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Analysis of Boundedness for Unknown Functions by a Delay Integral Inequality on Time Scales

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Abstract. In this paper, new explicit bound for unknown functions is derived by a new Volterra-Fredholm type delay integral inequality on time scales, which can be used as a hand tool in the investigation of qualitative properties as well as quantitative properties of delay dynamic equations.

Keywords: Delay integral inequality; Time scales; Dynamic equation; Bounded

1. Introduction

Integral inequalities play an important role in the research of qualitative properties of solutions of dynamic equations, and many integral inequalities as well as difference inequalities have been established since then, for example [1-10], and the references therein.

Our aim in this paper is to establish a new Volterra-Fredholm type delay integral inequality on time scales, which provides new bound for unknown functions.

In the rest of the paper, R denotes the set of real numbers and $R_+ = [0,\infty)$. T denotes an arbitrary time scale and $T_0 = [x_0,\infty)\bigcap T, \overline{T}_0 = [y_0,\infty)\bigcap T$, where $x_0, y_0 \in T$. The set T^κ is defined to be T if T does not have a left-scattered maximum, otherwise it is T without the left-scattered maximum. On T we define the forward and backward jump operators $\sigma \in (T,T)$ and $\rho \in (T,T)$ such that $\sigma(t) = \inf\{s \in T, s > t\}$, $\rho(t) = \sup\{s \in T, s < t\}$.

Definition 1: A point $t \in T$ with $t > infT$ is said to be left-dense if $\rho(t) = t$ and right-dense if $\sigma(t) = t$, left-scattered if $\rho(t) < t$ and right-scattered if $\sigma(t) > t$.

Definition 2: A function $f \in (T,R)$ is called rd-continuous if it is continuous in right-dense points and if the left-sided limits exist in left-dense points, while f is called regressive if $1 + \mu(t) f(t) \neq 0$, where $\mu(t) = \sigma(t) - t$. C_{rd} denotes the set of rd-continuous functions, while $\overline{R}$ denotes the set of all regressive and rd-continuous functions, and $\overline{R}^+ = \{f \mid f \in \overline{R}, 1 + \mu(t) f(t) > 0, \forall t \in T\}$.

Definition 3: For some $t \in T^\kappa$, and a function $f \in (T,R)$, the delta derivative of f is denoted by $f^\Delta(t)$, and satisfies $| f(\sigma(t)) - f(s) - f^\Delta(t)(\sigma(t) - s) | \le \varepsilon | \sigma(t) - s |$ for $\forall \varepsilon > 0$, where $s \in U$, and U is a neighborhood of t. The function f is called delta differential on T^κ

Similarly, for some $y \in T^\kappa$, and a function $f \in (T \times T, R)$, the partial delta derivative of f with respect to y is denoted by $(f(x,y))_y^\Delta$, and satisfies
$| f(x,\sigma(y)) - f(x,s) - (f(x,y))_y^\Delta(\sigma(y) - s) | \le \varepsilon | \sigma(y) - s |$ for $\forall \varepsilon > 0$, where $s \in U$, and U is a neighborhood of y.

Definition 4: For some $a, b \in T$ and a function $f \in (T,R)$, the Cauchy integral of f is defined by

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$\int_a^b f(t)\Delta t = F(b) - F(a)$, where $F^{\Delta}(t) = f(t), t \in T^{\kappa}$.

Similarly, for some $a, b \in T$ and a function $f \in (T \times T, R)$, the Cauchy partial integral of f with respect to y is defined by $\int_a^b f(x,y)\Delta y = F(x,b) - F(x,a)$, where $(F(x,y))_y^{\Delta} = f(x,y), y \in T^{\kappa}$.

2. Main Results

We will give some lemmas for further use.

Lemma 2.1 ([11], Gronwall's inequality): Suppose $X \in T_0$ is an arbitrarily fixed number, and $u(X,y)$, $b(X,y) \in C_{rd}$, $m(X,y) \in \overline{R}^{+}$ with respect to y, $m(X,y) \geq 0$, then

$$u(X,y) \leq b(X,y) + \int_{y_0}^{y} m(X,t)u(X,t)\Delta t, y \in \overline{T}_0$$

implies

$$u(X,y) \leq b(X,y) + \int_{y_0}^{y} e_m(y, \sigma(t)) b(X,t) m(X,t)\Delta t, y \in \overline{T}_0,$$

where $e_m(y, y_0)$ is the unique solution of the following equation

$$(z(X,y))_y^{\Delta} = m(X,y) z(X,y), z(X,y_0) = 1 .$$

Lemma 2.2 [12]: Assume that $a \geq 0, p \geq q \geq 0$, and $p \neq 0$, then for any $K > 0$,

$$a^{\frac{q}{p}} \leq \frac{q}{p} K^{\frac{q-p}{p}} a + \frac{p-q}{p} K^{\frac{q}{p}} .$$

