

Vectorization of Image Outlines Using Rational Spline and Genetic Algorithm

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Abstract. A rational spline approach has been introduced for the outline capture of the planar images. The idea of genetic algorithm(GA) has been incorporated to optimize the shape parameters in the description of the rational spline. The proposed approach has various phases including detecting digital outlines (contours), finding corner points on the digital outlines, and fitting the spline using GA. The proposed method ultimately produces optimal results for the approximate vectorization of the digital contours obtained from the generic shapes. Demonstrations and illustration of the results also make the essential part of the paper.

Keywords: vectorization, corner points, generic shapes, curve fitting, spline.

1. Introduction

Capturing and vectorizing outlines of images is one of the important problems of computer graphics, vision, and imaging. Various mathematical and computational phases are involved in the whole process. This is usually done by computing a curve close to the data point set. Computationally economical and optimally good solution is an ultimate objective to achieve the vectorized outlines of images for planar objects.

Curve modelling [1] is one of the important phases of capturing and vectorizing outlines of images. It plays a significant role in various applications. The representation of planar objects, in terms of curves, has many advantages. For example, scaling, shearing, translation, rotation and clipping operations can be performed without any difficulty. Although a good amount of work has been done in the area [3-6], it is still desired to proceed further to explore more advanced and interactive strategies. Most of the up-to-date research has tackled this kind of problem by curve subdivision or curve segmentation.

This work is a presentation of an approach using rational cubic spline interpolation. It is inspired by an optimization algorithm based on genetic algorithm (GA). In this paper, the data point set represents any generic shape whose outline is required to be captured. We present an iterative process to achieve our objectives. The algorithm comprises of various phases to achieve the target. First of all, it finds the contour [5] of the gray scaled bitmap images. Secondly, it uses the idea of corner points [2] to detect corners. That is, it detects the corner points on the digital contour of the generic shape under consideration. These phases are considered as preprocessing steps. Rational cubic spline interpolant is then used to vectorize the outline. The idea of genetic algorithm(GA) is used to fit a rational cubic spline which passes through the corner points. It globally optimizes the shape parameters in the description of the rational cubic spline to provide a good approximation to the digital curve. In case of poor approximation, the insertions of intermediate points are made as long as the desired approximation or fit is achieved.

The organization of the paper is as follows, Section 2 discusses about preprocessing steps which include finding the boundary of planar objects and detection of corner points. Section 3 is about the interpolant form

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of rational cubic spline curves. Overall methodology of curve fitting, using genetic algorithm, is explained in Section 4, it includes the idea of knot insertion as well as the algorithm design for the proposed vectorization scheme. Demonstration of the scheme is presented in Section 5. Finally, the paper is concluded in Section 6.

2. Preprocessing

The proposed schemes start with finding the boundary of the generic shape and then using the output to find the corner points. The image of the generic shapes can be acquired either by scanning or by some other mean. The aim of boundary detection is to produce an object's shape in graphical or non-scalar representation. Chain codes [7], in this paper, have been used for this purpose. Demonstration of the method can be seen in Figure 1(b) which is the contour of the bitmap image shown in Figure 1(a).

Corners in digital images give important clues for the shape representation and analysis. These are the points that partition the boundary into various segments. The strategy of getting these points is based on the method proposed in [1]. The demonstration of the algorithm is made on Figure 1(b). The corner points of the image are shown in Figure 1(c).

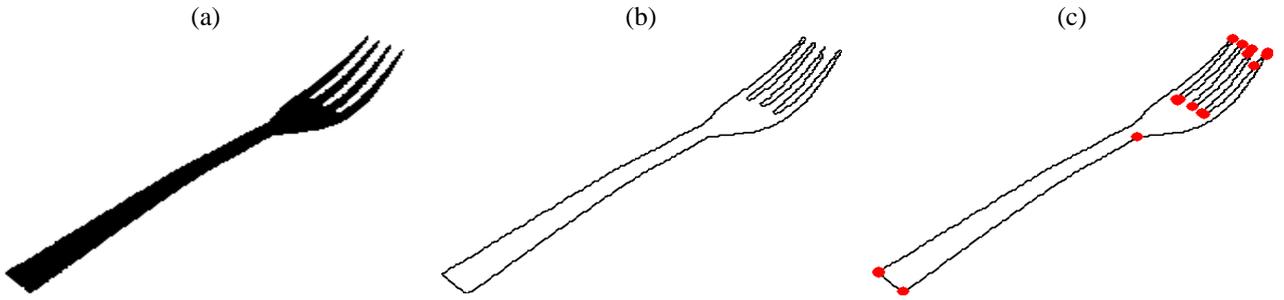


Fig. 1: Pre-processing Steps: (a) Original Image, (b) Outline of the image, (c) Corner points achieved, (d) Fitted Outline of the image.

