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An Optimal Transmit Power Allocation for Multiple-Access Relay Channel with Analog Network Coding

Mingfeng Zhao[*1], Yajian Zhou[*2], Yixian Yang[*2]

[*1] China Mobile Group Design Institute Co., Ltd. Sichuan Branch, Chengdu, China

[*2]National Engineering Laboratory for Disaster Backup and Recovery,
Beijing University of Posts and Telecommunications, Beijing

Abstract—This paper investigates the transmit power allocation of practical analog network coding (ANC) schemes for multiple-access relay channel (MARC), which consists of two source nodes, one relay node, and one destination node, all of which work in half-duplex mode with single antenna. The two symmetric source nodes transmit information to destination node with the help of relay node. For this scenario, we show that optimal transmit power allocation under the goal of maximizing the total mutual information is proposed. The numerical experimental results show that, the proposed method achieves better mutual information than uniform power allocation, and the total system mutual information has better performance than uniform power allocation.

Keywords-Analog Network Coding (ANC); Multiple-Access Relay Channel (MARC); Mutual Information (MI)

1. Introduction

Instead of treating interference as a nuisance to be avoided for the traditional wireless communication systems, analog network coding (ANC), which can make good use of interference with the "right mechanism"[1], has been attracting significant attention in wireless cooperative relay communication networks since it has a great potential to improve throughput and spectral efficiency. Additionally, considering the fading characteristic of the wireless channel, which impacts the system performance seriously, the cooperative communication is proposed to take advantage of users or relay each other cooperative transmit information to combat fading over wireless channels [2]. Except for source-destination pair direction transmission, utilizing ANC, the relay node just amplifies and forwards the mixed signal to the destination, which includes different information of two source nodes. The desired signal can be extract since the destination node knows its own contribution to the mixed signal from the direct transmission in different time slot for Multiple-Access Relay Channel (MARC). Therefore, it can obtain the full diversity gains and improve the efficiency of the cooperation [3, 4]. However, the main drawback of ANC is noise propagation in the simple amplify-and-forward scheme [5]. Consequently, signal-to-noise (SNR) reduces significantly at the destinations. Another disadvantage of ANC is that the signal transmitted by the relay is a sum over the received signals. Thus, from the perspective of a given destination node, a portion of the transmit power overhead of the relay is wasted in transmitting already known information [6].

Recently, many problems about two-way relay networks with power constraint to discuss the optimal power allocation, minimize the outage probability or maximize the total mutual information have been studied [7-10], and have obtained optimal power allocation scheme based on the different goal. In [11], under the goal of minimum outage probability, the impact of source power allocation on the performance of the

E-mail address: zhaomingfeng81@gmail.com, yajian@bupt.edu.cn, yxyang@bupt.edu.cn

MARC with ANC is analyzed. However, it assumes that channel gain is the same for every different channel. Obviously, the result is not accurate. It is necessary to study the transmit power allocation for MARC with ANC how to affect the total mutual information.

1.1. Main Contributions of This Paper

The main contributions of this paper are as follows.

- An optimal power allocation scheme, which is based on maximum system total mutual information for MARC with ANC, is proposed.
- The solution of optimal power allocation scheme is presented, and its existence is proved as well.
- The numerical results show that our proposed scheme can obtain optimal power allocation. Furthermore, we analyze the impact of relay location, power allocation factor and angle θ between *S-D* and *R-D* on the system mutual information.

1.2. Paper Organization

The rest of the paper is organized as follows. Firstly, we describe MARC model employing ANC scheme, and offer a detailed analysis for the signal aggregation and signal processing at the relay and signal extraction for the destination in Section 2. The optimal power allocation scheme for the purpose of maximum total mutual information is presented in Section 3. Numerical results and conclusion are presented in Section 4 and Section 5, respectively.

