A Novel Algorithm for DOA Estimation of Multi-path Channel

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Abstract. By analyzing the spatial smoothing MUSIC algorithm, we propose the transient smoothing algorithm and apply it in the presence of multi-path channel. It can be used in the estimation of DOA in Eigen-space. The relation between ratio of phase and time and the position of transmitter has been analyzed and concluded. By computer simulation, the performance of spatial and transient smoothing algorithm in the different assumptions has been compared and demonstrated.

Keywords: transient smoothing, DOA estimation, multi-path, space-smoothing

1. Introduction

Multi-path interference is generated mainly from the copies of emission signals. These signals are coherent or correlated when they are arriving at the receiver [1]. A wide variety of DOA estimators have been proposed and analyzed in the past few decades, such as the maximum likelihood (ML) algorithm[2,3], MUSIC algorithm[4], ESPRIT algorithm[5], weighted subspace fitting method (WSF)[6] and so on. All of these estimators or algorithms show excellent performance on some assumption or in some cases. The algorithms which are based on the subspace of signal or noise require that the covariance matrix of receiving signals is full rank. In the presence of multi-path channel, the receiving signals are coherent or correlated. This leads to singular covariance matrix of receiving signals. Many new algorithms such as spatial smoothing algorithm, front-rear spatial smoothing algorithm, array pattern have been proposed to increase the rank of covariance matrix of receiving signals [7]. These spatial smoothing covariance matrixes depend on the number of sub-array and space interval of DOA. More sub-arrays mean more antennas and the system is more complicated. According to the current array pattern of base station, we propose a transient smoothing method, which is similar to spatial smoothing method, to extract the coherence of sub-array which length is d. By this way, we can use any subspace algorithm to estimate DOA of coherent signals in fading channels. Simulation results have shown that the proposed algorithm is able to provide significant performance in the same condition of coherent sources

2. Multi-path model

Let $S(t)$ be the modulation signal of moving transmitter, follows as:

$$S(t) = s(t)e^{j(\omega_c t + \phi)} \tag{1}$$

where $s(t)$ is base-band signal, ω_c is frequency of carrier, ϕ is initial phase of carrier.

Here we assume that $\phi = 0$ and the distance of multi-path channel is q. The receiving terminal consists of M antennas. The received signal of given antenna element is:

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$$x_i^\alpha(t) = \sum_{l=1}^{q} \alpha_l s(t - \tau_l - (i-1)\frac{d}{c}\sin\theta_l)e^{j[\omega_c t + \phi_l(t) - (i-1)\frac{d}{c}\sin\theta_l]} + n_i(t)e^{j\omega_c t} \tag{2}$$

where $\phi_l(t) = (\omega_d \cos\psi_l)t - \omega_c \tau_l(t)$, α_l , τ_l , θ_l , d and c denote respectively correlation coefficient, delay of travel time, DOA, distance of antenna array and propagation velocity. ω_d , ψ_l and $n_i(t)$ denote respectively maximum Doppler frequency, angle between moving speed vector and lth scattering and white noise.

Transform (2) into base-band signal and assume the bandwidth of $s(t)$ is much smaller than the reciprocal of time delay in spanning array aperture, there be

$$x_i(t) = \sum_{l=1}^{q} \alpha_l s(t - \tau_l)e^{j\phi_l(t) - (i-1)\frac{d}{c}\sin\theta_l]} + n_i(t) \tag{3}$$

Transform (3) into the vector form and let $\gamma_l(t)$ be the product of α_l and time-varying phase term, then

$$x_i(t) = \sum_{l=1}^{q} \gamma_l(t)a(\theta_l)s(t - \tau_l) + n_i(t) \tag{4}$$

where $a(\bullet)$ is an array copy. In most space protocol, channel response is estimated by sending guidance or guidance sequences and the time of them must be short enough to guarantee θ_l continuous. If the transmitter is static then

$$x(t) = \sum_{l=1}^{q} \gamma_l(t)a(\theta_l)s(t - \tau_l) + n(t) \tag{5}$$

where $\gamma_l(t) = \alpha_l e^{-j\omega_c \tau_l}$ is continuous value and all scattering is static.

