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Three Round Adaptive Diagnosis with a Tree-Based Structure

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Abstract. The ability of identifying all the faulty devices in a multiprocessor system is known as diagnosability. The PMC model is the tested-based diagnosis with a processor performing the diagnosis by testing on the neighboring processors via the links between them. In this paper, we propose a tree-based structure for local diagnosis under PMC model. We design a three round adaptive local diagnosis algorithm in the tree-based structure.

Keywords: system diagnosis, diagnosability, PMC model.

1. Introduction

In multiprocess system, there are more than one processor and it can run many programs simultaneously. The reliability is crucial since even a few malfunctions may lead to a service unreliable. Whenever devices are found to be faulty, they should be replaced with fault-free ones as soon as possible to guarantee that the system can work properly. Identifying all the faulty devices in a system is known as system diagnosis. The maximum number of faulty devices that can be correctly identified is an important parameter, known as the diagnosability of a system. A system is said to be t-diagnosable if all its faulty device can be precisely pointed out provided that the total number of faulty device does not exceed t. In other words, the diagnosability of a system is just equal to the maximum integer t such that it can be t-diagnosable.

The problem of identifying faulty processors in a multiprocessor system has been widely addressed in [2,5,7]. Throughout this paper, the underlying topology of a multiprocessor system is modeled as a graph; each processor is represented by a vertex, and the communication bus, or fabric, is represented by a single edge between two vertices. A diagnosis testing signal is supposed to be delivered from one vertex to another one through the communication bus at one time. A system performs a so-called system-level diagnosis by making each processor act as a tester to test each of the directly connected ones. It is noticed that such a scheme contains no central test controller instead.

Several well-known approaches to system diagnosis have been developed. One classic approach, called the PMC diagnosis model (or PMC model for short), was first proposed by Preparata, Metze and Chien [8]. This model performs a diagnosis by sending a test signal from a processor to another linked one and then receiving a response in the reverse direction. According to the collection of all test results, the fault status of every processor can be identified. Hakimi and Amin [4] proved that a system is t-diagnosable if it is t-connected with at least 2t+1 vertices under the PMC model. They also gave a sufficient and necessary condition for verifying if a system is t-diagnosable. In [1], Dahbura and Masson proposed an O(N2.5) diagnosis algorithm to identify all faulty processors in a system with N processors. Fujita and Araki [3] proposed a scheme that completes a diagnosis in at most three test rounds under the PMC model if the number of faulty vertices is at most n in a binary n-cubes. It is optimal in the sense that three rounds are necessary for the adaptive diagnosis. In this paper, we concern with the local diagnosability. We propose a three-round adaptive local diagnosis algorithm under the PMC model in a tree-based structure. With our

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algorithm, the fault/fault-free status of a vertex can be identified correctly if the total number of faulty vertices does not exceed t, where t is the connectivity on the system.

2. The PMC Diagnosis Model

For the graph definitions and notation, we follow [6]. Let $G=(V,E)$ be a *graph* if V is a finite set and E is a subset of $\{\{u,v\} \mid \{u,v\}$ is an unordered pair of $V\}$. We say that V is the *vertex set* and E is the *edge set* of G. Two vertices u and v are *adjacent* if $\{u,v\}\in E$; we say u is a *neighbour* of v, and vice versa. We use $NG(u)$ to denote the neighborhood set $\{v \mid \{u,v\}\in E(G)\}$. The *degree* of a vertex v in a graph G, denoted by $\deg G(v)$, is the number of edges incident to v. A *path* is a sequence of vertices such that two consecutive vertices are adjacent. A path is represented by $<v_0,v_1,v_2,\ldots,v_n>$.