2. System Model and ANC Protocol

2.1. System Model

We first consider a MARC model with two sources, one relay and one destination. Every node works in a half-duplex mode and has only single antenna. All the channels in the MARC are Rayleigh flat fading. The path loss index is 4, and the channel gain $|h_{uv}|^2$ between two nodes is modeled as d_{uv}^{-4}, where d_{uv} is the distance between u and v. Furthermore, we assume that every channel gain is complex Gaussian random variables with zero mean and variances σ . The additive white Gaussian noise (AWGN) is assumed to be a complex Gaussian random variable with zero mean and unit variance for every channel in the MARC. We use S_1 and S_2, R and D to denote the two source nodes, relay node and destination node, respectively. Let x_1 and x_2 denote the information bearing symbols transmits from sources S_1 and S_2, respectively. We also assume that the destination and relay know channel gains and amplification factor.

2.2. MARC with ANC

In Fig. 1, the source S_1 and S_2 send information to desired destination node D independently under the help of cooperative relay node R, where ANC scheme is employed at the relay node R.

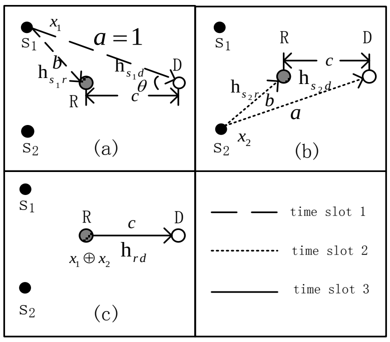

Fig.1. Sequence of transmission for MARC with ANC

We assume that both of destination node and relay node locate within the transmission range of the source nodes. In the first time slot, the source S_1 broadcasts signal x_1 to desired destination D, which is overheard by R; in the second time slot, the source S_2 broadcasts signal x_2 to destination D, which is also overheard by R; in the third time slot, the relay R perform a linear combination of all received signals, just called Analog network coding, then it amplifies and broadcasts the combined signals to destination D (see Fig 1(a)-1(c)). Finally, the destination extracts its desired signal by subtracting the overheard signals from the combined signal and direct transmission signals. For the MARC with ANC scheme, it needs just only three time slots to finish one loop transmission. However, for the traditional model, it needs four time slots in one loop transmission. When considering *n* source nodes, it needs at least 2*n* time slots, but ANC just needs *n*+1 time slots for one loop transmission. So ANC is one of best choices to enhance the system performance and improve spectral efficiency, meanwhile, it also can obtain the full diversity gain for each source node. We give a detail process for the MARC with ANC scheme.

The first stage, the sources S_1 and S_2 broadcast signal x_1 and x_2 in different two time slots, respectively. For the S_1 in the first time slot, the relay *R* and destination *D* received signal can be expressed as

$$\begin{aligned} y_{s_1 r} &= \sqrt{P_1} h_{s_1 r} x_1 + z_r \\ y_{s_1 d} &= \sqrt{P_1} h_{s_1 d} x_1 + z_d \end{aligned} \tag{1}$$

For the source node S_2 in the second time slot, the received signal can be expressed as

$$\begin{aligned} y_{s_2 r} &= \sqrt{P_2} h_{s_2 r} x_2 + z_r \\ y_{s_2 d} &= \sqrt{P_2} h_{s_2 d} x_2 + z_d \end{aligned} \tag{2}$$

where P_i is the transmission power at node *i*, z_v is background noise at node *v*.

The second stage, Relay *R* combines signal x_1 and x_2, and then amplifies by the factor α_r and broadcasts the combined signal to destination *D*. Considering the similarity, we only discuss the procedure of destination *D* on how to extracts its desired signal from source S_1. The received signal in the third time slot as

$$y_{rd} = \alpha_r \sqrt{P_3} h_{rd} [y_{s_1 r} + y_{s_2 r}] + z_d \, , \tag{3}$$

where the value of α_r can be calculated as [8]

$$\alpha_r^2 = \frac{1}{|h_{s_1 r}|^2 P_1 + |h_{s_2 r}|^2 P_2 + 2\sigma_r^2} \, ,$$

