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Classical Solutions for the DSSH Equation by A Generalized Sub-ODE Method

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Abstract. In this paper, we derive exact traveling wave solutions of DSSH equation by a proposed Bernoulli sub-ODE method. The method appears to be efficient in seeking exact solutions of nonlinear equations.

Keywords: Bernoulli sub-ODE method, traveling wave solutions, exact solution, evolution equation, DSSH equation

1. Introduction

The nonlinear phenomena exist in all the fields including either the scientific work or engineering fields, such as fluid mechanics, plasma physics, optical fibers, biology, solid state physics, chemical kinematics, chemical physics, and so on. It is well known that many nonlinear evolution equations (NLEEs) are widely used to describe these complex phenomena. Research on solutions of NLEEs is popular. So, the powerful and efficient methods to find analytic solutions and numerical solutions of nonlinear equations have drawn a lot of interest by a diverse group of scientists. Many efficient methods have been presented so far.

In this paper, we pay attention to the analytical method for getting the exact solution of some NLEES. Among the possible exact solutions of NLEEs, certain solutions for special form may depend only on a single combination of variables such as traveling wave variables. In the literature, Also there is a wide variety of approaches to nonlinear problems for constructing traveling wave solutions. Some of these approaches are the inverse scattering transform, the Darboux transform, the tanh-function expansion and its various extension, the Jacobi elliptic function expansion, the homogeneous balance method, the sine-cosine method, the rank analysis method, the exp-function expansion method and so on [1-22]. In this paper, we proposed a Bernoulli sub-ODE method to construct exact traveling wave solutions for NLEES.

The rest of the paper is organized as follows. In Section 2, we describe the known (G'/G) expansion method and the Bernoulli sub-ODE method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps for them. In the subsequent sections, we will apply the (G'/G) expansion method and the Bernoulli sub-ODE method to find exact traveling wave solutions of the DSSH equation. In the last Section, some conclusions are presented

2. Description of the (G'/G)-expansion Method and the Bernoulli Sub-ODE method

In this section we will describe the (G'/G)-expansion method for finding out the traveling wave solutions of nonlinear evolution equations.

Suppose that a nonlinear equation, say in three independent variables x, y and t , is given by

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$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy} ......) = 0 \tag{2.1}$$

where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the (G'/G)-expansion method.

Step 1. Combining the independent variables x, y and t into one variable $\xi = \xi(x, y, t)$, we suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{2.2}$$

the traveling wave variable (2.2) permits us reducing Eq. (2.1) to an ODE for $u = u(\xi)$

$$P(u, u', u'', ...) = 0 \tag{2.3}$$

Step 2. Suppose that the solution of (2.3) can be expressed by a polynomial in (G'/G) as follows:

$$u(\xi) = \alpha_m (\tfrac{G'}{G})^m + ... \tag{2.4}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0, \tag{2.5}$$

$\alpha_m, ...\lambda$ and μ are constants to be determined later, $\alpha_m \neq 0$ .The unwritten part in (2.4) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than m−1. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (2.3).

Step 3. Substituting (2.4) into (2.3) and using second order LODE (2.5), collecting all terms with the same order of $(\frac{G'}{G})$ together, the left-hand side of Eq. (2.3) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, ...\lambda$ and μ.

Step 4. Assuming that the constants $\alpha_m, ...\lambda$ and μ can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (2.5) have been well known for us, substituting $\alpha_m, ...$ and the general solutions of Eq. (2.5) into (2.4) we can obtain the traveling wave solutions of the nonlinear evolution equation (2.1).

In the following we will describe the main steps of Bernoulli sub-ODE method. First we consider the following ODE:

$$G' + \lambda G = \mu G^2, \tag{2.6}$$

where $\lambda \neq 0, G = G(\xi)$

When $\mu \neq 0$, Eq. (2.6) is the type of Bernoulli equation, and we can obtain the solution as

$$G = \frac{1}{\frac{\mu}{\lambda} + de^{\lambda\xi}}, \tag{2.7}$$

where d is an arbitrary constant.