Under the PMC diagnosis model, we assume that adjacent processors are capable of performing tests on each other. Let $G=(V,E)$ denote the underlying topology of a multiprocessor system. For any two adjacent vertices $u,v\in V(G)$, the ordered pair (u,v) represents a test that processor u is able to diagnose processor v. In this situation, u is a *tester*, and v is a *testee*. The outcome of a test (u,v) is 1 (respectively, 0) if u evaluates v to be faulty (respectively, fault-free). Since the faults considered here are permanent, the outcome of a test is *reliable* if and only if the tester is fault-free. A *test assignment* for system G is a collection of tests and can be modeled as a directed graph $T=(V,L)$, where $(u,v)\in L$ implies that u and v are adjacent in G. The collection of all test results from the test assignment T is called a *syndrome*. Formally, a syndrome of T is a mapping $\sigma: L \rightarrow \{0,1\}$. A faulty set F is the set of all faulty processors in G. It is noticed that F can be any subset of V. The process of identifying all faulty vertices is said to be the *system diagnosis*. Furthermore, the maximum number of faulty vertices that can be correctly identified in a system G is called the *diagnosability* of G, denoted by $\tau(G)$. We say a system G is *t-diagnosable* if all faulty vertices in G can be precisely pointed out with the total number of faulty vertices being at most t.

3. A Three Round Adaptive Diagnosis Algorithm with a Tree-Based Structure

In this section, we propose a tree-based structure, called a *DT-tree*, for local diagnosis and give a three-round adaptive local diagnosis algorithm under the PMC model. First, we give the definition of a DT-tree as follows.

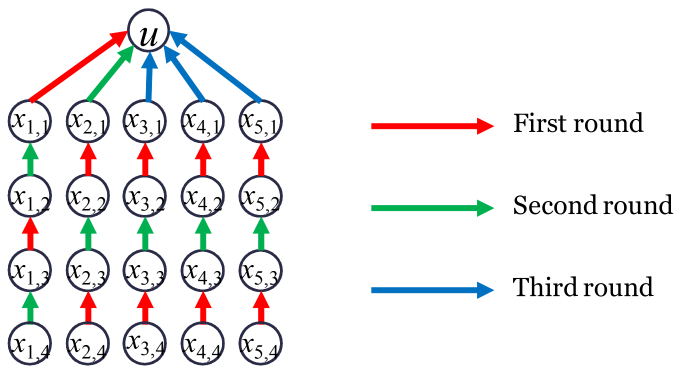

Fig. 1: A DT-tree $DT_X(u)$. (suppose that t=5)

Definition 1 (*DT-tree*) Let u be a vertex in a graph G with $\deg(u)=t$ and $X=\{x_{i,j} \mid x_{i,j}\in V(G)-\{u\}, 1\le i\le t, 1\le j\le t-1\}$. We say $DT_X(u)$ is a DT-tree which $V(DT_X(u))=\{u\}\cup X$, and $E(DT_X(u))=\{(x_{i,1},u) \mid 1\le i\le t\}\cup\{(x_{i,j+1},x_{i,j}) \mid 1\le i\le t, 1\le j\le t-2\}$. Figure 1 illustrates the $DT_X(u)$.

We describe the scheme that determining the fault status of a vertex u in a DT-tree with three test rounds under the PMC model in Figure 1. We use the red arrow, the green arrow and the blue arrow to represent the first, the second and the third test round, respectively. To diagnose a vertex in a DT-tree, we give the following two algorithms, **VOTE** and **ADA**. We propose the algorithm **ADA** (Adaptive-Local-Diagnosis) to identify the fault/fault-free status of a vertex u in $\mathrm{DT}_X(u)$ under the PMC model.

Algorithm VOTE(S, u)
Input: A vertex set S and a vertex u.
Output: The value is 0 or 1 if u is fault-free or faulty, respectively.
Begin
  $n_0 \leftarrow 0$;
  $n_1 \leftarrow 0$;
  if $|S| = 1$
    then let x be the vertex in S:
      return $\sigma(x, u)$;
  else
    $n_0 \leftarrow |\{i \mid \sigma(x_i, u) = 0$ for each $x_i \in S$ with $1 \le i \le |S|\}|$;
    $n_1 \leftarrow |\{i \mid \sigma(x_i, u) = 1$ for each $x_i \in S$ with $1 \le i \le |S|\}|$;
  if $n_0 \ge n_1$
    return 0;
  else
    return 1;
End