For the purpose of analysis [8], it is approximated as follows:

$$\alpha_r^2 \approx \frac{1}{|h_{s_1 r}|^2 P_1 + |h_{s_2 r}|^2 P_2} \tag{4}$$

where P_3, P_1, and P_2 are the transmission powers of node *R*, S_1 and S_2, respectively, σ_r is the variance of background noise at Relay *R*. By using (1) and (2), Eq. (3) can be re-written as

$$y_{rd} = \frac{\sqrt{P_3}\alpha_r h_{rd} h_{s_2 r}}{h_{s_2 d}} [y_{s_2 d} - z_d] + \sqrt{P_3}\alpha_r h_{rd} y_{s_1 r} + \sqrt{P_3}\alpha_r h_{rd} z_r + z_d \tag{5}$$

So it can get desired signal for *D* by subtracting useless signal, which can be got from the second time slot and multiply a factor. Just has

$$\begin{aligned} \hat{y}_{rd} &= y_{rd} - \frac{\sqrt{P_3}\alpha_r h_{rd} h_{s_2 r}}{h_{s_2 d}} y_{s_2 d} \\ &= \sqrt{P_3}\alpha_r h_{rd} (\sqrt{P_1} h_{s_1 r} x_1 + z_r) + \sqrt{P_3}\alpha_r h_{rd} z_r \\ &\quad + z_d - \frac{\sqrt{P_3}\alpha_r h_{rd} h_{s_2 r}}{h_{s_2 d}} z_d \end{aligned} \, , \tag{6}$$

Using [12], we denote new noise term as ANC noise, i.e.,

$$z_d^{new} = z_d + \sqrt{P_3}\alpha_r h_{rd} z_r - \frac{\sqrt{P_3}\alpha_r h_{rd} h_{s_2 r}}{h_{s_2 d}} z_d \, , \tag{7}$$

Note that z_d^{new} has a zero mean, and the variance of z_d^{new} is

$$\sigma_{z_d^{new}}^2 = \sigma_{z_d}^2 + P_3(\alpha_r h_{rd})^2 \sigma_{z_r}^2 + P_3 \left(\frac{\alpha_r h_{rd} h_{s_2 r}}{h_{s_2 d}}\right)^2 \sigma_{z_d}^2 \, , \tag{8}$$

which is larger than the noise variance δ_d^2 .

3. Optimal Power Allocation Scheme

We discuss the power allocation scheme under the constraint of total transmission power to obtain the maximum system total mutual information. Our goal is to find whether or not the optimal power allocation exists and how the location of relay impacts system performance for MARC with ANC. For the similarity and simplicity, we assume that source node S_1 and S_2 locate symmetrically, and have the same distance to relay and destination. The system maximum total mutual information is identical to maximum mutual information of any source node under the constraint of power. So we just only choose source node S_1 to discuss it. Its mutual information can be obtained as follows [12]

$$I_{system} = I(S_1, R, D) = \frac{1}{2}\log_2(1 + SNR_{s_1d} + SNR_{rd}) \tag{9}$$

where SNR_{s_1d}, SNR_{rd} are the effective signal-to-noise for direction transmission and amplify-and-forward relaying transmission with ANC, respectively. Just have

$$SNR_{rd} = \frac{P_3 P_1 \alpha_r^2 |h_{rd}|^2 |h_{s_1r}|^2}{4P_3\alpha_r^2 |h_{rd}|^2 \sigma_{z_r}^2 + \dfrac{P_3\alpha_r^2 |h_{rd}|^2 |h_{s_2r}|^2 \sigma_{z_d}^2}{|h_{s_2d}|^2} + \sigma_{z_d}^2}$$

$$SNR_{s_1d} = \frac{P_1}{\sigma_{z_d}^2}|h_{s_1d}|^2, \quad \sigma_{z_d}^2 = \sigma_{z_r}^2 = 1.$$

