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Single and Multiple Estimation in MIMO Rician Fading Channels

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Abstract. In this paper, the performance of the Single-Estimation (SE) and Multiple-Estimation (ME) is investigated in Multiple-Input Multiple-Output (MIMO) Rician fading channels using the Maximum Likelihood (ML) technique and the new Shifted Scaled Least Squares (SSLS) estimator. Analytical and numerical results show that both estimators have lower error in the case of ME than SE. Moreover, it is seen that increasing the channel Rice factor improves the performance of the SSLS estimator. However, the ML estimator cannot exploit the knowledge of the Rician fading channel. As a result, in fast fading MIMO channels with a short coherence time, the SSLS estimator can be used in SE mode. On the other hand, the ML estimator has only good results in the case of ME. This estimator is appropriate for slow fading MIMO channels with a long coherent time.

Keywords: Rician fading channel, Single Estimation (SE), Multiple Estimation (ME), MIMO, ML, SSLS.

1. Introduction

Training-Based Channel Estimation (TBCE) scheme is the most usual approach to channel identification [1-6]. This method is attractive because it decouples data detection from channel estimation at the receiver and hence it reduces complexity. TBCE methods can be optimal at high Signal-to-Noise Ratios (SNRs) [1]. In [2], the Mean Square Error (MSE) of the Least Squares (LS), Scaled LS (SLS), Minimum Mean Square Error (MMSE), and Relaxed MMSE (RMMSE) estimators has been compared analytically and numerically. In [3], the MMSE estimator is proposed to estimate the Rician fading Multiple-Input Multiple-Output (MIMO) channels. An interesting result in this paper is that the optimal training sequence length can be considerably smaller than the number of transmitter antennas in systems with strong spatial correlation. For MIMO Rician flat fading channels, the new Shifted Scaled Least Squares (SSLS) channel estimator is presented in [4]. It is seen that this estimator has the best performance among the LS-based estimators in Rician channel model.

In [5], the performances of the Time-Multiplexed (TM) and superimposed (SI) schemes have been compared in MIMO channel estimation. It is shown that in fast fading channels and/or for many receiver antennas, the SI scheme is better than TM but in other cases this scheme suffers from a higher estimation error. In part II of this paper [6], to improve the performance of the SI scheme a decision directed approach is applied.

In this paper, TBCE method is studied in the flat Rician fading MIMO channels. We investigate the Single-Estimation (SE) and Multiple-Estimation (ME) using the Maximum Likelihood (ML) technique and the new SSLS estimator. Analytical and numerical results show that the SSLS estimator is appropriate for Rician fading channels with a short coherence time (fast fading). For Rician fading channels with a long coherence time (slow fading), however, the ML estimator is better than SSLS. In practice, by considering the channel fading, one of the ML or SSLS estimators can be used.

The rest of this paper is organized as follows. Section 2 introduces the system model. The SE and ME methods in the Rician fading MIMO channels are investigated in Sections 3 and 4, respectively. Simulation results are presented in Section 5. Finally, concluding remarks are presented in Section 6.

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Notation: $(\cdot)^H$ is reserved for Hermitian, $(\cdot)^*$ for the complex conjugate, $(\cdot)^{-1}$ for the matrix inverse, $(\cdot)^T$ for the matrix transpose, $\otimes$ for the Kronecker product, $tr\{\cdot\}$ for the trace of a matrix. $E\{\cdot\}$ is the mathematical expectation, $\mathbf{I}_m$ denotes the $m \times m$ identity matrix, $\|\cdot\|_F$ denotes the Frobenius norm. $\mathrm{vec}(\cdot)$ stacks all the columns of its matrix argument into one tall column vector.