Algorithm ADA$(DT_X(u))$
Input: A DT-tree $DT_X(u)$.
Output: The value is 0 or 1 if u is fault-free or faulty, respectively.
Begin
  $S \leftarrow \emptyset$;
  $t \leftarrow \deg(u)$;
  $p \leftarrow |\{i \mid \sigma(x_{1,i}, x_{1,i-1}) = 1$ for $2 \le i \le t-1\}|$;
  $q \leftarrow |\{i \mid \sigma(x_{2,i}, x_{2,i-1}) = 1$ for $2 \le i \le t-1\}|$;
  for $j = 3$ to t **do**
    $\gamma_j \leftarrow |\{i \mid \sigma(x_{j,i}, x_{j,i-1}) = 1$ for $2 \le i \le t-1\}|$;
    if $\gamma_j = \min_{3 \le j' \le j} \{\gamma_{j'}\}$
      then $x' \leftarrow x_{j,1}$;
        $r \leftarrow \gamma_j$;
  if $r = 0$
    then if $p = 0$ and $q = 0$
        then $S \leftarrow \{x_{1,1}, x_{2,1}, x'\}$;
          return VOTE(S, u);
      else if $p = 0$ and $q \ne 0$
        then $S \leftarrow \{x_{1,1}, x'\}$;
          return VOTE(S, u);
      else if $p \ne 0$ and $q = 0$
        then $S \leftarrow \{x_{2,1}, x'\}$;
          return VOTE(S, u);
      else
        $S \leftarrow \{x'\}$;
        return VOTE(S, u);
  else
    if $p = 0$ and $q = 0$
      then $S \leftarrow \{x_{1,1}, x_{2,1}\}$;
        return VOTE(S, u);
    else if $p = 0$ and $q \ne 0$
      then $S \leftarrow \{x_{1,1}\}$;
        return VOTE(S, u);
    else if $p \ne 0$ and $q = 0$
      then $S \leftarrow \{x_{2,1}\}$;
        return VOTE(S, u);
    else
      return 0;
End

With the definition and the property of the PMC diagnosis model, we have the following two lemmas.

Lemma 1. Let $P=\langle p_1,p_2,p_3,\ldots,p_r\rangle$ be a path. If p_1 is faulty, and $\sigma(p_i,p_{i-1})=0$ for every $2 \le i \le r$, then all the r vertices in P are faulty.

Lemma 2. Let $P=\langle p_1,p_2,p_3,\ldots,p_r\rangle$ be a path. If $\sigma(p_i,p_{i-1})=1$ for some i in $2 \le i \le r$, then there exists at least one fault vertex in P.

Theorem 1. A vertex u in a DT-tree $\mathrm{DT}_X(u)$ can be diagnosed correctly with the algorithm **ADA** under the PMC model.

Proof. For $1 \le i \le t$, let $\sigma_i=(\sigma(x_{i,2},x_{i,1}),\sigma(x_{i,3},x_{i,2}),\ldots,\sigma(x_{i,t-1},x_{i,t-2}))$ and $e_0=(z_1,z_2,\ldots,z_{t-2})$ where $z_{i'}=0$ for every $1 \le i' \le t-2$. We consider the following cases.

Case 1: Suppose that there exists some k such that $\sigma_k=e_0$ for $3 \le k \le t$.

Subcase 1.1: Suppose that $\sigma_1=e_0$ and $\sigma_2=e_0$. We claim that u not in F if at least two of $\sigma(x_{1,1},u)$, $\sigma(x_{2,1},u)$ and $\sigma(x_{k,1},u)$ are zero. Without loss of generality, suppose that $\sigma(x_{1,1},u)=0$ and $\sigma(x_{2,1},u)=0$. We assume that $u\in F$ by contradiction. Thus $\{x_{1,1},x_{2,1}\}\subset F$. By Lemma 1, $|F|\geq 2(t-1)+1>t$ if $t\geq 3$, which contradicts the assumption that $|F|\leq t$. Now we assume that u not in F, and at least two of $\sigma(x_{1,1},u)$, $\sigma(x_{2,1},u)$ and $\sigma(x_{k,1},u)$ are one. Without loss of generality, suppose that $\sigma(x_{1,1},u)=1$ and $\sigma(x_{2,1},u)=1$. Thus $\{x_{1,1},x_{2,1}\}\subset F$. By Lemma 1, $|F|\geq 2(t-1)>t$ if $t\geq 3$, which contradicts the assumption that $|F|\leq t$.

