2012 International Conference on Image, Vision and Computing (ICIVC 2012)
IPCSIT vol. 50 (2012) © (2012) IACSIT Press, Singapore
DOI: 10.7763/IPCSIT.2012.V50.48

Improving Edge-detection Algorithm based on Fractional Differential Approach

XueHui Chen+ and XiangDong Fei

School of Computer Science and Technology, Sichuan University, Chengdu, China

Abstract. Fractional differential approach can detect textural features of digital image, makes it non-linear enhancement and suppresses the noise to a certain extent. And the traditional Roberts algorithm can locate relative accurately for edge detection, it could get thin but not clear enough edge, which is largely affected by noise. This paper mainly discusses how to use the advantages of fractional differential method to improve the shortcomings of Roberts. Firstly, both the Roberts and fractional differential theory are clearly explained respectively. Secondly, it discusses in detail the structures and parameters of 3*3 any order fractional differential mask on eight orientations. Lastly,it puts forward a new method based on combination fractional differential and Roberts operator. Experiments show that it can detect edges with high accuracy, more details, fine distinction. This also can provide reference for the thin edge needed detection in some specific applications.

Keywords: fractional differential; Roberts algorithm; edge-detection; fractional differential Mask;gradient method

1. Introduction

Edges are boundaries between different textures which also can be defined as discontinuities in image intensity from one pixel to another. The edges of a given image are always important characteristics that offer an indication for a higher frequency. Consequently, edge detection with one image is not only helpful to achieve image segmentation, data compression; but also applicable in well matching, such as image reconstruction [1]. Now, there have been a number of solutions proposed to extract edges, such as Roberts, Sobel, Prewitt and Laplacian operators. Moreover,it can be noticed that all of these are integer-order differential algorithms.

Fractional differential, non-integer order differential as well, attracts more attention compared with integer-order differential algorithms currently. As for the definition of fractional differential, there are three alternatives which named G-L, R-L and Caputo respectively [2].

Both in industrial application and research community, fractional differential has devoted great contribution, especially to digital image processing, which drives our motivation to conduct a deep investigation into this topic. This paper presents fractional differential characteristic of enhancing image texture detail features, which helps improve the traditional Roberts operator. Finally, an ideal observation was collected through experiments.

+ Corresponding author. Tel.: +15928828241.
E-mail address: huizi86054@126.com.

2. Theoretical Concepts

2.1. Integer Order Differential

There are many ways to perform edge detection. However, most of them can be grouped into two categories: search-based and zero-crossing based. The search-based methods detect edges by first computing a measure of edge strength, usually a first-order derivative expression, such as the gradient magnitude, sequentially, search local directional maxima of the gradient magnitude using a computed estimate of the local orientation of the edge, generally the gradient direction taken. The zero-crossing based ones look through zero crossings in a second-order derivative expression that computed from the image in order to find edges, mostly the zero-crossings of the Laplacian or the zero-crossings of a non-linear differential expression considered [3].

The most common type of edge detection process employs a gradient operator which detects the edges by looking for the maximum and minimum in the first derivative of the image. As for duration of two-dimensional digital image $f(x,y)$, the gradient is a vector at point $f(x,y)$, it can be defined as [3]:

$$\nabla f(x,y) = \begin{bmatrix} G_x & G_y \end{bmatrix}^T = \begin{bmatrix} \frac{\partial f}{\partial x} & \frac{\partial f}{\partial y} \end{bmatrix}^T$$

(1)

2.2. Roberts Edge Detector

Any pair of mutually perpendicular directions difference can be considered as the gradient approximation. The idea is behind experimental result which is observed by computing the sum of the squares of the differences between diagonally adjacent pixels. Roberts proposed the following equation [3]:

$$g(x,y) = \left|\nabla f(x,y)\right| = \left\{ \begin{array}{l} \left[f(x,y+1) - f(x+1,y)\right]^2 \\ + \left[f(x+1,y+1) - f(x,y)\right]^2 \end{array} \right\}^{\frac{1}{2}}$$

(2)

In order to perform edge detection with the Roberts operator, we convolve the original image with the following two kernels:

1	0
0	-1

(a)

0	1
-1	0

(b)

Fig. 1. Roberts mask on two orientation.(a) G_x (b) G_y .