2. The System Model

Let us consider a MIMO system with t transmitter and r receiver antennas. It is assumed the block fading model for flat MIMO channels. It means that the channel response is fixed within one block. Such a channel can change from one block to another one randomly. Each transmitted block has N sub-blocks which contain training and data symbols as shown in Fig. 1. The frame structure is the same for all Tx antennas. Training and data symbols are located in the first and end part of the sub-blocks, respectively. In practice, the channel is estimated using training symbols in the training phase. Then, the results are used for data detection. We consider the ideal case in which the antenna elements at both transmitter and receiver are sufficiently far apart so that the fading corresponding to different antenna elements is uncorrelated. To estimate the MIMO channel in each sub-block, it is required that $n_p \geq t$ training signals are transmitted by each transmitter antenna. The $t \times n_p$ complex received signal matrix can be expressed as

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{V} \tag{1}$$

where $\mathbf{X}$ and $\mathbf{V}$ are the complex t-vector of transmitted sequences on the t transmit antennas and r-vector of additive receiver noise, respectively. The elements of noise matrix are independently and identically distributed (i.i.d.) complex Gaussian random variables as $\mathcal{CN}(0, 1)$.

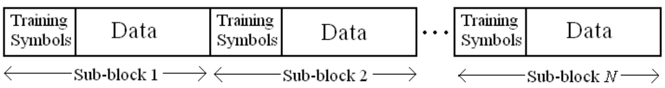

Fig. 1: Frame structure for each Tx antenna in a MIMO channel

In MIMO Rician fading channels with K as Rice factor, the $r \times t$ matrix of channel, $\mathbf{H}$, is defined in the following form:

$$\mathbf{H} = \sqrt{\frac{1}{\mathrm{K}+1}}\,\mathbf{H}_{\mathrm{Ray}} + \sqrt{\frac{\mathrm{K}}{\mathrm{K}+1}}\,\mathbf{H}_{\mathrm{LOS}} \tag{2}$$

The matrix $\mathbf{H}_{\mathrm{Ray}}$ explains the Rayleigh component of the channel and the matrix $\mathbf{H}_{\mathrm{LOS}}$ describes the channel mean value or the Line of Sight (LOS) component of the channel. The elements of the matrix $\mathbf{H}_{\mathrm{Ray}}$ are i.i.d. complex Gaussian random variables with the zero mean and the unit variance.

The MIMO channel model of (1) can be expressed in the following vector form:

$$\mathbf{y} = \tilde{\mathbf{X}}\mathbf{h} + \mathbf{v} \tag{3}$$

where $\mathbf{y} = \mathrm{vec}(\mathbf{Y})$, $\mathbf{v} = \mathrm{vec}(\mathbf{V})$, $\tilde{\mathbf{X}} = \mathbf{X}^T \otimes \mathbf{I}_r$ and $\mathbf{h} = \mathrm{vec}(\mathbf{H}) = \sqrt{1/(1+\mathrm{K})}\,\mathbf{h}_{\mathrm{Ray}} + \sqrt{\mathrm{K}/(1+\mathrm{K})}\,\mathbf{h}_{\mathrm{LOS}}$.
It is notable that $\mathbf{h}_{\mathrm{Ray}} = \mathrm{vec}(\mathbf{H}_{\mathrm{Ray}})$, $\mathbf{h}_{\mathrm{LOS}} = \mathrm{vec}(\mathbf{H}_{\mathrm{LOS}})$ and equation $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\,\mathrm{vec}(\mathbf{B})$ is applied.

It is straightforward to show that $\mathbf{h}$ has the following $rt \times rt$ correlation matrix:

$$\mathbf{R}_{\mathbf{h}} = E\{\mathbf{h}\mathbf{h}^H\} = \frac{1}{1+\mathrm{K}}\begin{bmatrix} 1+\mathrm{K} & \mathrm{K} & \mathrm{K} & \cdots & \mathrm{K} \\ \mathrm{K} & 1+\mathrm{K} & \mathrm{K} & \cdots & \mathrm{K} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \mathrm{K} & \mathrm{K} & \mathrm{K} & \cdots & 1+\mathrm{K} \end{bmatrix} \tag{4}$$

Using equations $\mathbf{h} = \mathrm{vec}(\mathbf{H}) = \sqrt{1/(1+\mathrm{K})}\,\mathbf{h}_{\mathrm{Ray}} + \sqrt{\mathrm{K}/(1+\mathrm{K})}\,\mathbf{h}_{\mathrm{LOS}}$ and (4), the $rt \times rt$ co-variance matrix of the Rician fading MIMO channel can be written as $\mathbf{C}_{\mathbf{h}} = \mathbf{R}_{\mathbf{h}} - E\{\mathbf{h}\}E\{\mathbf{h}\}^H = (1/(1+\mathrm{K}))\,\mathbf{I}_{rt}$.

