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The Comparative Study of the Convergence Method Based on Unstructured Grid System

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Abstract. This article discusses the application of different implicit methods on unstructured grid computing through the data structure of unstructured grid. In this article, we investigated three methods which are Guass－Seidel，LU－SGS（Lower-Upper Symmetric Gauss-Seidel）and GMRES (Generalized Minimum Residual) with pre-processing and two kinds of process technologies, including all the none-zero element Jacobi matrix and approximate treatment Jacobi matrix multiplication. Our overriding concern is the computation efficiency and memory overheads of different methods in research.

Keywords: unstructured grid; implicit method; Multi-grid; Convergence

1. Introduction

The digital computation method and dependent program have been developed tremendously due to the remarkable ability of unstructured grid to describe complex configuration, and applied extensively as well in the field of flight vehicle design, industry application as far as automobile design. Of recent years, some famous business calculation program (such as Star-CD etc.) regards the unstructured grid as the new developing direction.

A set of rather impeccable method was developed in allusion to the structure grid during the process of hydromechanical calculation. Nonetheless, in the calculating of unstructured grid, algorithm (such as high accuracy format and implicit method etc.) which is applied widely in structure grid cannot be expanded directly in the unstructured grid calculation owing to the randomness of unstructured grid data organization. It is necessary to be organized by establishing high efficient data structure.

Developing high efficient convergence method has great superiority for utilizing the unstructured grid preferably, and more meaningful in application.

2. Numerical Method and Data Structure

2.1. Numerical method

The equation of two dimension conservation mode is written as follows, based on the Descartes coordinate system (each item will not be repeated any more).

$$\frac{\partial Q}{\partial t}+\frac{\partial E_i}{\partial x}+\frac{\partial F_i}{\partial y}=\frac{1}{\mathrm{Re}}\left(\frac{\partial E_v}{\partial x}+\frac{\partial F_v}{\partial y}\right) \tag{1}$$

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where, Q is conservative variables, E_i is invisid flux vector in x direction, F_i is invisid flux vector in y direction, E_v is viscous flux vector in x direction, F_v is viscous flux vector in y direction, R_e is Renaulds number.

Adopting median-dual control volume of C-V(Control Volume) type, variable is saved at the node. The method of approximate Riemann of Roe's is to calculate the discrete of non-viscosity flux[1]. The accuracy will be at second order by using the interpolation, meanwhile, fluctuation be limited by limiter[2]. The numerical value $[\overrightarrow{H_{inv}}]_j$ of non-viscosity flux for the interface j can be described as follows:

$$[\overrightarrow{H_{inv}}] = \frac{1}{2}[(f^R + f^L) - \left|\bar{A}\right|\Delta Q] \cdot \left|d\vec{S}\right| \tag{2}$$

The calculating of viscosity flux is also on the surface of median-dual, adopting the method which is similar to finite element method (FEM).

Time item, adopting implicit discrete:

$$\frac{Q_i^{n+1} - Q_i^n}{\Delta t} V_i = -R_i^{n+1} \tag{3}$$

Linearizing the right item, we can obtain：

$$R_i^{n+1} = R_i^n + \frac{\partial R_i^n}{\partial Q}\Delta Q \tag{4}$$

$\Delta Q = Q^{n+1} - Q^n$, $\frac{\partial R_i^n}{\partial Q}$ is the Jacobi matrix produced in the linearize process. In order to reduce the connecting node in the calculating, this article will adopt the upwind discrete of the first order for non-viscosity flux in $\frac{\partial R_i^n}{\partial Q}$, thus, the solving procedure will be simplified.

$$A^+ = R\Lambda^+ R^{-1} \quad , \quad A^- = R\Lambda^- R^{-1} \tag{5}$$

R is the right characteristic vector matrix in Jacobi matrix of convection item $A_i = \frac{\partial f_i}{\partial Q}$. Λ^+ is the diagonal matrix which only includes positive characteristic value. On the contrary, Λ^- is the diagonal matrix which only includes negative characteristic value, in addition, $\Lambda = \Lambda^+ + \Lambda^-$ and the maximum characteristic value is for division.

the format of the complete discrete is described as follows:

$$\left[\frac{V_i}{\Delta t} I + \sum_{f=1}^{nf} A^+ \cdot \overrightarrow{dS_j}\right]\Delta Q_i + \sum_{f=1}^{nf} A^- \cdot \overrightarrow{dS_j}\Delta Q_k = -R_i^n \tag{6}$$

or

$$A \cdot \Delta Q = R \tag{7}$$

2.2. Data structure[3]

For a solution domain the overall points of which is N, A is block matrix of N×N (subblock is : equation number × equation number, such as : two dimensions is 4×4, three dimensions is 5×5). Therefore, the storage of big sparseness Jacobi matrix A need to use the special data structure. The edge-base data structure [3,4] (regard the Nedge as the total amount of the edge)is adopted in this article in order to make the practical storage as : opposite angle of numbers of N ＋2×Nedge block, and the total amount is about 7N block for two dimensions, furthermore, adding the established data index for up and down triangular matrix respectively, the implicit calculation can be provided with all information.