3. Curve Fitting and Spline

The motive of finding the corner points, in Section 2, was to divide the contours into pieces. Each piece contains the data points in between two subsequent corners inclusive. This means that if there are m corner points cp_1, cp_2, \dots, cp_m then there will be m pieces pi_1, pi_2, \dots, pi_m . We treat each piece separately and fit the spline to it. In general, the i^{th} piece contains all the data points between cp_i and cp_{i+1} inclusive. After breaking the contour of the image into different pieces, we fit the spline curve to each piece. To construct the parametric spline interpolant on the interval $[t_0, t_n]$, we have $F_i \in R^m$, $i = 0, 1, \dots, n$, as interpolation data, at knots t_i , $i = 0, 1, \dots, n$.

The curve fitted by a rational cubic spline is a candidate of best fit, but it may not be a desired fit. This leads to the need of introducing some shape parameters in the description of the rational cubic spline. This section deals with a form of rational cubic spline. It introduces shape parameters v 's in the description of rational cubic spline defined as follows:

$$P(t) = \frac{P_i(1-\theta)^3 + v_i V_i \theta(1-\theta)^2 + v_i W_i \theta^2(1-\theta) + P_{i+1} \theta^3}{(1-\theta)^3 + v_i \theta(1-\theta)^2 + v_i \theta^2(1-\theta) + \theta^3}, \quad (1)$$

where

$$V_i = P_i + \frac{h_i D_i}{v_i}, \quad W_i = P_{i+1} - \frac{h_i D_{i+1}}{v_i}.$$

D_i and D_{i+1} are the corresponding tangents at corner points P_i and P_{i+1} of the i^{th} piece. For open curves, the tangent vectors are calculated as follows:

$$\left. \begin{aligned} D_0 &= 2(P_1 - P_0) - \frac{(P_2 - P_0)}{2} \\ D_i &= a_i(P_i - P_{i-1}) + (1 - a_i)(P_{i+1} - P_i) \\ D_n &= 2(P_n - P_{n-1}) - \frac{(P_n - P_{n-2})}{2} \end{aligned} \right\}, \text{where } a_i = \frac{\|P_{i+1} - P_i\|}{\|P_{i+1} - P_i\| + \|P_i - P_{i-1}\|}.$$

For closed curves, the conditions are as follows:

$$\left. \begin{aligned} F_{-1} &= F_{n-1}, \quad F_{n+1} = F_1 \\ D_i &= a_i(P_i - P_{i-1}) + (1 - a_i)(P_{i+1} - P_i), \quad i = 0, 1, \dots, n \end{aligned} \right\}$$

Obviously, the parameters v_i 's, when equal to 3, provide the special case of cubic Hermite interpolation. If v_i 's are too large, then the rational cubic function (1) converges to the linear interpolant. This paper proposes an evolutionary technique, namely genetic algorithm (GA), to optimize these parameters so that the curve fitted is optimal.

4. Proposed Approach

In this Section, the proposed scheme to the curve fitting problem is described. It includes the phases of problem matching with Genetic Algorithm using rational cubic spline function, description of parameters used for GA and curve fitting.

4.1. Problem mapping

Since, the objective of the paper is to come up with optimal technique which can provide decent curve fit to the digital data. Therefore, the interest would be to compute the curve in such a way that the sum square error of the computed curve with the actual curve (digitized contour) is minimized. Mathematically, the sum squared distance is given by:

$$S_i = \sum_{j=1}^{m_i} [P_i(t_{i,j}) - P_{i,j}]^2, \quad t_{i,j} \in [t_i, t_{i+1}], \quad i = 0, 1, \dots, n-1,$$

where $P_{i,j} = (x_{i,j}, y_{i,j})$, $j = 1, 2, \dots, m_i$, are the data points of the i th segment on the digitized contour. The parameterization over t 's is in accordance with the chord length parameterization. Thus the curve fitted in this way will be a candidate of best fit.