3. Single Channel Estimation

In this section, it is supposed that the number of sub-blocks used for channel estimation is 1. First, the ML channel estimator is probed. Then, the performance of the SSLS channel estimator, appropriate for Rician fading MIMO channel, is investigated.

3.1. ML Channel Estimator

In classical estimation, the channel is assumed to be unknown deterministic. For linear model of (3), the ML estimator which maximizes the joint probability distribution function (pdf) of (5) is optimal [7].

$$P(\mathbf{y};\mathbf{h}) = \frac{1}{\pi^{(r n_p)} \det(\mathbf{C_v})} \exp[-(\mathbf{y}-\tilde{\mathbf{X}}\mathbf{h})^H \mathbf{C_v}^{-1} (\mathbf{y}-\tilde{\mathbf{X}}\mathbf{h})] \tag{5}$$

Clearly, for noise vector in (3), the co-variance matrix is

$$\mathbf{C_v} = \mathbf{R_v} = \mathrm{E}\{\mathbf{v}\mathbf{v}^H\} = \mathbf{I}_{r n_p} \tag{6}$$

Therefore, the ML estimator which is equal with the LS estimator in our interested model can be defined in the following form:

$$\hat{\mathbf{h}}_{ML} = \arg\min_{\mathbf{h}} \quad (\mathbf{y}-\tilde{\mathbf{X}}\mathbf{h})^H (\mathbf{y}-\tilde{\mathbf{X}}\mathbf{h}) \tag{7}$$

By differentiating $(\mathbf{y}-\tilde{\mathbf{X}}\mathbf{h})^H (\mathbf{y}-\tilde{\mathbf{X}}\mathbf{h})$ with respect to $\mathbf{h}$ and setting the result equal to zero, we have

$$\hat{\mathbf{h}}_{ML} = (\tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^H \mathbf{y} \tag{8}$$

Using equation $\mathrm{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\,\mathrm{vec}(\mathbf{B})$, (8) can be expressed in the following matrix form:

$$\hat{\mathbf{H}}_{ML} = \mathbf{Y}\mathbf{X}^H (\mathbf{X}\mathbf{X}^H)^{-1} \tag{9}$$

The error of this estimator is $J_{ML} = E\{\|\mathbf{h}-\hat{\mathbf{h}}_{ML}\|_F^2\} = r\, tr\{(\mathbf{X}\mathbf{X}^H)^{-1}\}$. It is shown that by applying a properly normalized sub-matrix of the Discrete Fourier Transform (DFT) matrix as optimal training the error of ML estimator is minimized as follows (see [2, 4]):

$$J_{ML\,(\min)} = \frac{t^2 r}{p} \tag{10}$$

where p is a given constant value as the total power of training matrix $\mathbf{X}$. This estimator achieves the classical Cramér-Rao Lower Bound (CRLB), hence, it is efficient. However, the ML estimator utilizes only received signals and transmitted symbols that are given at the receiver. It has no knowledge about the channel.

3.2. SSLS Channel Estimator

Consider (3), the SSLS channel estimator can be expressed in the following form

$$\hat{\mathbf{h}}_{SSLS} = \gamma \mathbf{h}_{LS} + \mathbf{b} \tag{11}$$

where $\hat{\mathbf{h}}_{LS}$ is the LS estimation of the channel. The SSLS estimator is the shifted type of SLS [2] which has been proposed in [4]. The scaling factor, γ, and the shifting vector, $\mathbf{b}$, have to be obtained so that the MSE $J_{SSLS} = E\{\|\mathbf{h}-\hat{\mathbf{h}}_{SSLS}\|_F^2\}$ is minimized.