Consider the following inequality

$$u^p(x,y) \leq C + \int_{y_0}^{y}\int_{x_0}^{x} L(s,t,u(\tau_1(s),\tau_2(t)))\Delta s\Delta t + \int_{y_0}^{y}\int_{x_0}^{x}\int_{y_0}^{t}\int_{x_0}^{s} h_1(\xi,\eta) u^q(\tau_1(\xi),\tau_2(\eta))\Delta\xi\Delta\eta\Delta s\Delta t$$

$$+ \int_{y_0}^{N}\int_{x_0}^{M} L(s,t,u(\tau_1(s),\tau_2(t)))\Delta s\Delta t + \int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta) u^q(\tau_1(\xi),\tau_2(\eta))\Delta\xi\Delta\eta\Delta s\Delta t \quad (1)$$

with the initial condition

$$\begin{cases} u(x,y) = \phi(x,y), x \in [\alpha, x_0] \cap T, or, y \in [\beta, y_0] \cap T \\ \phi(\tau_1(x), \tau_2(y)) \leq C^{\frac{1}{p}}, \tau_1(x) \leq x_0, or, \tau_2(y) \leq y_0 \end{cases}, \quad (2)$$

where $u, h_i \in C_{rd}(T_0 \times \overline{T}_0, R_+), i = 1,2$, p, q, C m, C are constants, and $p \geq q \geq 0$, $p \neq 0, C > 0$, $\tau_1 \in (T_0, T), \tau_1(x) \leq x$, $-\infty < \alpha = \inf\{\tau_1(x), x \in T_0\} \leq x_0$, $\tau_2 \in (\overline{T}_0, T), \tau_2(y) \leq y, -\infty < \beta =$, $\inf\{\tau_2(y), y \in \overline{T}_0\} \leq y_0$ $\phi \in C_{rd}(([\alpha, x_0] \times [\beta, y_0]) \cap T^2, R_+)$, $M \in T_0, N \in \overline{T_0}$ are two fixed numbers.

Theorem 2.1: If for $(x,y) \in ([x_0, M] \cap T \times ([y_0, N] \cap T)$, $u(x,y)$ satisfies (1), and $K > 0$ is an arbitrary constant, then the following inequality holds

$$u(x,y) \leq \{[\frac{\overline{C} + B_6}{1 - B_5}] B_3(x,y) + B_4(x,y)\}^{\frac{1}{p}}, (x,y) \in ([x_0, M] \cap T) \times ([y_0, N] \cap T) \quad (3)$$

provided that $B_5 < 1$, where

$$\overline{C}=C+\int_{y_0}^{N}\int_{x_0}^{M}[L(s,t,\frac{p-1}{p}K^{\frac{1}{p}})+\int_{y_0}^{t}\int_{x_0}^{s}h_2(\xi,\eta)\frac{p-m}{p}K^{\frac{m}{p}}\Delta\xi\Delta\eta]\Delta s\Delta t$$

$$B_1(x,y)=\int_{y_0}^{y}\int_{x_0}^{x}[L(s,t,\frac{p-1}{p}K^{\frac{1}{p}})+\int_{y_0}^{t}\int_{x_0}^{s}h_1(\xi,\eta)\frac{p-q}{p}K^{\frac{q}{p}}\Delta\xi\Delta\eta]\Delta s\Delta t$$

$$B_2(x,y)=\int_{x_0}^{x}[A(s,y,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}+\int_{y_0}^{y}\int_{x_0}^{s}h_1(\xi,\eta)\frac{q}{p}K^{\frac{q-p}{p}}\Delta\xi\Delta\eta]\Delta s\cdot$$

$$B_3(x,y)=1+\int_{y_0}^{y}e_{B_2}(y,\sigma(t))B_2(x,t)\Delta t$$

$$B_4(x,y)=B_1(x,y)+\int_{y_0}^{y}e_{B_2}(y,\sigma(t))B_2(x,t)B_1(x,t)\Delta t$$

$$B_5=\int_{y_0}^{y}\int_{x_0}^{x}A(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}B_3(s,t)\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s}h_2(\xi,\eta)\frac{q}{p}K^{\frac{q-p}{p}}B_3(\xi,\eta)\Delta\xi\Delta\eta]\Delta s\Delta t\cdot$$

$$B_6=\int_{y_0}^{y}\int_{x_0}^{x}A(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}B_4(s,t)\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s}h_2(\xi,\eta)\frac{q}{p}K^{\frac{q-p}{p}}B_4(\xi,\eta)\Delta\xi\Delta\eta]\Delta s\Delta t\cdot$$