Subcase 1.2: Suppose that $\sigma_1=e_0$ and $\sigma_2\neq e_0$. We claim that u not in F if at least one of $\sigma(x_{1,1},u)$ and $\sigma(x_{k,1},u)$ is zero. Without loss of generality, suppose that $\sigma(x_{1,1},u)=0$. We assume that $u\in F$ by contradiction. Thus $x_{1,1}\in F$. By Lemma 1 and Lemma 2, $|F|\geq(t-1)+1+1>t$, which contradicts the assumption that $|F|\leq t$. Now we assume that u not in F, $\sigma(x_{1,1},u)=1$ and $\sigma(x_{k,1},u)=1$. Thus $\{x_{1,1},x_{2,1}\}\subset F$. By Lemma 1 and Lemma 2, $|F|\geq 2(t-1)+1>t$ if $t\geq 3$, which contradicts the assumption that $|F|\leq t$.

Subcase 1.3: Suppose that $\sigma_1\neq e_0$ and $\sigma_2=e_0$. This case is similar as Subcase 1.2 by interchanging the roles of $x_{1,1}$ and $x_{2,1}$.

Subcase 1.4: Suppose that $\sigma_1\neq e_0$ and $\sigma_2\neq e_0$. We claim that u not in F if $\sigma(x_{k,1},u)=0$. We assume that $u\in F$ by contradiction. Thus $x_{k,1}\in F$. By Lemma 1 and Lemma 2, $|F|\geq(t-1)+2+1>t$, which contradicts the assumption that $|F|\leq t$. Now we assume that u not in F and $\sigma(x_{k,1},u)=1$. Thus $x_{k,1}\in F$. By Lemma 1 and Lemma 2, $|F|\geq(t-1)+2>t$, which contradicts the assumption that $|F|\leq t$.

Case 2: Suppose that $\sigma_k\neq e_0$ for $3\leq k\leq t$..

Subcase 2.1: Suppose that $\sigma_1=e_0$ and $\sigma_2=e_0$. We claim that u not in F if at least one of $\sigma(x_{1,1},u)$ and $\sigma(x_{2,1},u)$ is zero. Without loss of generality, suppose that $\sigma(x_{1,1},u)=0$. We assume that $u\in F$ by contradiction. Thus $x_{1,1}\in F$. By Lemma 1 and Lemma 2, $|F|\geq(t-1)+1+1>t$, which contradicts the assumption that $|F|\leq t$. Now we assume that u not in F, $\sigma(x_{1,1},u)=1$ and $\sigma(x_{2,1},u)=1$. Thus $\{x_{1,1},x_{2,1}\}\subset F$. By Lemma 1 and Lemma 2, $|F|\geq 2(t-1)+1>t$ if $t\geq 3$, which contradicts the assumption that $|F|\leq t$.

Subcase 2.2: Suppose that $\sigma_1=e_0$ and $\sigma_2\neq e_0$. We claim that u not in F if $\sigma(x_{1,1},u)=0$. We assume that $u\in F$ by contradiction. Thus $x_{1,1}\in F$. By Lemma 1 and Lemma 2, $|F|\geq(t-1)+2+1>t$, which contradicts the assumption that $|F|\leq t$. Now we assume that u not in F and $\sigma(x_{1,1},u)=1$. Thus $x_{1,1}\in F$. By Lemma 1 and Lemma 2, $|F|\geq(t-1)+2>t$, which contradicts the assumption that $|F|\leq t$.

Subcase 2.3: Suppose that $\sigma_1\neq e_0$ and $\sigma_2=e_0$. This case is similar as Subcase 2.2 by interchanging the roles of $x_{1,1}$ and $x_{2,1}$.

Subcase 2.4: Suppose that $\sigma_1\neq e_0$ and $\sigma_2\neq e_0$. By Lemma 2, there exist t fault vertices in $\{x_{i,j} \mid 1\leq i\leq t, 2\leq j\leq t-1\}$. With the assumption that $|F|\leq t$, the vertex u not in F. Thus the theorem proved.

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