In (5) the covariance matrix of $x(t)$'s sampling is

$$R = E[x(t)x^*(t)] = APA^* + \sigma^2 I \tag{6}$$

Where $A = [a(\theta_1), a(\theta_2), \cdots, a(\theta_q)]$,

$$P = \begin{bmatrix} \|\gamma_1\|^2 E[\|s(t-\tau_1)\|^2] & \cdots & \gamma_1\gamma_q^* E[s(t-\tau_1)s^*(t-\tau_q)] \\ \cdots & \cdots & \cdots \\ \gamma_q\gamma_1^* E[s(t-\tau_q)s^*(t-\tau_1)] & \cdots & \|\gamma_q\|^2 E[\|s(t-\tau_q)\|^2] \end{bmatrix}. \tag{7}$$

If $\tau_1 = \tau_2 = \cdots = \tau_q$, then

$$P = E[\|s(t-\tau_1)\|^2] \begin{bmatrix} \|\gamma_1\|^2 & \cdots & \gamma_1\gamma_q^* \\ \cdots & \cdots & \cdots \\ \gamma_q\gamma_1^* & \cdots & \|\gamma_q\|^2 \end{bmatrix}$$

Now we turn to the DOA $\phi_l(t)$. If the transmitter is moving, the first term of $\phi_l(t)$ should attribute to Doppler Effect and the second term be time delay of propagation. In the Rayleigh fading channel, let $\phi_l(t)$, $l = 1, 2, \ldots, q$ be the uniform random variable in $(-\pi, \pi]$ time interval. Based on this assumption, the transmitter must be moving to make sure the $\phi_l(t)$ is changing with time. If $\phi_l(t)$ is random variable, P will be as follows:

$$P = \begin{bmatrix} E[\|\gamma_1(t)\|^2 \|s(t-\tau_1)\|^2] & \cdots & E[\gamma_1(t)\gamma_q^*(t)s(t-\tau_1)s^*(t-\tau_q)] \\ \cdots & \cdots & \cdots \\ E[\gamma_q(t)\gamma_1^*(t)s(t-\tau_q)s^*(t-\tau_1)] & \cdots & E[\|\gamma_q\|^2 \|s(t-\tau_q)\|^2] \end{bmatrix} \tag{8}$$

If $\gamma_1(t)$, $\gamma_m(t)$ and $s(t-\tau_1)s(t-\tau_m)$ is uncorrelated and $\phi(t)$ is uniform random variable in $(-\pi, \pi]$ adding to statistical independence between $\phi(t)$ and α_l , $E[\gamma_1(t)] = 0$ will transform into diagonal matrix:

$$P = E[\|s(t)\|^2] \begin{bmatrix} \|\alpha_1\|^2 & \cdots & \gamma_1\gamma_q^* \\ \cdots & \cdots & \cdots \\ \gamma_q\gamma_1^* & \cdots & \|\alpha_q\|^2 \end{bmatrix} \tag{9}$$

Based on the above theory we can obtain a DOA estimation algorithm. Through the diagonal matrix, conventional MUSIC algorithm can achieve the same performance.

Transient smoothing method applies the MUSIC algorithm to sample data. Be sure that the sampling interval is longer enough, so the fading channel phase term $\gamma_1(t)$ is independent stochastic process.

Due to P in (7) is diagonal matrix, so the covariance matrix R can be decompose into signal subspace and noise subspace

$$\lambda_1 = \begin{cases} p_{ii} + \sigma^2, i = 1, \cdots, q \\ \sigma^2, i = q+1, \cdots, M \end{cases} \tag{10}$$

The MUSIC algorithm needs at least one noise eigen-vector to project the copy of array into noise subspace. Based on the above theory we can obtain DOAs of m source signals through m+1 antenna. However, space smoothing algorithm needs 2m antennas and front-rear space smoothing 3m/2.