Using (4), (10), and (11), the MSE of the SSLS estimator can be computed as follows

$$\begin{aligned} J_{SSLS} &= \mathrm{E}\left\{ \left\| \mathbf{h}-\gamma\,\hat{\mathbf{h}}_{LS}-\mathbf{b}\right\|_F^2 \right\} = tr\left\{\mathrm{E}\{(\mathbf{h}-\gamma\,\mathbf{h}_{LS}-\mathbf{b})(\mathbf{h}-\gamma\,\mathbf{h}_{LS}-\mathbf{b})^H\}\right\} \\ &= (1-\gamma)^2\, rt + (\gamma-1)\, tr\{\mathbf{m}\mathbf{b}^H + \mathbf{b}\mathbf{m}^H\} + tr\{\mathbf{b}\mathbf{b}^H\} + \gamma^2 \frac{t^2 r}{p} \end{aligned} \tag{12}$$

where

$$\mathbf{m} = E\{\mathbf{h}\} = \sqrt{\frac{\mathrm{K}}{\mathrm{K}+1}}\,\mathbf{h}_{\mathrm{LOS}} \tag{13}$$

By differentiating (12) with respect to γ and **b** and setting the results equal to zero, we have

$$-2\,r\,t\,(1-\gamma)+tr\{\mathbf{m}\,\mathbf{b}^{H}+\mathbf{b}\,\mathbf{m}^{H}\}+2\gamma\,\frac{t^{2}r}{p}=0 \tag{14}$$

$$(\gamma-1)\,\mathbf{m}^{*}+\mathbf{b}^{*}=0 \tag{15}$$

Using (15), the SSLS estimator of (11) can be rewritten as $\hat{\mathbf{h}}_{SSLS}=\gamma\mathbf{h}_{LS}+(1-\gamma)\,\mathbf{m}$ and using (13), (14), and (15), we have

$$\gamma=\frac{p}{t\,(1+\mathrm{K})+p} \tag{16}$$

According to [4], optimal training for LS (that is equal with ML in this paper) and SSLS estimators is identical. This fact is considered in (12), (14), and (16). Then, using (13), (15), and (16), the MSE (12) under optimal training minimizes as follows:

$$J_{SSLS\ (\min)}=\frac{t^{2}r}{p+t\,(1+\mathrm{K})} \tag{17}$$

It is seen that the MSE of the SSLS channel estimator decreases when K is increased.

4. Multiple Channel Estimation

In order to improve the performance of the estimators, we combine the multiple estimates of the channel during received N sub-blocks. In this section, it is assumed that the channel response is fixed within N sub-blocks. In other words, the coherent time of the channel is enough to use N sub-blocks for channel estimation. Suppose that N estimates $\hat{\mathbf{h}}_1,\ldots,\mathbf{h}_N$ of the MIMO channel are obtained using one of the ML or SSLS estimators in Section 3. The results of ME are combined in the following linear method:

$$\hat{\mathbf{h}}_{\mathrm{ME}}=\sum_{n=1}^{N}a_n\,\mathbf{h}_n \tag{18}$$

where $a_1,\ldots,a_N$ have to be obtained so that the MSE (19) is minimized.

$$J_{ME}=E\left\{\left\|\mathbf{h}-\sum_{n=1}^{N}a_n\,\hat{\mathbf{h}}_n\right\|_F^2\right\} \tag{19}$$

We obtain $a_1,\ldots,a_N$ subject to $\sum_{n=1}^{N}a_n=0$. Then, the optimization problem of (20) will be solved:

$$\min_{a_1,\ldots,a_N}\ E\left\{\left\|\mathbf{h}-\sum_{n=1}^{N}a_n\,\hat{\mathbf{h}}_n\right\|_F^2\right\}\qquad S.T\qquad \sum_{n=1}^{N}a_n=1 \tag{20}$$