2.3. Convergence iteration method [3,4,5,6,7]

For the linear system, seen equation (7), this article discusses the solving procedure through iteration method: especially for methods of Guass-Seidel、LU-SGS[6] and GMRES with pre-processing.

During the procedure of calculating, there is a great need for memory if the Jacobi matrix A_i is calculated and stored in every step, especially for three dimensions problem. In order to reduce the memory need, and the same time, considering that the A_i is always appeared as the model of arithmetic product in solving procedure, so, $A_i \cdot \Delta Q$ can utilize the simplified calculating [3,4,6] in solving procedure:

$$A_i \cdot \Delta Q \approx \frac{f(Q_i + \varepsilon \Delta Q) - f(Q_i)}{\varepsilon}, \quad \varepsilon \text{ is decimals} \tag{8}$$

or, $A_i \cdot \Delta Q \approx A(Q_i) \cdot (Q_i + \Delta Q - Q_i)$, the Roe format is for approximate calculating. Through this approximate treatment, it is only need to storage the cater-cornered Jacobi matrix finally, no need for the up and down triangle Jacobi matrix. Actually, the memory of the implicit calculating is only more than explicit calculating for N pieces of opposite angles block ＋ the data index of the up and down triangle, at last, the memory overheads is reduced greatly.

Combing the multiple grid technology and implicit method can generate the giant convergence effect.

3. Numerical Result and Conclusion

The numerical calculation is performed in allusion to the non-viscosity and streaming flux of the two dimensions cylinder in this paper, the calculating conditions are: Ma(Mach Number)＝0.4,CFL(Courant-Friedrichs-Lewy)＝100, so as to the four methods of displaying, Jacobi, LU-SGS and the GMRES (inner iteration 10 times) with pre-processing. The grid and curve of calculating convergence is showed in diagram 1,2.

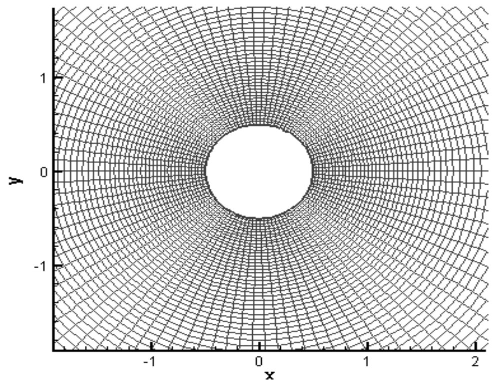


Fig. 1. Schematic of the grid (9800 points, 9702 quadrilateral unit)

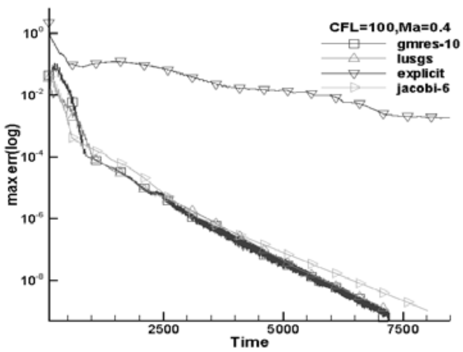


Fig. 2. Convergence curve

The Euler equation is for calculating steaming flux of the cylinder. When the mach number is 0.4, there is no shock wave in the flow field which should be bilateral symmetry. The isogram of pressure and density is showed in diagram 3,4.