Suppose that a nonlinear equation, say in two or three independent variables x, y and t , is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}......) = 0 \tag{2.8}$$

where u = u(x, y, t) is an unknown function, P is a polynomial in u = u(x, y, t) and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. By using the solutions of Eq. (2.6), we can construct a serials of exact solutions of nonlinear equations:.

Step 1. We suppose that

$$u(x, y, t) = u(\xi), \xi = \xi(x, y, t) \tag{2.9}$$

the traveling wave variable (2.9) permits us reducing Eq. (2.8) to an ODE for $u = u(\xi)$

$$P(u, u', u'', ...) = 0 \tag{2.10}$$

Step 2. Suppose that the solution of (2.10) can be expressed by a polynomial in G as follows:

$$u(\xi) = \alpha_m G^m + \alpha_{m-1} G^{m-1} + ... \tag{2.11}$$

where $G = G(\xi)$ satisfies Eq. (2.1), and $\alpha_m, \alpha_{m-1}...$ are constants to be determined later, $\alpha_m \neq 0$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and non-linear terms appearing in (2.10).

Step 3. Substituting (2.11) into (2.10) and using (2.6), collecting all terms with the same order of G together, the left-hand side of Eq. (2.10) is converted into another polynomial in G. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, \alpha_{m-1}, ...\lambda, \mu$.

Step 4. Solving the algebraic equations system in Step 3, and by using the solutions of Eq. (2.6), we can construct the traveling wave solutions of the nonlinear evolution equation (2.10).

3. Application Of Bernoulli Sub-ODE Method For DSSH Equation

In this section, we will consider the following DSSH equation:

$$u_{xxxxxx} - 9u_x u_{xxxx} - 18u_{xx}u_{xxx} + 18u_x{}^2 u_{xx} - \frac{1}{2}u_{tt} + \frac{1}{2}u_{xxxt} = 0 \tag{3.1}$$

In order to obtain the traveling wave solutions of Eq. (3.1), we suppose that

$$u(x,t) = u(\xi), \xi = x - ct, \tag{3.2}$$

Where c are constants that to be determined later.

By using (3.2), (3.1) is converted into an ODE

$$u^{(6)} - 9u'u^{(4)} - 18u''u''' + 18(u')^2 u'' - \frac{1}{2}c^2 u'' - \frac{1}{2}cu^{(4)} = 0 \tag{3.3}$$

Integrating (3.3) once it follows:

$$u^{(5)} - 9u'u''' - \frac{9}{2}(u'')^2 + 6(u')^3 - \frac{1}{2}c^2 u' - \frac{1}{2}cu''' = g \tag{3.4}$$

where g is the integration constant.

Suppose that the solution of (3.4) can be expressed by a polynomial in G as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i G^i \tag{3.5}$$

where a_i are constants, G. satisfies Eq. (2.6). Balancing the order of $u^{(5)}$ and $(u')^3$ in Eq. (3.3), we have $m+5 = 3m+3 \Rightarrow m = 1$. So (3.5) can be rewritten as

$$u(\xi) = a_0 + a_1 G, \tag{3.6}$$

where a_1, a_0 are constants to be determined later.

Substituting (3.6) into (3.4) and collecting all the terms with the same power of G together and equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$G^0 : -g = 0$$

$$G^1 : \frac{1}{2}c^2 a_1 \lambda - a_1 \lambda^5 + \frac{1}{2} c a_1 \lambda^3 = 0$$

$$G^2 : -\frac{1}{2}c^2 a_1 \mu - \frac{7}{2} c\mu a_1 \lambda^2 + 31\mu a_1 \lambda^4 - \frac{27}{2}\lambda^4 a_1^{\ 2} = 0$$

$$G^3 : 99 a_1^{\ 2} \mu \lambda^3 + 6c\lambda a_1 \mu^2 - 180 a_1 \mu^2 \lambda^3 - 6 a_1^{\ 3} \lambda^3 = 0$$

$$G^4 : 390 a_1 \mu^3 \lambda^2 - \frac{459}{2} \mu^2 a_1^{\ 2} \lambda^2 + 18 a_1^{\ 3} \mu \lambda^2 - 3c a_1 \mu^3 = 0$$