The formulation of the problem can be represented as

$$Max\, I_{system} = I(S_1, R, D)$$
$$P_1 + P_2 + P_3 \le P_{total}$$

According to the assumption, we know that the power of S_1 and S_2 is the same $P_1 = P_2$. Let $P_1 = xP_{total}$ $x \in (0, 0.5)$. From (4), (9) and assumption, we have

$$MaxI_{system} = \frac{1}{2}\log_2[1 + xP_{total}(\frac{1}{a^4} + \frac{1}{4b^4 + a^4 - c^4 + c^4/(1-2x)})]$$ which is equivalent to

$$Subject\ to\ 0 < x < 0.5, b + c \ge a, 0 \le b \le a, 0 \le c \le a$$

$$Max\ f(x) = x(\frac{1}{a^4} + \frac{1}{4b^4 + a^4 - c^4 + c^4/(1-2x)}) \tag{10}$$

$$Subject\ to\ 0 < x < 0.5, b + c \ge a, 0 \le b \le a, 0 \le c \le a$$

Theorem 1: When the source node S_1 and S_2 are in symmetrical location and $P_1 + P_2 + P_3 = P_{total}$, the optimum power allocation that maximizes the system total mutual information of the ANC protocol for MARC is given by

$$P_1 = P_2 = \mu P_{total} \tag{11}$$

$$P_3 = (1 - 2\mu)P_{total} \tag{12}$$

where

$$\mu = \frac{(a^4 + c^4)m + m^2 + a^4c^4 - a^2c^2(a^4c^4 + (a^4 + c^4)m + m^2)^{1/2}}{2a^4m + 2m^2}$$ *Proof:* See Appendix A.

$$m = 4b^4 + a^4 - c^4$$

4. Numerical Results

To facilitate analysis and comparison, we assume that relay node R is in line with the destination node D, the distance of S_1 to D is fixed at 1(a=1), the total transmit power is 3Watt, the angle between S-D and R-D is θ ($0 \le \theta \le \pi/2$), which is used to show the relation between a, b and c. We plot the effect of power allocation factor x on the system mutual information under relay node R locating in different position for Multiple-Access Relay Channel with ANC scheme in Fig.2. We can find that the optimal power allocation exists to maximize system mutual information, when the power allocation factor is at 0.45, relay locates at 0.5(c=a/2) and $\theta = \pi/4$, we can obtain the maximum system mutual information. The result is consistent with in [13], when considering the direction transmission and diversity gain, the more power must allocate to source nodes to obtain maximum capacity (mutual information).

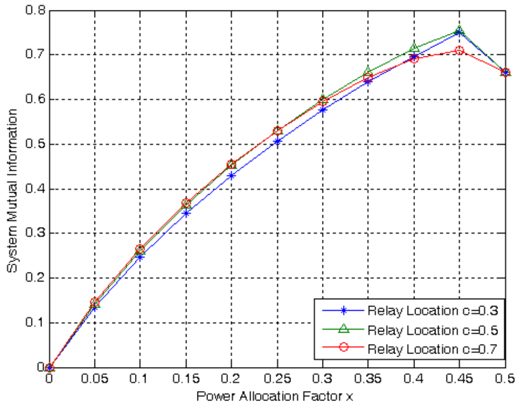

Fig.2. Impact on power allocation factor *x* for *MI*

The relay location *c* impacts the system mutual information under different power allocation factor can be illustrated in Fig.3. We can see that employing optimal power allocation scheme is significant beneficial for mutual information under the same constraint for MARC with ANC, compared to uniform power allocation, there exists maximum mutual information when the relay locates suitable position under given power allocation factor.