4.1. ME-ML Channel Estimator

By combining (3) and (8), the ML estimator can be rewritten as $\hat{\mathbf{h}}_{ML}=\mathbf{h}+(\tilde{\mathbf{X}}^{H}\tilde{\mathbf{X}})^{-1}\,\tilde{\mathbf{X}}^{H}\mathbf{v}$. Using this equation, (6), and the constraint in (20), the MSE of ME-ML estimator will be written as

$$\begin{aligned}
J_{ME-ML}&=E\left\{\left\|\mathbf{h}-\sum_{n=1}^{N}a_n\,\hat{\mathbf{h}}_n\right\|_F^2\right\}=E\left\{\left\|\mathbf{h}-\sum_{n=1}^{N}a_n\,(\mathbf{h}+(\tilde{\mathbf{X}}_n^{H}\tilde{\mathbf{X}}_n)^{-1}\,\tilde{\mathbf{X}}_n^{H}\mathbf{v}_n)\right\|_F^2\right\}=E\left\{\left\|\sum_{n=1}^{N}a_n\,(\tilde{\mathbf{X}}_n^{H}\tilde{\mathbf{X}}_n)^{-1}\,\tilde{\mathbf{X}}_n^{H}\mathbf{v}_n\right\|_F^2\right\}\\
&=E\{tr\{(\sum_{n=1}^{N}a_n(\tilde{\mathbf{X}}_n^{H}\tilde{\mathbf{X}}_n)^{-1}\tilde{\mathbf{X}}_n^{H}\mathbf{v}_n)(\sum_{m=1}^{N}a_m(\tilde{\mathbf{X}}_m^{H}\tilde{\mathbf{X}}_m)^{-1}\tilde{\mathbf{X}}_m^{H}\mathbf{v}_m)^{H}\}\}\\
&=tr\{\sum_{n=1}^{N}\sum_{m=1}^{N}a_n\,a_m^{*}\,(\tilde{\mathbf{X}}_n^{H}\tilde{\mathbf{X}}_n)^{-1}\tilde{\mathbf{X}}_n^{H}\,E\{\mathbf{v}_n\mathbf{v}_m^{H}\}\,\tilde{\mathbf{X}}_m(\tilde{\mathbf{X}}_m^{H}\tilde{\mathbf{X}}_m)^{-1}\}=tr\{\sum_{n=1}^{N}\left|a_n\right|^2(\tilde{\mathbf{X}}_n^{H}\tilde{\mathbf{X}}_n)^{-1}\}
\end{aligned} \tag{21}$$

where the latter one is obtained using $E\{\mathbf{v}_n\mathbf{v}_m^{H}\}=\mathbf{I}_{rn_p}$; if $n=m$ and 0; if $n\neq m$. Then, the problem (20) reduces to

$$\min_{a_1,\ldots,a_N}\ tr\left\{\sum_{n=1}^{N}\left|a_n\right|^2\mathrm{E}_n\right\}\qquad S.T\qquad \sum_{n=1}^{N}a_n=1 \tag{22}$$

where $\mathrm{E}_n = (\tilde{\mathbf{X}}_n^H \tilde{\mathbf{X}}_n)^{-1}$. The ML estimator is unbiased. The constraint in (22) guarantees that the ME is also unbiased. To solve (22), the Lagrange multiplier method is used. The problem can be written as

$$L(a_1,...,a_N,\eta) = tr\left\{\sum_{n=1}^{N} \left|a_n\right|^2 \mathrm{E}_n\right\} + \eta\left\{\sum_{n=1}^{N} a_n - 1\right\} \tag{23}$$

To find $a_1,..., a_N$, the partial derivatives of (23) with respect to $a_1,..., a_N$ are computed. Then, the result is set equal to zero. Finally, we have

$$a_n = \frac{1}{tr\{\mathrm{E}_n\}\sum_{l=1}^{N} 1/tr\{\mathrm{E}_l\}} \quad ; \quad n = 1,...,N \tag{24}$$

