction and user's guide to the IEEE Smart Grid Web Portal. *Innovative Smart Grid Technologies Conf. Europe (ISGT Europe)* 2010;1–5, .
- [2] Xiu-Fen Ye, Zhe-Hui Zhang, Peter X. Liu, Hong-Ling Guan. Sonar image segmentation based on GMRF and level-set models Original Research Article. *Ocean Engineering*, Volume 37, Issue 10, July 2010, Pages 891-901.
- [3] 2. Pin Zhang, Rong Li, Jun Li. Segmentation of holographic images using the level set method Original Research Article *Optik - International Journal for Light and Electron Optics*, Volume 123, Issue 2, January 2012, Pages 132-136
- [4] 3. Yunjie Chen, Jianwei Zhang, Jim Macione. An improved level set method for brain MR images segmentation and bias correction *Computerized Medical Imaging and Graphics*, Volume 33, Issue 7, October 2009, Pages 510-519
- [5] 4. Hui Gao, Oksam Chae . Individual tooth segmentation from CT images using level set method with shape and intensity prior Original Research Article *Pattern Recognition*, Volume 43, Issue 7, July 2010, Pages 2406-2417
- [6] 5. Mariano G. Uberti, Michael D. Boska, Yutong Liu. A semi-automatic image segmentation method for extraction of brain volume from in vivo mouse head magnetic resonance imaging using Constraint Level Sets Original Research Article *Journal of Neuroscience Methods*, Volume 179, Issue 2, 15 May 2009, Pages 338-344
- [7] 6. Yao-Tien Chen .A level set method based on the Bayesian risk for medical image segmentation Original Research Article. *Pattern Recognition*, Volume 43, Issue 11, November 2010, Pages 3699-3711
- [8] 7. Ye Yuan, Chuanjiang He .Variational level set methods for image segmentation based on both L^2 and Sobolev gradients Original Research Article *Nonlinear Analysis: Real World Applications*, Volume 13, Issue 2, April 2012, Pages 959-966
- [9] 8. LI C, Xu C, Gui C ,etal. Level set evolution without re-initialization : a new variational formulation[C]. *Proceedings of the 2005 IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, 2005, 1:1-7
- [10] 9 Image segmentation and selective smoothing by using Mumford-Shah model
Gao, Song (Department of Computer Science, Concordia University, Montreal, Que. H3G 1M8, Canada); Bui, Tien D. Source: *IEEE Transactions on Image Processing*, v 14, n 10, p 1537-1549, October 2005
- [11] 10. Improvements in level set segmentation of 3D small animal imagery
Price, Jeffery R. (Oak Ridge National Laboratory, Oak Ridge, TN); Aykac, Deniz; Wall, Jonathan Source: *Progress in Biomedical Optics and Imaging - Proceedings of SPIE*, v 6512, n PART 2, 2007, Medical Imaging 2007: Image Processing
- [12] 11. Law, Yan Nei; Lee, Hwee Kuan; Yip, Andy M. A multiresolution stochastic level set method for Mumford-Shah image segmentation. Source: *IEEE Transactions on Image Processing*, v 17, n 12, p 2289-2300, 2008