2.3. Fractional Differential to Digital Image Processing

When $\forall v \in R$, suppose the integral part is $[v]$. When $s(t) \in [a,t](a < t, a \in R, t \in R)$, it has $m+1$ order continuous derivative; When $v > 0$, minimum m is $[v]$, and the fractional order Grümwald-Letnikov definition can be expressed as[4]

$$ {}_a^G D_t^v s(t) = \lim_{h->0} s_h^{(v)}(t) = \lim_{\substack{h->0 \\ nh=t-a}} h^{-v} \sum_{r=0}^{n} \begin{bmatrix} -v \\ r \end{bmatrix} s(t-rh)$$

(3)

When $\begin{bmatrix} -v \\ r \end{bmatrix} = \frac{(-v)(-v+1)\cdots(-v+r-1)}{r!}$. to make $s_n^{(-v)}(t)$ meet its non-zero limit, it must have $n \to \infty$, when $h \to 0$. If $h = \frac{t-a}{n}$, it has $n = \left[\frac{t-a}{h}\right]$.

As for any quadratic integrability energy type signal that $s(t) \in L^2(R)$, the v-order fractional differential Fourier Transform is [5]

$$D^v s(t) = D_v s(t) = \frac{d^v s(t)}{dt^v} \Leftrightarrow (\hat{D}_v s)(w) = (iw)^v \cdot \hat{s}(w)$$
$$= \hat{d_v}(w)\,\hat{s}(w), v \in R^+$$
(4)

where v order differential operator that $D^v = D_v$ is v order differential multipliable operator of function $\hat{d}(w) = (iw)^v$. The fractional differential filter function can be expressed as [7]

$$\begin{cases} \hat{d}(w) = (iw)^v = a_v\hat{(w)} \cdot \exp(i\theta_v(w)) = a\hat{(w)} \cdot p_v\hat{(w)} \\ a_v\hat{(w)} = |w|^v \\ \theta_v\hat{(w)} = \frac{v\pi}{2}\operatorname{sgn}(w) \end{cases}$$
(5)

Referring to eqs.(5)[6-8], in particular, when $0 < v < 1$, within $w > 1$ section, fractional differential can enhance high frequency of signal less than first-order and second-order differential, so does high frequency edge component of image signal. However, in $0 < w < 1$ low frequency area, fractional differential non-linear attenuates low frequency of signals unlike first derivative, which makes it substantially linear attenuation. As the differential order v decreases, the decay rate for low frequency components of fractional differential reduces, especially $v \to 0$, there is no attenuation.

The smooth area of an image is that the values of neighboring pixels are almost the same, which is corresponding to low frequency of signals; Texture region that is the values of adjacent pixels have small change corresponding to the medium frequency; Edges and noise are high frequent, the surrounding pixel values of which change sharply. Therefore, texture detail of smooth area will have substantially linear attenuation if we use traditional operators, such as Sobel operator (based on first-order derivative) and the Gauss-Laplace operator (based on the second derivative) to deal with the image smooth area, and the result approaches zero. It verified that the edge detection methods based on integer order differential cannot well detect texture details in image smooth regions. In contrast, fractional differential method non-linear preserves low frequency of signals [6, 7]. And the oder v can continuously change, we can obtain the best image edge information by adjusting the value of v in our experiments. Thus, fractional differential is more favorable than the integer-order differential for extracting image edge.

2.4. Fractional Differential Mask

Referring to eq. (3), suppose the duration of monadic signal $f(t)$ is $t \in [a,t]$, and then averagely divide the signal's duration as h=1. Thus, it has $n = \left[\frac{t-a}{h}\right] = [t-a]$. The difference of monadic fractional differential of signal $f(t)$ can be expressed as [7-10]

$$\frac{d^v f(t)}{dt^v} \approx f(t) + (-v)f(t-1) + \frac{(-v)(-v+1)}{2!} f(t-2) + \cdots + \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} f(t-n) \quad (6)$$

In a similar way, the backward difference of fractional partial differential on negative $x-$ and $y-$coordinate for duration of two-dimensional digital image $f(x,y)$ can be expressed as[9]

$$\frac{\partial^v f(x,y)}{\partial x^v} \approx f(x,y) + (-v)f(x-1,y) + \frac{(-v)(-v+1)}{2} f(x-2,y) + \cdots + \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} f(x-n,y) \quad (7)$$

$$\frac{\partial^v f(x,y)}{\partial y^v} \approx f(x,y) + (-v) f(x,y-1) + \frac{(-v)(-v+1)}{2} f(x,y-2) + \cdots + \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} f(x,y-n) \tag{8}$$

According to observations, the corresponding coefficient of the approximation of fractional partial differential on negative $x-$ and $y-$coordinate is the same. Moreover, only the first coefficient is constant "1", the other is nonzero, and the function of fractional differential order v. The non-zero coefficients are 1, $-v$, $\frac{(-v)(-v+1)}{2}$, ..., $\frac{\Gamma(-v+1)}{(n-1)!\Gamma(-v+n)}$. It can be proven that the summation of the nonzero coefficients is not zero, which is the distinct difference between fractional differential based processing and an integral based one.