Under optimal training $tr\{\mathrm{E}_n\} = tr\{(\tilde{\mathbf{X}}_n^H \tilde{\mathbf{X}}_n)^{-1}\} = r\,t^2 / p_n$ where p_n is the total power of training matrix $\mathbf{X}_n$ which is used during the training phase in the sub-block *n*. In special case where $p_1 = ... = p_N = p_{tot} / N = p$, (24) reduces to

$$a_n = \frac{1}{N} \quad ; \quad n = 1,...,N \tag{25}$$

Using $tr\{\mathrm{E}_n\} = r\,t^2 / p_n$ and (25), under optimal training, the MSE of (21) is minimizes as follows

$$J_{ME-ML_{(\min)}} = tr\{\sum_{n=1}^{N} \left|a_n\right|^2 (\tilde{\mathbf{X}}_m^H \tilde{\mathbf{X}}_m)^{-1}\} = tr\{\sum_{n=1}^{N} \frac{1}{N^2} (\tilde{\mathbf{X}}_m^H \tilde{\mathbf{X}}_m)^{-1}\} = \frac{1}{N^2} \times N \frac{r\,t^2}{p} = \frac{r\,t^2}{N\,p} \tag{26}$$

Comparing (26) and (10), it is seen that the error decreases in the ME case by the number of sub-blocks *N* which is used for channel estimation.

4.2. ME-SSLS Channel Estimator

The SSLS channel estimator can be rewritten as $\hat{\mathbf{h}}_{SSLS} = \gamma\mathbf{h} + \gamma(\tilde{\mathbf{X}}^H \tilde{\mathbf{X}})^{-1} \tilde{\mathbf{X}}^H \mathbf{v} + (1-\gamma)\mathbf{m}$. Using this equation, the MSE of ME-SSLS estimator is expressed as

$$\begin{aligned} J_{ME-SSLS} &= E\left\{\left\|\mathbf{h} - \sum_{n=1}^{N} a_n \hat{\mathbf{h}}_n\right\|_F^2\right\} = E\left\{\left\|\mathbf{h} - \sum_{n=1}^{N} a_n (\gamma_n \mathbf{h} + \gamma_n \mathrm{E}_n \tilde{\mathbf{X}}_n^H \mathbf{v}_n + (1-\gamma_n)\mathbf{m})\right\|_F^2\right\} \\ &= E\left\{\left\|(1 - \sum_{n=1}^{N} a_n \gamma_n)\mathbf{h} - \sum_{n=1}^{N} a_n \gamma_n \mathrm{E}_n \tilde{\mathbf{X}}_n^H \mathbf{v}_n - \sum_{n=1}^{N} a_n (1-\gamma_n)\mathbf{m}\right\|_F^2\right\} \\ &= E\{tr\{((1 - \sum_{n=1}^{N} a_n \gamma_n)\mathbf{h} - \sum_{n=1}^{N} a_n \gamma_n \mathrm{E}_n \tilde{\mathbf{X}}_n^H \mathbf{v}_n - \sum_{n=1}^{N} a_n (1-\gamma_n)\mathbf{m}) \\ &\quad \times((1 - \sum_{m=1}^{N} a_m \gamma_m)\mathbf{h} - \sum_{m=1}^{N} a_m \gamma_m \mathrm{E}_m \tilde{\mathbf{X}}_m^H \mathbf{v}_m - \sum_{m=1}^{N} a_m (1-\gamma_m)\mathbf{m})^H\}\} \end{aligned} \tag{27}$$

Using (4), (6), and (13), and with some calculation the result is

$$\begin{aligned} J_{ME-SSLS} &= r\,t\,(1 - \sum_{m=1}^{N} a_m^* \gamma_m - \sum_{n=1}^{N} a_n \gamma_n + \sum_{n=1}^{N} \sum_{m=1}^{N} a_m^* a_n \gamma_n \gamma_m) + r\,t \frac{\mathrm{K}}{1+\mathrm{K}} (-\sum_{n=1}^{N} (a_n + a_n^*)(1-\gamma_n) \\ &\quad + \sum_{n=1}^{N} \sum_{m=1}^{N} a_n a_m^* (1 - \gamma_n \gamma_m)) + tr\{\sum_{n=1}^{N} \left|a_n\right|^2 \gamma_n^2 (\tilde{\mathbf{X}}_n^H \tilde{\mathbf{X}}_n)^{-1}\} \end{aligned} \tag{28}$$

The optimization problem is

$$\min_{a_1,...,a_N} J_{ME-SSLS} \qquad S.T \qquad \sum_{n=1}^{N} a_n = 1 \tag{29}$$