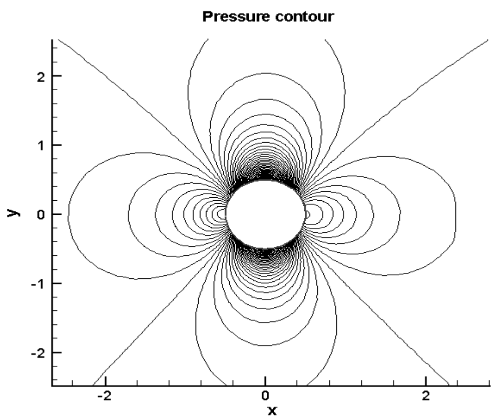


Fig. 3. Isoline of pressure

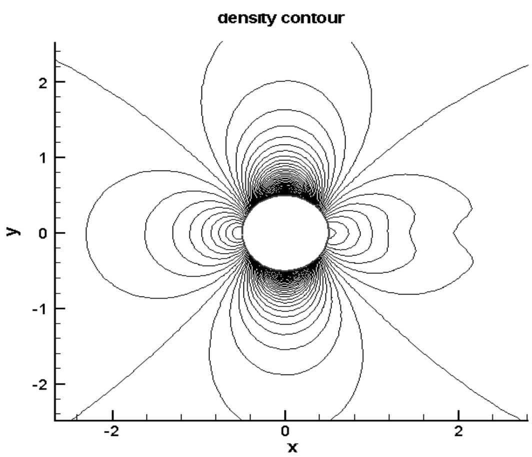
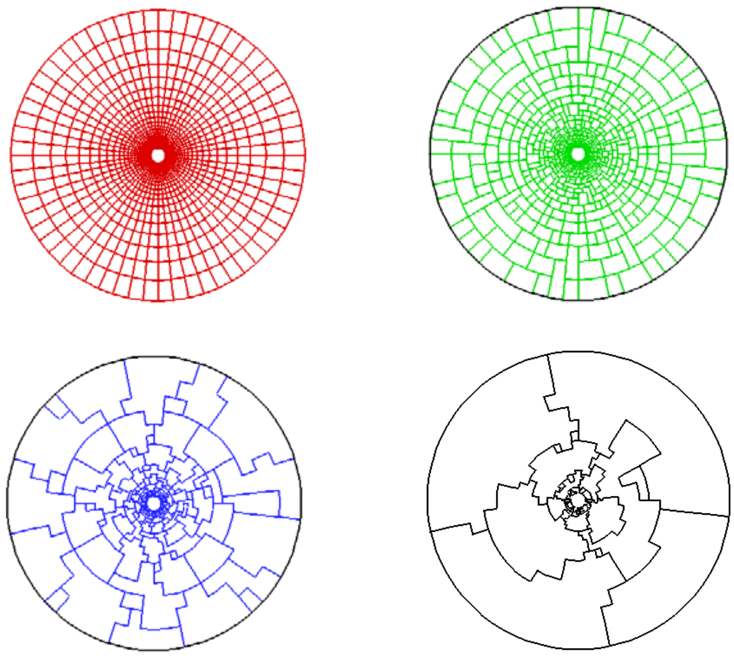


Fig. 4. Isoline of density

During calculating, the implicit method has great advantage in convergence comparing with the explicit method. The convergence process of the three implicit method differs rarely. The memory of the LU-SGS and the CPU(Central Processing Unit) time of every step is the least; the Jacobi method is relative to the inner iteration, the calculating overhead is the same magnitude to the LU-SGS; the GMERS method is relative to the number of the inner iterations and convergence condition of the inner iteration, the calculating overhead of which is greater than other method when the inner iteration number is big and the higher demand for the inner iteration convergence. When taking the limit large CFL, the convergence is not sensible to the method, for the information transmission in flow field is relative to the $U \cdot \Delta t$(U is characteristic speed), while the Δt is relative to CFL. Meanwhile, due to using the division of the maximum characteristic value, the information of the Jacobi matrix is broken, so, the convergence condition is influenced correspondingly.

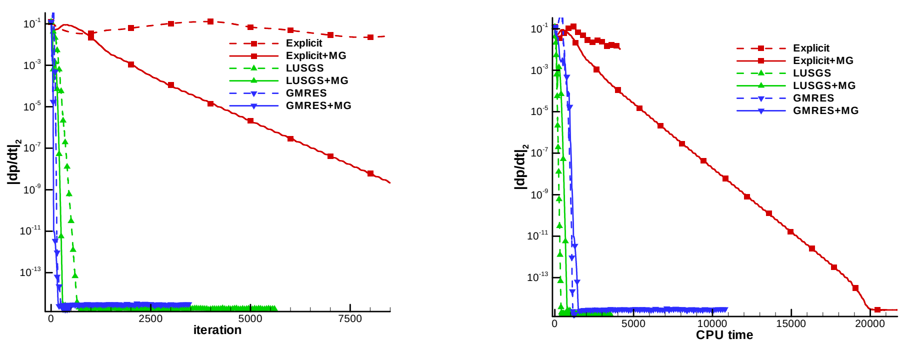
According the above result, the implicit method possessed higher efficiency than the explicit method, but in the limited CFL, the efficiency among the implicit method differs little. It is very necessary to apply the multi-Grid method if you want the great convergence effect this moment.

The viscosity low calculating result of the incompressible cylinder of Re=40 is showed in diagram 5,6, adopting the virtual compressing method combined with the different implicit method and the multi-grid technology. It is obvious that the convergence step of the GMRES method is the least, whereas the cost of the each step is the largest, therefore, a very well convergence method should achieve equilibrium between them.



(a) The thinnest grid（7200 cells）; (b) The first gathering (974 cells); (c) The second gathering ; (d)The third gathering

Fig. 5. Schematic of gathering effect of multiple grid



(a) convergence curve of pressure residul error along with iterative steps ; (b) CPU time curve of convergence procedur

Fig. 6. Schematic of gathering effect of multiple grid

4. References

[1] P.L.Roe. Approximate Riemann Solvers,Parameter Vectors, and Difference Schemes, J.C.P v43:357-372, 1981

[2] T.J.Barth. Recent Developments in High Order K-Exact Reconstruction on Unstructured Meshes, AIAA-93-0668

[3] V.Venkatakrishnan. Implicit Solvers for Unstructured Meshes, J.C.P v105:83-91, 1993

[4] T.J.Barth. An Unstructured Mesh Newton Solver for Compressible Fluid Flow and Its Parallel Imlementation, AIAA-95-0221

[5] Alberto Valli,etc. Numerical Approximation of Partial Differential Equations, Springer-Verlag

[6] Luo H etc. A fast, matrix-free implicit method for compressible flows on unstructured grid, J.C.P. v146:664-690,1998

[7] Dayong Cai, Fengshan Bai. Advanced numerical analysis, Tsinghua University.