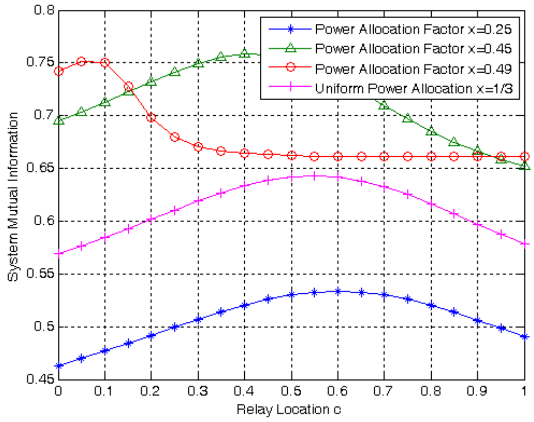

Fig.3. Impact on relay location *c* for *MI*

The angle θ between *S-D* and *R-D* impacts the system mutual information can be illustrated in Fig.4. We can conclude that the system mutual information decreases with increasing of angle θ, under the same constraints, the optimal power allocation scheme is better than the uniform power allocation for the goal of mutual information.

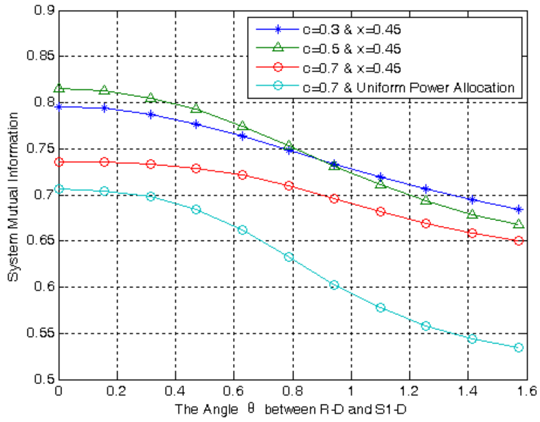

Fig.4. Impact on angle θ for *MI*

5. Conclusions

In this paper, the optimal power allocation scheme is presented for multiple-access relay channel with analog network coding. Compared with uniform power allocation, we have shown that our proposed scheme always obtain better system mutual information. Furthermore, we discuss the effect of power allocation factor, relay location and the angle θ between *S-D* and *R-D* on the system mutual information under certain conditions, respectively. As a future work, we will extend our proposed scheme to discuss general scenarios, whether or not exists simultaneously minimum outage probability and maximum the mutual information of ANC protocol.

6. Appendix a Proof of theorem 1

Let $m = 4b^4 + a^4 - c^4$, from Eq. (9) and Eq. (10), we just need to prove that there exists one solution under the constraints to maximize $f(x)$. We follow the method given in advanced algebra to solve the maximize problem. Thus, we has

$$\partial f(x) / \partial x = 0 \tag{A.1}$$

The solutions of equation are

$$x_1 = \frac{(a^4+c^4)m + m^2 + a^4c^4 + a^2c^2(a^4c^4 + (a^4+c^4)m + m^2)^{1/2}}{2a^4m + 2m^2} \tag{A.2}$$

$$x_2 = \frac{(a^4+c^4)m + m^2 + a^4c^4 - a^2c^2(a^4c^4 + (a^4+c^4)m + m^2)^{1/2}}{2a^4m + 2m^2} \tag{A.3}$$

Obviously, x_1>0.5, but $0 < x < 0.5$, it is abandoned.

$$x_2 = \frac{(a^4+c^4)m + m^2 + a^4c^4 - a^2c^2(a^4c^4 + (a^4+c^4)m + m^2)^{1/2}}{2a^4m + 2m^2}$$ So we just only need to prove
$$= 0.5 + \frac{c^4m + a^4c^4 - a^2c^2(a^4c^4 + (a^4+c^4)m + m^2)^{1/2}}{2a^4m + 2m^2}$$

$$-1 < \frac{c^4m + a^4c^4 - a^2c^2(a^4c^4 + (a^4+c^4)m + m^2)^{1/2}}{a^4m + m^2} < 0 \cdot \tag{A.4}$$

For $m = 4b^4 + a^4 - c^4$, $0 \le b \le a, 0 \le c \le a$, $m \ge 4b^4$ and Eq. (A.3), we can obtain the result of (A.4). Furthermore, we also can verify the solution is maximum value point. Therefore, we has $0 < x_2 < 0.5$, it is the solution for Eq. (10).

7. Acknowledgements

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