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New Results on the Solutions of a Nonlinear Evolution Equation

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Abstract. In this paper, we derive exact traveling wave solutions of Kadomtsev-Petviashvili equation by a proposed Bernoulli sub-ODE method. The method appears to be efficient in seeking exact solutions of nonlinear equations

Keywords: Bernoulli sub-ODE method, traveling wave solutions, exact solution, evolution equation, Kadomtsev-Petviashvili equation

1. Introduction

The nonlinear phenomena exist in all the fields including either the scientific work or engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and so on. It is well known that many nonlinear evolution equations (NLEEs) are widely used to describe these complex phenomena. Research on solutions of NLEEs is popular. So, the powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far.

In this paper, we pay attention to the analytical method for getting the exact solution of some NLEES. Among the possible exact solutions of NLEEs, certain solutions for special form may depend only on a single combination of variables such as traveling wave variables. In the literature, Also there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the inverse scattering transform, the Darboux transform, the tanh-function expansion and its various extension, the Jacobi elliptic function expansion, the homogeneous balance method, the sine-cosine method, the rank analysis method, the exp-function expansion method and so on [1-20]. In this paper, we proposed a Bernoulli sub-ODE method to construct exact traveling wave solutions for NLEES.

The rest of the paper is organized as follows. In Section 2, we describe the known (G'/G) expansion method and the Bernoulli sub-ODE method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps for them. In the subsequent sections, we will apply the (G'/G) expansion method and the Bernoulli sub-ODE method to find exact traveling wave solutions of the Kadomtsev-Petviashvili equation. In the last Section, some conclusions are presented.

2. Description of the Bernoulli Sub-ODE method

In this section we present the solutions of the following ODE:

$$G' + \lambda G = \mu G^2, \tag{2.1}$$

where $\lambda \neq 0, G = G(\xi)$

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When $\mu \neq 0$, Eq. (2.1) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \tag{2.2}$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t , is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}......) = 0 \tag{2.3}$$

where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. By using the solutions of Eq. (2.1), we can construct a serials of exact solutions of nonlinear equations:.

Step 1. We suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{2.4}$$

the traveling wave variable (2.4) permits us reducing Eq. (2.3) to an ODE for $u = u(\xi)$

$$P(u, u', u'', ...) = 0 \tag{2.5}$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in G as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + ... \tag{2.6}$$

where $G = G(\xi)$ satisfies Eq. (2.1), and $\alpha_m, \alpha_{m-1}...$ are constants to be determined later, $\alpha_m \neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the same order of G together, the left-hand side of Eq. (2.5) is converted into another polynomial in G. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, ...\lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.1), we can construct the traveling wave solutions of the nonlinear evolution equation (2.5).

In the subsequent section we will illustrate the proposed method in detail by applying it to Kadomtsev-Petviashvili equation.

3. Application of the Bernoulli Sub-ODE Method for Kadomtsev-Petviashvili Equation

In this section, we will consider the following Kadomtsev-Petviashvili equation:

$$(u_t + 6uu_x + u_{xxx})_x + 3\sigma u_{yy} = 0 \tag{3.1}$$

Suppose that

$$u(x, y, t) = u(\xi), \xi = x + y - ct \tag{3.2}$$

c is a constant that to be determined later.

By (3.2), (3.1) is converted into an ODE

$$-cu'' + 6(u')^2 + 6uu'' + u^{(4)} + 3\sigma u'' = 0 \tag{3.3}$$

Integrating the ODE (3.3) once, we obtain

$$(-c+3\sigma)u'+u^{(3)}+6uu'=g, \qquad (3.4)$$

where g is the integration constant that can be determined later.

Suppose that the solution of (3.4) can be expressed by a polynomial in G as follows:

$$u(\xi)=\sum_{i=0}^{m} a_i G^i, \qquad (3.5)$$

where a_i are constants, and $G=G(\xi)$ satisfies Eq.(2.1).

Balancing the order of uu' and u''' in Eq.(3.4), we have $m+m+1=m+3 \Rightarrow m=2$ .So Eq.(3.5) can be rewritten as

$$u(\xi)=a_2G^2+a_1G+a_0, a_2\neq 0, \qquad (3.6)$$

where a_2, a_1, a_0 are constants to be determined later.

Substituting (3.6) into (3.4) and collecting all the terms with the same power of G together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$G^0: -g=0$

$G^1: ca_1\lambda-6a_0a_1\lambda-a_1\lambda^3-3\sigma a_1\lambda=0$

$G^2: -6\sigma a_2\lambda-8a_2\lambda^3+2ca_2\lambda+7\mu a_1\lambda^2-12a_0a_2\lambda+3\sigma a_1\mu -ca_1\mu-6a_1{}^2\lambda+6a_0a_1\mu=0$

$G^3: 12a_0a_2\mu+6\sigma a_2\mu-2ca_2\mu-18a_1a_2\lambda-12\mu^2a_1\lambda +6a_1{}^2\mu+38a_2\mu\lambda^2=0$

$G^4: -54a_2\mu^2\lambda+18a_1a_2\mu-12a_2{}^2\lambda+6\mu^3a_1=0$

$G^5: 24a_2\mu^3+12a_2{}^2\mu=0$

Solving the algebraic equations above, yields:

$$a_2=-2\mu^2, a_1=2\mu\lambda, a_0=a_0, c=\lambda^2+3\sigma+6a_0, g=0 \qquad (3.7)$$

where a_0 is an arbitrary constants.

Substituting (3.7) into (3.6), we get that

$$u(\xi)=-2\mu^2G^2+2\mu\lambda G+a_0,\ \xi=x+y-(\lambda^2+3\sigma+6a_0)t \qquad (3.8)$$

Combining with Eq. (2.2), we can obtain the traveling wave solutions of (3.1) as follows:

$$u(\xi)=-2\mu^2\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)^2+2\mu\lambda\left(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}}\right)+a_0, \qquad (3.9)$$

where a_0, d are arbitrary constants.

Since $\xi=x+y-(\lambda^2+3\sigma+6a_0)t$, we can obtain:

$$u(\xi)=-2\mu^2\left\{\frac{1}{\frac{\mu}{\lambda}+de^{\lambda[x+y-(\lambda^2+3\sigma+6a_0)t]}}\right\}^2+2\mu\lambda\left\{\frac{1}{\frac{\mu}{\lambda}+de^{\lambda[x+y-(\lambda^2+3\sigma+6a_0)t]}}\right\}+a_0. \qquad (3.10)$$

Remark : Our result (3.10) is a new family of exact traveli-ng wave solutions for Eq. (3.1)

4. Conclusions

We have seen that some new traveling wave solutions of Kadomtsev-Petviashvili equation are successfully found by using the Bernoulli sub-ODE method. The main points of the method are that assuming the solution of the ODE reduced by using the traveling wave variable as well as integrating can be expressed by an m-th degree polynomial in G, where $G = G(\xi)$ is the general solutions of a Bernoulli sub-ODE equation. The positive integer m can be determined by the general homogeneous balance method, and the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations. Also this method can be used to many other nonlinear problems.

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