3. Time-varying phase and transient smoothing algorithm

To define the condition of time-varying phase term $\phi_l(t)$ and estimate the DOAs of coherent signals, we analyze the non-diagonal terms in (7), there will be $p_{ij} = E[\alpha_i \alpha_j^* e^{j[\phi_i(t) - \phi_j(t)]}]$. If the non-coherent term is a uniform random variable in $(-\pi, \pi]$, p_{ij} should be 0.

Assume that a two-path model consists of direct travel path and feedback path and the two paths form an equilateral triangle. The transmitter moves along the path that is vertical to the receiving antennas.

The phase term (DOA) $\phi_l(t)$ will be

$$\phi_2(t) - \phi_1(t) = \omega_d(\cos\psi_2 - \cos\psi_1)t - \omega_c(\tau_2(t) - \tau_1(t)) \tag{11}$$

Combining the above assumption, we can obtain:

$$\omega_c(\tau_2(t) - \tau_1(t)) = \omega_c\left(\frac{2r(t)}{c} - \frac{d(t)}{c}\right) \tag{12}$$

As $\cos\psi_1 = -1$, to derivate the two sides of (11),

$$\phi_2(t) - \phi_1(t) = \omega_d\left(\frac{v}{2r(t)} - \frac{vd^2(t)}{8r^3(t)}\right)t - \omega_d\left(\frac{d(t)}{r(t)} - 2\right) \tag{13}$$

where v denotes the moving speed.

When $t = 0$,

$$\phi_2(t) - \phi_1(t) = \omega_d\left(2 - \frac{d(t)}{r(t)}\right) \tag{14}$$

Equation (14) shows the ratio of phase to time. If the reflector is close to direct path, i.e. the feedback path is close to direct path, the ratio will be small. In contrast, if the reflector is far to direct path, the ratio will be big and achieve its maximum value $2\omega_d$. This will lead to p_{12} in p_{ij} close to 0 and the estimation of DOA is easier and more precise.

4. Simulation result

In this section, we present the simulation results of the proposed theory, comparing with the spatial smoothing MUSIC algorithm. Assume that the direction of sending signals is vertical to moving direction of transmitter and the receiver consists of uniform antennas. The distance of transmitter and receiver is 1.5 km. The DOA of signals which follow the feedback path is $30°$ and $38°$. The SNR in receiver is 18dB. The sampling rate of spatial smoothing algorithm is 130 and transient smoothing is 100.

Simulation 1：In this simulation, the transmitter is static. Fig.1 shows the simulation result. At first, we assume that the array consists of 10 antennas and the number of subspace is 4. (a) gives the spatial spectrums of signals in three different direct paths. The algorithm is spatial smoothing MUSIC. It is obvious that the DOAs of signals can be estimated accurately.