Referring to eqs.(7)and(8), we can structure the fractional differential mask of 3*3 on the eight central symmetric directions, which are negative x -coordinate, negative y -coordinate, positive x -coordinate, positive y -coordinate, left downward diagonal, right upward diagonal, left upward diagonal, and right downward diagonal. It is shown in Fig. 2[8, 9].

0	$\frac{v^2-v}{2}$	0
0	$-v$	0
0	1	0

(a)

0	0	0
$\frac{v^2-v}{2}$	$-v$	1
0	0	0

(b)

$\frac{v^2-v}{2}$	0	0
0	$-v$	0
0	0	1

(c)

0	0	1
0	$-v$	0
$\frac{v^2-v}{2}$	0	0

(d)

$\frac{v^2-v+2}{2}$	$\frac{v^2-v+2}{2}$	$\frac{v^2-v+2}{2}$
$\frac{v^2-v+2}{2}$	$-8v$	$\frac{v^2-v+2}{2}$
$\frac{v^2-v+2}{2}$	$\frac{v^2-v+2}{2}$	$\frac{v^2-v+2}{2}$

(e)

Fig. 2Fractional differential mask on eight orientations. (a) negative x -coordinate. (b) positive y -coordinate. (c)right downward diagonal. (d)left upward diagonal .(e)the final mask.

3. Combination of Roberts and Fractional Order Differential Operator

Roberts edge detector uses diagonal direction difference between two adjacent pixels to detect gradient magnitude, it is better for detecting edges of horizontal and vertical directions rather than slash direction ones. Due to position accuracy and edge detection of a small number of pixels, the edge line is smaller, but not sharp enough, and sensitive to noise. The fractional derivative can enhance the image texture, if they are combined together, it should recognize the edges of small and prominent. Combination formula as following:

$$D^v[g(x,y)] = \frac{\partial^v g(x,y)}{\partial x^v} + \frac{\partial^v g(x,y)}{\partial y^v} \tag{9}$$

$$\begin{aligned}
\frac{\partial^v g(x,y)}{\partial x^v} &\approx g(x,y) + (-v)g(x-1,y) + \frac{(-v)(-v+1)}{2} g(x-2,y) \\
&+ \cdots + \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} g(x-n,y) \\
&= \begin{Bmatrix} [f(x,y+1) - f(x+1,y)]^2 \\ + [f(x+1,y+1) - f(x,y)]^2 \end{Bmatrix}^{\frac{1}{2}} + \\
&(-v)\begin{Bmatrix} [f(x-1,y+1) - f(x,y)]^2 \\ + [f(x,y+1) - f(x-1,y)]^2 \end{Bmatrix}^{\frac{1}{2}} + \\
&\frac{(-v)(-v+1)}{2}\begin{Bmatrix} [f(x-2,y+1) - f(x-1,y)]^2 \\ + [f(x-1,y+1) - f(x-2,y)]^2 \end{Bmatrix}^{\frac{1}{2}} \\
&+ \cdots + \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)}\begin{Bmatrix} [f(x-n,y+1) - f(x-n+1,y)]^2 \\ + [f(x-n+1,y+1) - f(x-n,y)]^2 \end{Bmatrix}^{\frac{1}{2}}
\end{aligned} \tag{10}$$

$$\begin{aligned}
\frac{\partial^v g(x,y)}{\partial y^v} &\approx g(x,y) + (-v)g(x,y-1) + \frac{(-v)(-v+1)}{2} g(x,y-2) \\
&+ \cdots + \frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)} g(x,y-n) \\
&= \begin{Bmatrix} [f(x,y+1) - f(x+1,y)]^2 \\ + [f(x+1,y+1) - f(x,y)]^2 \end{Bmatrix}^{\frac{1}{2}} + \\
&(-v)\begin{Bmatrix} [f(x,y) - f(x+1,y-1)]^2 \\ + [f(x+1,y) - f(x,y-1)]^2 \end{Bmatrix}^{\frac{1}{2}} + \\
&\frac{(-v)(-v+1)}{2}\begin{Bmatrix} [f(x,y-1) - f(x+1,y-2)]^2 \\ + [f(x+1,y-1) - f(x,y-2)]^2 \end{Bmatrix}^{\frac{1}{2}} + \cdots + \\
&\frac{\Gamma(-v+1)}{n!\Gamma(-v+n+1)}\begin{Bmatrix} [f(x,y-n+1) - f(x+1,y-n)]^2 \\ + [f(x+1,y-n+1) - f(x,y-n)]^2 \end{Bmatrix}^{\frac{1}{2}}
\end{aligned} \tag{11}$$

Among them, $f(x,y)$ is the the original image, $g(x,y)$ is the processed image by Roberts.