The SSLS estimator is biased. The constraint in (29) guarantees that the ME is also biased. Using the Lagrange multiplier method, we have

$$L(a_1,...,a_N,\eta) = J_{ME-SSLS} + \eta\left\{\sum_{n=1}^{N} a_n - 1\right\} \tag{30}$$

By differentiating (30) with respect to $a_1,..., a_N$ and setting the results equal to zero, in the case of equal powers $p_1 = ... = p_N = p_{tot} / N = p$, the result is same as (25). Finally, the MSE of (28) is minimized under optimal training as

$$J_{ME-SSLS_{(\min)}} = rt(1-\gamma)^2 - rt\frac{\mathrm{K}}{1+\mathrm{K}}(1-\gamma)^2 + \frac{\gamma^2}{N}\frac{rt^2}{p} = \frac{rt^2(Nt(1+\mathrm{K})+p)}{N(t(1+\mathrm{K})+p)^2} \quad (31)$$

It is seen that in the ME case the error decreases when the number of sub-blocks N increases. Note that when N=1, (31) reduces to the special case of (17) for single channel estimation with the SSLS estimator.

5. Simulation Results

In order to compare the performance of the ML and SSLS estimators in the case of SE and ME, we consider the channel MSE, normalized by the average channel energy as $NMSE = E\{\|\mathbf{h} - \hat{\mathbf{h}}\|_F^2\} / E\{\|\mathbf{h}\|_F^2\}$.

Fig. 2 shows Normalized MSE (NMSE) of the ML channel estimator with optimal training versus SNR in the case of SE and ME. According to this figure, increasing the number of the sub-blocks N results in a lower error of the estimation. In other words, the performance of the ML estimator in ME case is better than SE case. Clearly, the performance of the ML estimator is independent of K.

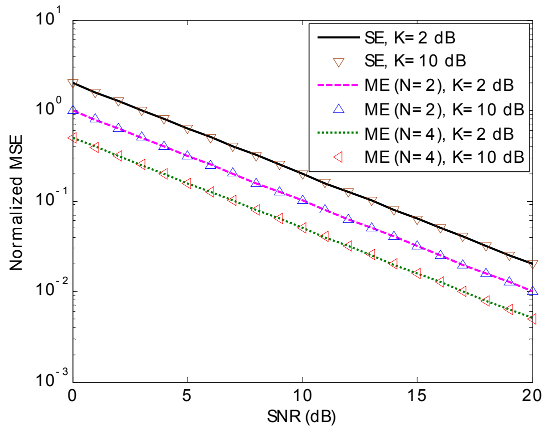

Fig. 2: NMSE of SE (N = 1) and ME (N = 2, 4) with ML estimator (r = t = 2, K = 2, 10 dB)

Figs. 3 and 4 indicate the NMSE of the SSLS channel estimator in the case of SE and ME for K= 2, 10 dB, respectively. As depicted in these figures, the SSLS estimator has better performance in ME case than SE especially at high SNRs and low Rice factors. However, at low SNRs, the NMSEs of the estimator for various numbers of sub-blocks N are analogous particularly for higher values of K.

In Figs. 5 and 6, the performance of the ML and SSLS estimators is compared for various SNRs, Rice factors and the number of sub-blocks N. It is seen that at low SNRs and high Rice factors and also for small numbers of N the SSLS estimator is better than ML. On the other hand, at high SNRs and low Rice factors and for large numbers of N, the ML estimator is better than SSLS. Therefore, in the Rician channels with a long coherence time, and hence large N, the ML estimator is an appropriate method but in channels with a short coherence time, and hence small N, the SSLS is better than ML. Note that increasing K results in a lower error of the SSLS channel estimation.

In practice, to obtain the best result in channel estimation, one of the ML or SSLS methods can be used considering the values of K, SNR, the number of antennas and N (or channel coherent time) in (26) and (31).

6. Conclusion

The performance of ML and SSLS estimators in the case of SE and ME is probed for a MIMO Rician flat fading channel. For each of these methods, the channel estimation error is obtained under optimal training.