$$G^5 : 216 a_1^{\ 2} \mu^3 \lambda - 360 \mu^4 a_1 \lambda - 18 a_1^{\ 3} \mu^2 \lambda = 0$$

$$G^6 : -72 \mu^4 a_1^{\ 2} + 6\mu^3 a_1^{\ 3} + 120 a_1 \mu^5 = 0$$

Solving the algebraic equations above, yields:

$$a_1 = 2\mu, a_0 = a_0, c = \lambda^2, g = 0 \tag{3.7}$$

Substituting (3.7) into (3.6), we have

$$u(\xi) = a_0 + 2\mu G,\ \xi = x - \lambda^2 t \tag{3.8}$$

Combining with Eq. (2.7) and (3.8), we can obtain the traveling wave solutions of (3.1) as follows:

$$u(x,t) = a_0 + 2\mu\left(\frac{1}{\frac{\mu}{\lambda} + d e^{\lambda(x + \frac{2}{\lambda^2 - 2}t)}}\right) \tag{3.9}$$

where d, a_0 are an arbitrary constants

4. Application Of the (G'/G)-expansion Method For DSSH Equation

In this section, we will apply the (G'/G) expansion method to solve DSSH equation.

Suppose that the solution of (3.4) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i \left(\frac{G'}{G}\right)^i \tag{4.1}$$

where a_i are constants, G. satisfies Eq. (2.2). Balancing the order of $u^{(5)}$ and $(u')^3$ in Eq. (4.3), we have $m+5=3m+3 \Rightarrow m=1$. So (4.5) can be rewritten as

$$u(\xi)=a_0+a_1G \tag{4.2}$$

a_1, a_0 are constants to be determined later.

Substituting (4.2) into (3.4) and collecting all the terms with the same power of $(\frac{G'}{G})$ together and equating each coefficient to zero, yields a set of simultaneous algebraic equations. Solving the algebraic equations above, yields:

$$a_1=-2, a_0=a_0, c=\lambda^2-4\mu, g=0 \tag{4.3}$$

Substituting (4.7) into (4.6), we have

$$u(\xi)=a_0-2G,\ \xi=x-(\lambda^2-4\mu)t \tag{4.4}$$

Combining with Eq. (2.2) and (4.4), we can obtain the traveling wave solutions of (3.1) as follows:

Case 1:

When $\lambda^2-4\mu>0$

$$u_1(\xi)=a_0+\lambda-\sqrt{\lambda^2-4\mu}.\,(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}) \tag{4.5}$$

where λ, a_0 are an arbitrary constants.

Since $\xi=x-(\lambda^2-4\mu)t$, then furthermore we have

$$u_1(x,t)=a_0+\lambda-\sqrt{\lambda^2-4\mu}.\,(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}(x-(\lambda^2-4\mu)t)+C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}(x-(\lambda^2-4\mu)t)}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}(x-(\lambda^2-4\mu)t)+C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}(x-(\lambda^2-4\mu)t)}) \tag{4.6}$$

Case 2:

When $\lambda^2-4\mu<0$

$$u_1(\xi)=a_0+\lambda-\sqrt{4\mu-\lambda^2}.\,(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}) \tag{4.7}$$

where λ, a_0 are an arbitrary constants.

Since $\xi=x-(\lambda^2-4\mu)t$, then furthermore we have

$$u_1(\xi)=a_0+\lambda-\sqrt{4\mu-\lambda^2}.\,(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}(x-(\lambda^2-4\mu)t)+C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}x-(\lambda^2-4\mu)t}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}x-(\lambda^2-4\mu)t+C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}x-(\lambda^2-4\mu)t}) \tag{4.8}$$

Remark: From Section III and Section IV we can see the solutions derived by the Bernoulli sub-ODE method are different from those by (G'/G) expansion method

5. Conclusions

We have seen that some new traveling wave solution of DSSH equation is successfully found by using the Bernoulli sub-ODE method. Also we make a comparison between the Bernoulli sub-ODE method and the known (G'/G) expansion method. One can see the method is concise and effective. This method can be used to many other nonlinear problems.

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