Proof : Let the right side of (1) be $v(x,y)$. Then

$$u(x,y)\le v^{\frac{1}{p}}(x,y),(x,y)\in([x_0,M]\cap T)\times([y_0,N]\cap T) \tag{4}$$

From (2) we have

$$u(\tau_1(x),\tau_2(y))\le v^{\frac{1}{p}}(x,y),(x,y)\in([x_0,M]\cap T)\times([y_0,N]\cap T) \tag{5}$$

Given a fixed $X\in[x_0,M]\cap T$, and $x\in[x_0,X]\cap T, y\in[y_0,N]\cap T$, then

$$v(x,y)\le v(X,y), x\in[x_0,X]\cap T, y\in[y_0,N]\cap T \tag{6}$$

Furthermore, considering

$$v(x_0,y_0)=C+\int_{y_0}^{N}\int_{x_0}^{M}L(s,t,u(\tau_1(s),\tau_2(t)))\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s}h_2(\xi,\eta)u^q(\tau_1(\xi),\tau_2(\eta))\Delta\xi\Delta\eta\Delta s\Delta t \tag{7}$$

so we have

$$v(X,y)=C+\int_{y_0}^{y}\int_{x_0}^{X}L(s,t,u(\tau_1(s),\tau_2(t)))\Delta s\Delta t+\int_{y_0}^{y}\int_{x_0}^{X}\int_{y_0}^{t}\int_{x_0}^{s}h_1(\xi,\eta)u^q(\tau_1(\xi),\tau_2(\eta))\Delta\xi\Delta\eta\Delta s\Delta t$$

$$+\int_{y_0}^{N}\int_{x_0}^{M}L(s,t,u(\tau_1(s),\tau_2(t)))\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s}h_2(\xi,\eta)u^q(\tau_1(\xi),\tau_2(\eta))\Delta\xi\Delta\eta\Delta s\Delta t$$

$$\le C+\int_{y_0}^{y}\int_{x_0}^{X}L(s,t,v^{\frac{1}{p}}(s,t))\Delta s\Delta t+\int_{y_0}^{y}\int_{x_0}^{X}\int_{y_0}^{t}\int_{x_0}^{s}h_1(\xi,\eta)v^{\frac{q}{p}}(\xi,\eta)\Delta\xi\Delta\eta\Delta s\Delta t$$

$$+\int_{y_0}^{N}\int_{x_0}^{M} L(s,t,v^{\frac{1}{p}}(s,t))\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta)u^q(\tau_1(\xi),\tau_2(\eta))\Delta\xi\Delta\eta\Delta s\Delta t$$

$$=v(x_0,y_0)+\int_{y_0}^{y}\int_{x_0}^{X} L(s,t,v^{\frac{1}{p}}(s,t))\Delta s\Delta t+\int_{y_0}^{y}\int_{x_0}^{X}\int_{y_0}^{t}\int_{x_0}^{s} h_1(\xi,\eta)v^{\frac{q}{p}}(\xi,\eta)\Delta\xi\Delta\eta\Delta s\Delta t \qquad (8)$$

From Lemma 2.2, we have

$$\begin{cases} v^{\frac{q}{p}}(x,y)\le \frac{q}{p}K^{\frac{q-p}{p}}v(x,y)+\frac{p-q}{p}K^{\frac{q}{p}} \\ v^{\frac{1}{p}}(x,y)\le \frac{1}{p}K^{\frac{1-p}{p}}v(x,y)+\frac{p-1}{p}K^{\frac{1}{p}} \end{cases} \qquad (9)$$

Combining (8), (9) we have

$$v(X,y)=v(x_0,y_0)+\int_{y_0}^{y}\int_{x_0}^{X} L(s,t,(\frac{1}{p}K^{\frac{1-p}{p}}v(s,t)+\frac{p-1}{p}K^{\frac{1}{p}}))\Delta s\Delta t+\int_{y_0}^{y}\int_{x_0}^{X}\int_{y_0}^{t}\int_{x_0}^{s} h_1(\xi,\eta)(\frac{q}{p}K^{\frac{q-p}{p}}v(\xi,\eta)+\frac{p-q}{p}K^{\frac{q}{p}})\Delta\xi\Delta\eta\Delta s\Delta t$$