In the second case, the array consists of 6 antennas and the number of subspace is 4. (b) shows the spatial spectrums of signals in three different paths when the algorithm is transient smoothing MUSIC. The DOAs of signals can not be estimated accurately. If we add the number of antennas to 10, the performance of the algorithm is also bad. So we can conclude that spatial smoothing MUSIC performs better than transient smoothing MUSIC when the transmitter is static.
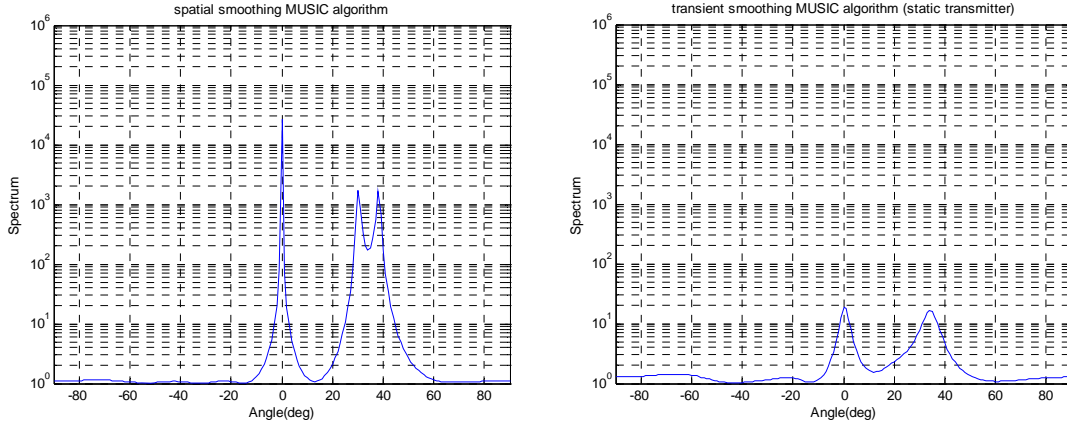
Fig.1. (a) Estimation of DOA for spatial smoothing MUSIC when transmitter is static; (b) Estimation of DOA for transient smoothing MUSIC when transmitter is static.

Simulation 2: In this simulation, the array consists of 6 antennas and the number of subspace is 4. Fig.2 shows the spatial spectrums of three different signals in the condition of direct path for transient smoothing MUSIC. (a) shows the result when transmitter is moving at a speed of 0.5km/h. (b) shows the spatial spectrums when the speed of transmitter is 10km/h.
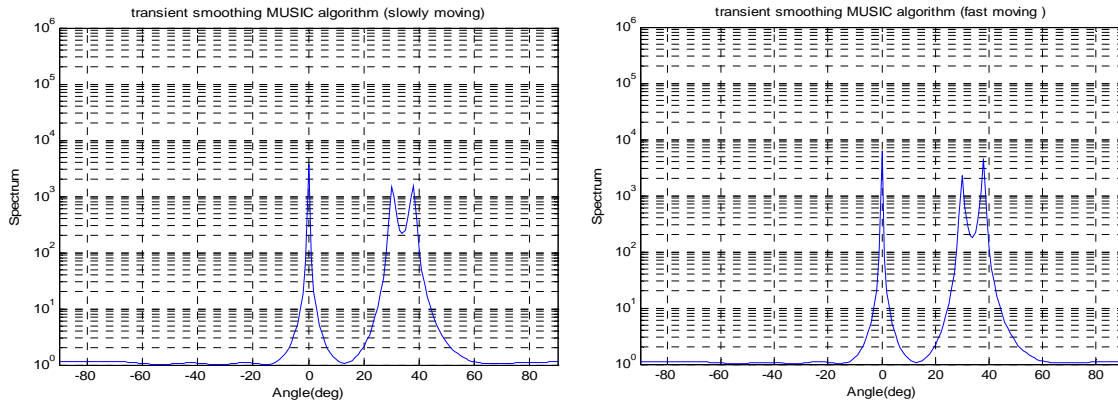

Fig.2. (a) Estimation of DOA for transient smoothing MUSIC when the speed of transmitter is 0.5km/h; (b) Estimation of DOA for transient smoothing MUSIC when the speed of transmitter is 10km/h.

Obviously, even the signals are moving slowly the DOAs can be estimated accurately using transient smoothing MUSIC. When the transmitter moves at high speed, the spectrum peaks needed to tell are more and the estimation should be more precise.

5. Conclusion

In this paper, we propose the transient smoothing MUSIC algorithm and apply it on estimating algorithm of DOA in the presence of multi-path channel. By analyzing and deducing seriously, we transplant the spatial smoothing algorithm into the eigen-space and obtain the transient smoothing algorithm to estimate the DOA in the condition of coherent sources. Through extensive computer simulations, the relation between ratio of phase and time and the position of transmitter has been demonstrated. Besides, we have compared the performance of spatial and transient smoothing algorithm in the different assumptions.

6. References

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