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Soling a Nonlinear Evolution Equation by A Proposed Bernoulli Sub-ODE Method

Bin Zheng+

School of Science, Shandong University of Technology, Zhangzhou Road 12, Zibo, Shandong, China, 255049

Abstract. In this letter, a generalized sub-ODE method is proposed to construct exact solutions of nonlinear Schrödinger (NLS) equation. As a result, some new exact traveling wave solutions are found.

Keywords: Bernoulli sub-ODE method, traveling wave solutions, exact solution, evolution equation, nonlinear Schrödinger equation

1. Introduction

Research on solutions of NLEEs is popular. So, the powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far.

Recently, seeking the exact solutions of nonlinear equations has getting more and more popular. Many approaches have been presented so far such as the homogeneous balance method [1,2], the hyperbolic tangent expansion method [3,4], the trial function method [5], the tanh-method [6-8], the nonlinear transform method [9], the inverse scattering transform [10], the Backlund transform [11,12], the Hirotas bilinear method [13,14], the generalized Riccati equation [15,16], the theta function method [17-19], the sine-Ccosine method [20], the Jacobi elliptic function expansion [21,22], the complex hyperbolic function method [23-25], and so on.

In this paper, we proposed a generalized Bernoulli sub-ODE method, and present an application for this method to nonlinear evolution equations.

The rest of the paper is organized as follows. In Section 2, we describe the Bernoulli sub-ODE method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method. In the subsequent sections, we will apply the method to find exact traveling wave solutions of the nonlinear Schrödinger equation. In the last Section, some conclusions are presented.

2. Description of the Bernoulli Sub-ODE method

In this section we present the solutions of the following ODE:

$$G' + \lambda G = \mu G^2, \tag{2.1}$$

where $\lambda \neq 0, G = G(\xi)$

When $\mu \neq 0$, Eq. (2.1) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \tag{2.2}$$

+ Corresponding author.
E-mail address: zhengbin2601@126.com

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t , is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy} ......) = 0 \tag{2.3}$$

where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. By using the solutions of Eq. (2.1), we can construct a serials of exact solutions of nonlinear equations:.

Step 1. We suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{2.4}$$

the traveling wave variable (2.4) permits us reducing Eq. (2.3) to an ODE for $u = u(\xi)$

$$P(u, u', u'', ...) = 0 \tag{2.5}$$

Step 2. Suppose that the solution of (2.5) can be expressed by a polynomial in G as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + ... \tag{2.6}$$

where $G = G(\xi)$ satisfies Eq. (2.1), and $\alpha_m, \alpha_{m-1}...$ are constants to be determined later, $\alpha_m \neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.5).

Step 3. Substituting (2.6) into (2.5) and using (2.1), collecting all terms with the same order of G together, the left-hand side of Eq. (2.5) is converted into another polynomial in G . Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, ...\lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.1), we can construct the traveling wave solutions of the nonlinear evolution equation (2.5).

In the subsequent sections we will illustrate the proposed method in detail by applying it to NLS equation.

3. Application Of the Bernoulli Sub-ODE Method For NLS Equation

In this section, we will consider the following NLS equation:

$$i\phi_t - \phi_{xx} + 2(|\phi|^2 - \rho^2)\phi = 0 \tag{3.1}$$

where ϕ is complex wave function and ρ is a constant.

Since $\phi = \phi(x, t)$ in Eq. (3.1) is a complex function, we suppose that

$$\phi = u(\xi)\exp[i(\alpha x + \beta t)], \xi = k(x + 2\alpha t) \tag{3.2}$$

where the constants α, β, k can be determined later.

By using (3.2), (3.1) is converted into an ODE

$$(-\beta + \alpha^2 - 2\rho^2)u + 2u^3 - k^2 u'' = 0 \tag{3.3}$$

Suppose that the solution of (3.4) can be expressed by a polynomial in G as follows:

$$u(\xi)=\sum_{i=0}^{m}a_iG^i \tag{3.4}$$

where a_i are constants, and $G=G(\xi)$ satisfies Eq. (2.1).

Balancing the order of u^3 and u'' in Eq.(3.3), we obtain that $3m=m+2\Rightarrow m=1$. So Eq. (3.4) can be rewritten as

$$u(\xi)=a_1G+a_0, a_1\neq 0 \tag{3.5}$$

where a_1, a_0 are constants to be determined later.

Substituting (3.5) into (3.3) and collecting all the terms with the same power of G together, the left-hand side of Eq.(3.3) is converted into another polynomial in G. Equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0: -\beta a_0+\alpha^2a_0-2\rho^2a_0+2a_0^{\ 3}=0$$

$$G^1: -\beta a_1-2\rho^2a_1+\alpha^2a_1-k^2a_1\lambda^2+6a_0^{\ 2}a_1=0$$

$$G^2: 6a_1^{\ 2}a_0+3k^2\lambda a_1\mu=0$$

$$G^3: 2a_1^{\ 3}-2k^2a_1\mu^2=0$$

Solving the algebraic equations above, yields:

$$\text{Case 1: } a_1=-k\mu, a_0=\frac{1}{2}k\lambda, \beta=\alpha^2-2\rho^2+2a_0^{\ 2} \tag{3.6}$$

Substituting (3.8) into (3.7), we have

$$u_1(\xi)=-k\mu G+\frac{1}{2}k\lambda,\ \ \xi=k(x+2\alpha t) \tag{3.7}$$

Combining with Eq. (2.2) and considering $u=v^{-\frac{1}{n-1}}$, we can obtain the traveling wave solutions of (3.1) as follows:

$$\phi_1(\xi)=[-k\mu(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda\xi}})+\frac{1}{2}k\lambda].\ \exp[i(\alpha x+(\alpha^2-2\rho^2+2a_0^{\ 2})t)] \tag{3.8}$$

where d is an arbitrary constant.
Then we have

$$\phi_1(x,t)=[-k\mu(\frac{1}{\frac{\mu}{\lambda}+de^{\lambda k(x+2\alpha t)}})+\frac{1}{2}k\lambda].\ \exp[i(\alpha x+(\alpha^2-2\rho^2+2a_0^{\ 2})t)] \tag{3.9}$$

$$\text{Case 2: } a_1=k\mu, a_0=-\frac{1}{2}k\lambda, \beta=\alpha^2-2\rho^2+2a_0^{\ 2} \tag{3.10}$$

Substituting (3.8) into (3.7), we have

$$u_2(\xi)=k\mu G-\frac{1}{2}k\lambda,\ \xi=k(x+2\alpha t) \tag{3.11}$$

Combining with Eq. (2.2) and considering $u = v^{-\frac{1}{n-1}}$, we can obtain the traveling wave solutions of (3.1) as follows:

$$\phi_2(\xi) = [k\mu(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}) - \frac{1}{2}k\lambda].\ \exp[i(\alpha x + (\alpha^2 - 2\rho^2 + 2{a_0}^2)t)] \quad (3.12)$$

where d is an arbitrary constant. Then we have

$$\phi_2(x,t) = [-k\mu(\frac{1}{\frac{\mu}{\lambda} + de^{\lambda k(x+2\alpha t)}}) + \frac{1}{2}k\lambda].\ \exp[i(\alpha x + (\alpha^2 - 2\rho^2 + 2{a_0}^2)t)] \quad (3.13).$$

4. Conclusions

In the present work, we propose a new Bernoulli sub-ODE method, and then test its power by finding some new traveling wave solutions of NLS equation. Being concise and simple, this method is one of the most effective approaches handling nonlinear evolution equations, and can be used to many other nonlinear problems.

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