The Genetic Algorithm formulation of the problem discussed in this paper is described in detail. For the best fitting of the curve to given data, the values of parameters v_i 's are required so that the sums S_i 's are minimal. Genetic Algorithm is used to optimize these values for the fitted curve. We start with initial population of values of v_i 's chosen randomly. Successive applications of search operations to this population leads to optimal values of v_i 's.

4.2. Initialization

Once we have the bitmap image shown in Figure 1(a) and Figure 3(a), the boundary of the image can be extracted (see Figure 1(b) and Figure 3(b)) using the method described in Section 2. After the boundary points of the image are found, the next step is to detect corner points as mentioned in Section 2. The corner detection technique assigns a measure of 'corner strength' to each of the points on the boundary of the image. This step helps to divide the boundary of the image into n segments. Each of these segments is then approximated by interpolating spline described in Section 3. The initial solution of spline parameters v 's is randomly selected. Figure 1(c) and Figure 3(c) show boundary of the bitmap images together with detected corner points. Table 1 gives number of contour points and initial corner points of the images.

4.3. Algorithm for rational cubic spline using GA

The overall scheme can be explained in the form of an algorithm. The summary of the algorithm, designed for optimal curve design using rational cubic interpolant, is as follows:

Step AG2.1: Input the image.

Step AG2.2: Extract the contours from the image in Step AG2.1.

- Step AG2.3:** Compute the corner points from the contour points in Step AG2.2 using the method in Section 2.
- Step AG2.4:** Compute the derivative values at the corner / intermediate points.
- Step AG2.5:** Compute the best optimal values of the shape parameters v_i 's using GA.
- Step AG2.6:** Fit the spline curve method, of Section 3, to the corner / intermediate points achieved in Step AG2.2.
- Step AG2.7:** If the curve, achieved in Step AG2.6, is optimal then GO TO Step AG2.10, ELSE locate the appropriate intermediate points (points with highest deviation) in the undesired curve pieces.
- Step AG2.8:** Enhance and order the list of the corner / intermediate points achieved in Step AG2.3 and AG2.7.
- Step AG2.9:** GO TO Step AG2.4.
- Step AG2.10:** STOP.

5. Demonstrations

The proposed curve scheme has been implemented successfully in this section. We evaluate the performance of the system by fitting parametric curves to different binary images.

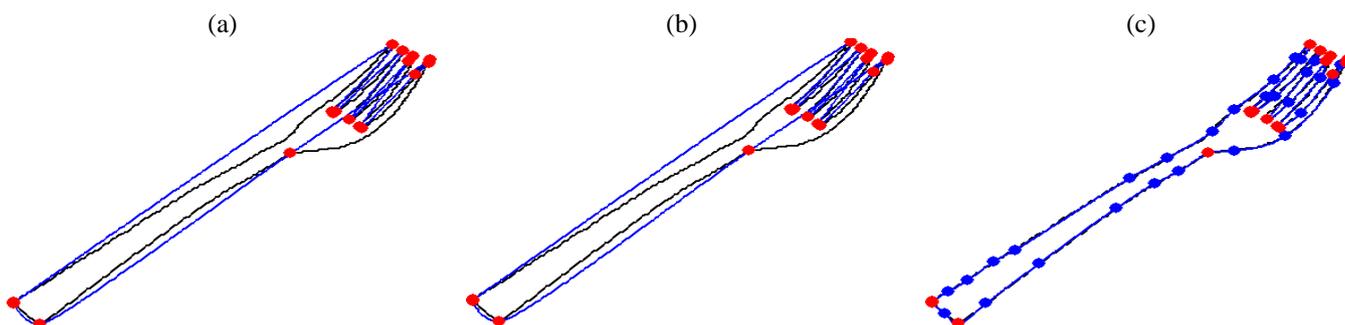


Fig. 2: Results of the curve fitting: (a) Cubic Hermite fitted to corners of the outline of the image, (b) Fitted rational cubic for the 1st iteration of GA, (c) Fitted rational cubic for the 5th iteration of GA with intermediate points.