$$=v(x_0,y_0)+\int_{y_0}^{y}\int_{x_0}^{X}[L(s,t,(\frac{1}{p}K^{\frac{1-p}{p}}v(s,t)+\frac{p-1}{p}K^{\frac{1}{p}}))-L(s,t,\frac{p-1}{p}K^{\frac{1}{p}})+L(s,t,\frac{p-1}{p}K^{\frac{1}{p}})]\Delta s\Delta t$$

$$+\int_{y_0}^{y}\int_{x_0}^{X}\int_{y_0}^{t}\int_{x_0}^{s} h_1(\xi,\eta)(\frac{q}{p}K^{\frac{q-p}{p}}v(\xi,\eta)+\frac{p-q}{p}K^{\frac{q}{p}})\Delta\xi\Delta\eta\Delta s\Delta t$$

$$\le v(x_0,y_0)+\int_{y_0}^{y}\int_{x_0}^{X} A(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}v(s,t)\Delta s\Delta t+\int_{y_0}^{y}\int_{x_0}^{X} L(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\Delta s\Delta t$$

$$+\int_{y_0}^{y}[\int_{x_0}^{X}\int_{y_0}^{t}\int_{x_0}^{s} h_1(\xi,\eta)\frac{q}{p}K^{\frac{q-p}{p}}\Delta\xi\Delta\eta\Delta s]v(X,t)\Delta t+\int_{y_0}^{y}\int_{x_0}^{X}\int_{y_0}^{t}\int_{x_0}^{s} h_1(\xi,\eta)\frac{p-q}{p}K^{\frac{q}{p}}\Delta\xi\Delta\eta\Delta s\Delta t$$

$$\le v(x_0,y_0)+\int_{y_0}^{y}[\int_{x_0}^{X} A(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}\Delta s]v(X,t)\Delta t+\int_{y_0}^{y}\int_{x_0}^{X} L(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\Delta s\Delta t$$

$$+\int_{y_0}^{y}[\int_{x_0}^{X}\int_{y_0}^{t}\int_{x_0}^{s} h_1(\xi,\eta)\frac{q}{p}K^{\frac{q-p}{p}}\Delta\xi\Delta\eta\Delta s]v(X,t)\Delta t+\int_{y_0}^{y}\int_{x_0}^{X}\int_{y_0}^{t}\int_{x_0}^{s} h_1(\xi,\eta)\frac{p-q}{p}K^{\frac{q}{p}}\Delta\xi\Delta\eta\Delta s\Delta t$$

$$=v(x_0,y_0)+B_1(X,y)+\int_{y_0}^{y} B_2(X,t)v(X,t)\Delta t$$

By Lemma 2.2 we obtain

$$v(X,y)\le v(x_0,y_0)+B_1(X,y)+\int_{y_0}^{y} e_{B_2}(y,\sigma(t))B_2(X,t)(v(x_0,y_0)+B_1(X,t))\Delta t$$

$$=v(x_0,y_0)[1+\int_{y_0}^{y} e_{B_2}(y,\sigma(t))B_2(X,t)\Delta t]+B_1(X,y)+\int_{y_0}^{y} e_{B_2}(y,\sigma(t))B_2(X,t)(v(x_0,y_0)+B_1(X,t))\Delta t,\ y\in[y_0,N]\cap T \qquad (10)$$

Combining (6), (10), it follows

$$v(x,y)\le v(x_0,y_0)+B_1(X,y)+\int_{y_0}^{y} e_{B_2}(y,\sigma(t))B_2(X,t)(v(x_0,y_0)+B_1(X,t))\Delta t$$

$$=v(x_0,y_0)[1+\int_{y_0}^{y} e_{B_2}(y,\sigma(t))B_2(X,t)\Delta t]+B_1(X,y)+$$

$$\int_{y_0}^{y} e_{B_2}(y,\sigma(t))B_2(X,t)(v(x_0,y_0)+B_1(X,t))\Delta t \quad, \ x\in[x_0,X]\cap T, y\in[y_0,N]\cap T \tag{11}$$

Setting $x=X$ in (11), considering X is selected from $[x_0,M]\cap T$ arbitrarily, substituting X with x, yields

$$v(x,y)\le v(x_0,y_0)+B_1(x,y)+\int_{y_0}^{y} e_{B_2}(y,\sigma(t))B_2(x,t)(v(x_0,y_0)+B_1(x,t))\Delta t$$

$$=v(x_0,y_0)[1+\int_{y_0}^{y} e_{B_2}(y,\sigma(t))B_2(x,t)\Delta t]+B_1(X,y)+$$

$$\int_{y_0}^{y} e_{B_2}(y,\sigma(t))B_2(x,t)(v(x_0,y_0)+B_1(x,t))\Delta t, \ x\in[x_0,X]\cap T, y\in[y_0,N]\cap T \tag{12}$$

that is,

$$v(x,y)\le v(x_0,y_0)B_3(x,y)+B_4(x,y) \ x\in[x_0,X]\cap T, y\in[y_0,N]\cap T \tag{13}$$