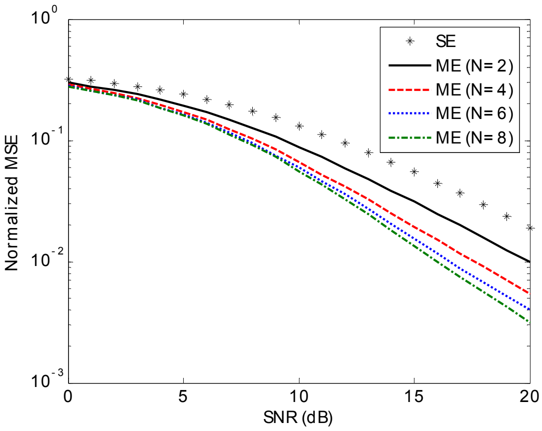

Fig. 3: NMSE of SE ($N = 1$) and ME ($N = 2, 4, 6, 8$) with SSLS estimator ($r = t = 2$, K = 2 dB)

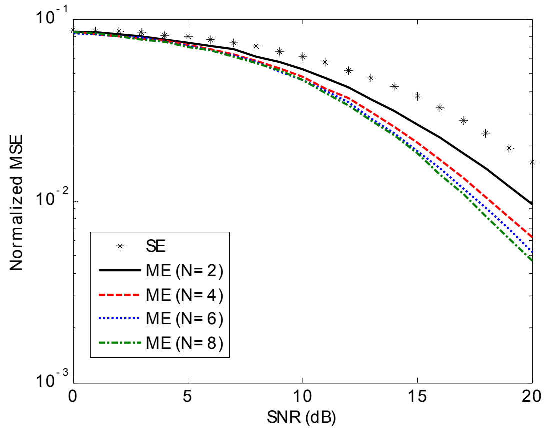

Fig. 4: NMSE of SE ($N = 1$) and ME ($N = 2, 4, 6, 8$) with SSLS estimator ($r = t = 2$, K = 10 dB)

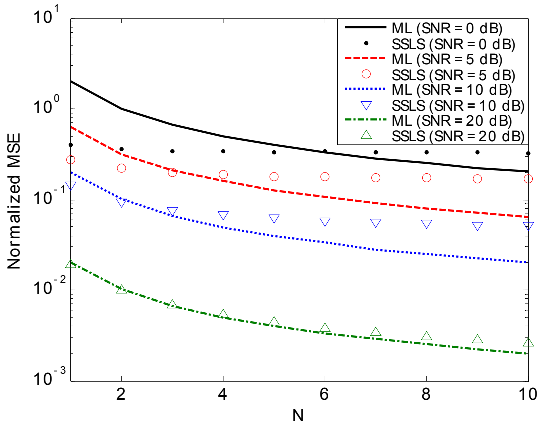

Fig. 5: NMSE of ML and SSLS estimators vs. the number of sub-blocks for various SNRs ($r = t = 2$, K =0 dB)

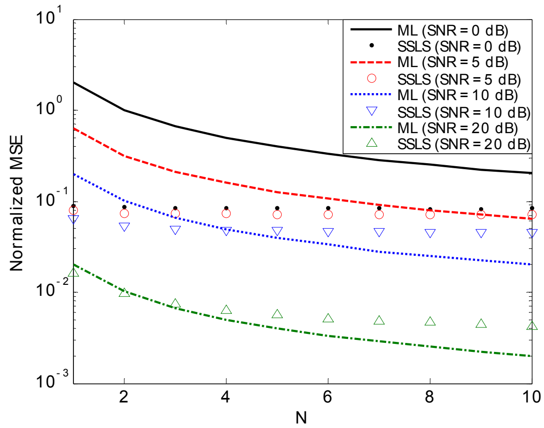

Fig. 6: NMSE of ML and SSLS estimators vs. the number of sub-blocks for various SNRs ($r = t = 2$, K =10 dB)

Analytical and numerical results show that the MSE of both estimators decreases when the number of sub-blocks N is increased. It is shown that for small values of N, suitable for estimation of the channel with fast fading, the SSLS estimator is better than ML especially at low SNRs and high Rice factors. However, for large values of N, proper for estimation of the channel with slow fading, the ML estimator is better than SSLS especially at high SNRs and low Rice factors.

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