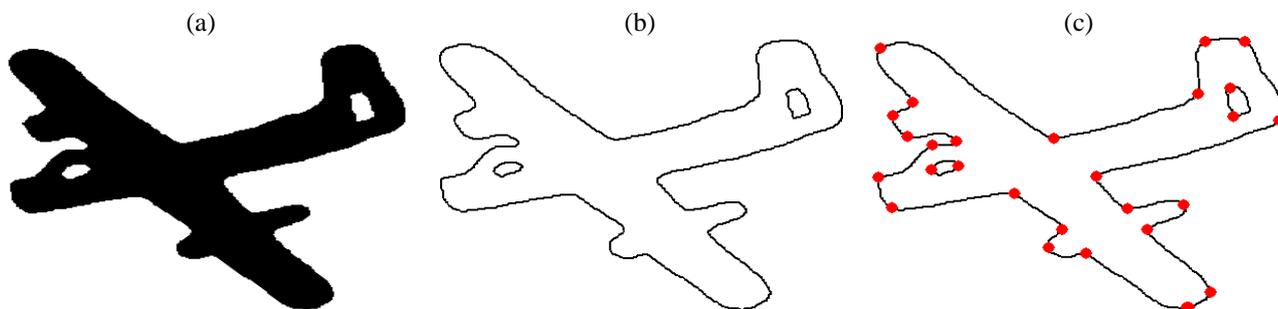


Fig. 3: Pre-processing Steps: (a) Original Image, (b) Outline of the image, (c) Corner points achieved.

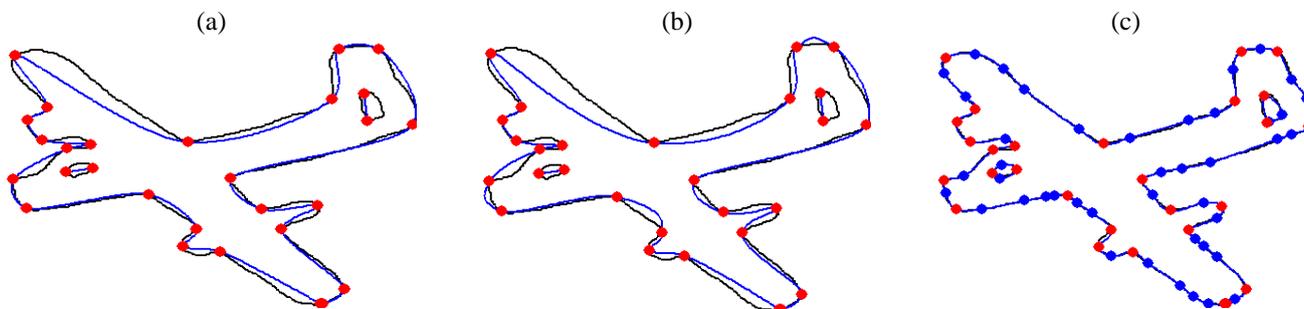


Fig. 4: Results of the curve fitting: (a) Cubic Hermite fitted to corners of the outline of the image, (b) Fitted rational cubic for the 1st iteration of GA, (c) Fitted rational cubic for the 5th iteration of GA with intermediate points.

Figure 2 shows the implementation results of the algorithm for the image “Fork” in Figure 1(a). Figures 2(a), 2(b) and 2(c) are the results for the scheme, respectively, for the cubic Hermite spline (default curve),

1st iteration of the GA, and the 5th iteration of the GA respectively. One can see the insertion of intermediate points in Figure 2(c).

Table. 1: Names and contour details of images

Image	Name	# of Contours	# of Contour Points	# of Initial Corner Points
	Fork	1	673	15
	Plane	3	915+36+54	28

Figures 3 and 4 show the implementation results of a “Plane” image. Figures 3(a), 3(b), 3(c) are respectively the original image of the Plane, its outline, outline together with the corner points detected. Figure 4 shows the implementation results of the algorithm for the “Plane” image in Figures4(a-c). Figures 4(a), 4(b) and 4(c) are the results for the scheme, respectively, for the cubic Hermite spline (default curve), 1st iteration of the GA, and the 5th iteration of the GA respectively. One can see the insertion of intermediate points in Figure 4(c).

6. Concluding Remarks

An optimization technique is proposed for the outline capture of planar images. It uses the GA to optimize a rational cubic spline to the digital outline of planar images. The idea of GA has been used to optimize the shape parameters in the description of a rational cubic spline introduced. The experimental study shows that the method ultimately produces optimal results for the approximate vectorization of the digital contours obtained from the generic shapes. The scheme provides an optimal fit with an efficient computation cost as far as curve fitting is concerned. The proposed algorithm is fully automatic and requires no human intervention.

7. References

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