On the other hand, from (5), (7) (9) we have

$$v(x_0,y_0)=C+\int_{y_0}^{N}\int_{x_0}^{M} L(s,t,v^{\frac{1}{p}}(s,t))\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta)v^{\frac{q}{p}}(\xi,\eta)\Delta\xi\Delta\eta\Delta s\Delta t$$

$$\le C+\int_{y_0}^{N}\int_{x_0}^{M} L(s,t,\frac{1}{p}K^{\frac{1-p}{p}}v(s,t)+\frac{p-1}{p}K^{\frac{1}{p}})\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta)(\frac{q}{p}K^{\frac{q-p}{p}}v(\xi,\eta)+\frac{p-q}{p}K^{\frac{q}{p}})\Delta\xi\Delta\eta\Delta s\Delta t$$

$$\le C+\int_{y_0}^{N}\int_{x_0}^{M}[L(s,t,\frac{1}{p}K^{\frac{1-p}{p}}v(s,t)+\frac{p-1}{p}K^{\frac{1}{p}})-L(s,t,\frac{p-1}{p}K^{\frac{1}{p}})+L(s,t,\frac{p-1}{p}K^{\frac{1}{p}})]\Delta s\Delta t$$

$$+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta)(\frac{q}{p}K^{\frac{q-p}{p}}v(\xi,\eta)+\frac{p-q}{p}K^{\frac{q}{p}})\Delta\xi\Delta\eta\Delta s\Delta t\le C+\int_{y_0}^{N}\int_{x_0}^{M} A(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}v(s,t)\Delta s\Delta t$$

$$+\int_{y_0}^{N}\int_{x_0}^{M} L(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta)(\frac{q}{p}K^{\frac{q-p}{p}}v(\xi,\eta)+\frac{p-q}{p}K^{\frac{q}{p}})\Delta\xi\Delta\eta\Delta s\Delta t$$

$$=\overline{C}+\int_{y_0}^{N}\int_{x_0}^{M} A(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}v(s,t)\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta)(\frac{q}{p}K^{\frac{q-p}{p}}v(\xi,\eta)+\frac{p-q}{p}K^{\frac{q}{p}})\Delta\xi\Delta\eta\Delta s\Delta t \tag{14}$$

Then using (13) in (14) yields

$$v(x_0,y_0)\le\overline{C}+\int_{y_0}^{N}\int_{x_0}^{M} A(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}[v(x_0,y_0)B_3(s,t)+B_4(s,t)]\Delta s\Delta t$$

$$+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta)(\frac{q}{p}K^{\frac{q-p}{p}}[v(x_0,y_0)B_3(\xi,\eta)+B_4(\xi,\eta)]+\frac{p-q}{p}K^{\frac{q}{p}})\Delta\xi\Delta\eta\Delta s\Delta t$$

$$=\overline{C}+v(x_0,y_0)\{\int_{y_0}^{y}\int_{x_0}^{x} A(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}B_3(s,t)\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta)\frac{q}{p}K^{\frac{q-p}{p}}B_3(\xi,\eta)\Delta\xi\Delta\eta]\Delta s\Delta t\}$$

$$+\int_{y_0}^{y}\int_{x_0}^{x} A(s,t,\frac{p-1}{p}K^{\frac{1}{p}})\frac{1}{p}K^{\frac{1-p}{p}}B_4(s,t)\Delta s\Delta t+\int_{y_0}^{N}\int_{x_0}^{M}\int_{y_0}^{t}\int_{x_0}^{s} h_2(\xi,\eta)\frac{q}{p}K^{\frac{q-p}{p}}B_4(\xi,\eta)\Delta\xi\Delta\eta]\Delta s\Delta t$$

$$=\overline{C}+v(x_0,y_0)B_5+B_6 \quad (15)$$

which is followed by

$$v(x_0,y_0)\le\frac{\overline{C}+B_6}{1-B_5} \quad (16)$$

Combining (4), (13) and (16) we can obtain the desired inequality (3).

3. Conclusions

In this paper, new explicit bound for unknown functions is derived by use of a new Volterra- Fredholm type integral inequality on time scales, which provides a handy tool in the qualitative analysis of solutions of dynamic equations on time scales.

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