4. Experiments and Result Analysis

Experiments are conducted on Visual C++ 6.0 platform, with several methods to handle two classic images respectively to comparison. From the experimental results, we obtained that the improved algorithm has the advantages of Roberts, that is, thin edge, besides allowing edge enhancement, i.e clear and obvious. Comparatively, Sobel and Laplacian operators identified thick edges, which may make targets obscure easily in some special applications which require thin and delicate edge detection.

(a) Original Lena

(b) Roberts

(c) Improved operator (v=0.8)

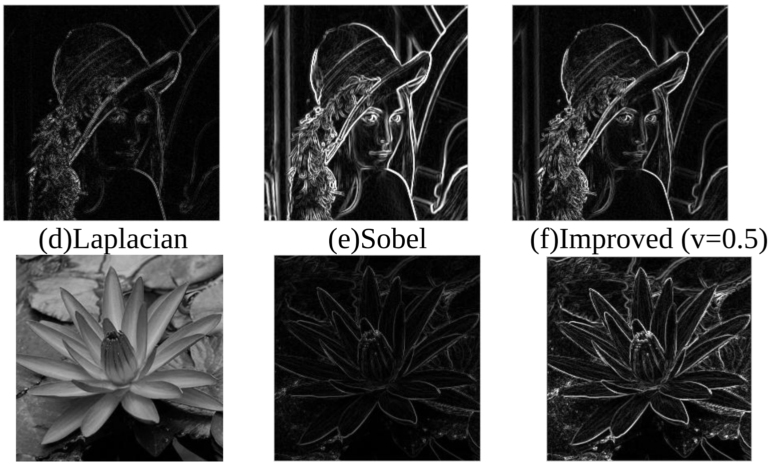

Fig. 3 Experimental results

5. Conclusions

How to get a better edge on special occasions in image processing is an interesting research issue. This paper intends to apply a newly mathematic approach to improve the edge detection methods based on the traditional integer order differential and achieves the better experimental results. It provides reference for the thin edge detection needed and also promotes the application of fractional differential.

6. Acknowledgment

This paper is supported and finaced by the National Natural Science Foundation of China(Grant No.60736046).

7. References

[1] Zheng Youyi, Rao Jilai, Wu Lei.analyzing and comparing several edge detection methods of digital image (in Chinese). Mechanical Engineering &Automation.2010.

[2] Pu Yifei, Yuan Xiao, Liao Ke, et al. Five numerical algorithms of fractional calculus applied in modern signal analyzing and processing (in Chinese with English abstract). Journal of Sichuan University: Engineering Science Edition, 2005, 37(5): 118-124.

[3] Xie Fengying, Zhao Danpei. Visual C++ digital image processing [M].Beijing: Electronic Indu- stry Press, 2008.

[4] Oldham K B, Spanier J.The fractional calculus [M]. New York: Academic Press, 1974.

[5] Falconer K[Writer], Zeng Wen-Qu, Liu Shi-Yao[Translator]. Fractal Geometry Mathematical Foundations and Applications. Liaoning: Northeast Industrial Press, 1991

[6] Pu Yifei. Application of fractional differential approach to digital image processing (in Chinese with English abstract). Sichuan Univ (Eng Sci Ed)(in Chinese), 2007, 39(3): 124-132.

[7] Pu Yifei, Wang Weixing. The fractional differential mask of digital image and its numerical implementation algorithm [J]. Acta Autom Sin(in Chinese), 2007, 33(11): 1128-1135

[8] Pu Yifei, Wang WeiXing, Zhou Jiliu,et al.Fractional differential approach to detecting textural features of digital image and its fractional differential filter implementation[J]. SCIENCE CHINA PRESS.2008.

[9] Pu Y F. Fractional Differential Filter of Digital Image. Invention Patent of China, No.200610021702.2006

[10] Pu Y F. Research on Application of Fractional Calculus to Latest Signal Analysis and Processing [Ph.D.dissertation], Sichuan University, 2006.