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Decentralized Controller for Constrained Nonlinear Water Quality Control in Streams

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Abstract. In this paper, stream water quality control process is considered. The system is represented by a constrained nonlinear interconnected dynamical model with time delay. By decomposing the system into a set of subsystems, a partially closed loop decentralized controller is developed which leads to a suboptimal system performance while satisfying system constraints. The developed procedure is applied to a typical nonlinear stream water quality control system and the obtained results show the effectiveness of the presented approach.

Keywords: Decentralized control; optimal control theory; constrained nonlinear optimization problems.

1. Introduction

Distributed interconnected systems form an important class of problems that control engineers are faced with. These systems are often characterized by their nonlinear dynamic behavior, large dimensionality, time delay and system constraints on the states and/or inputs. Decomposing such systems into smaller interconnected subsystems provides a less complex and more efficient way to deal with the overall problem. A special type of this class of systems are those characterized by their sequential nature (e.g. traffic networks, production lines, rivers, etc.), in which the response of the system at any section is directly affected by the behavior of the preceding section(s).

Under the assumption that optimal control problems associated with these systems have a feasible solution, it is obvious that controllers designed in a global sense lead to an optimal performance. However, for geographically spread systems, the initial cost of implementing globally optimal controllers tends to be quite high. More importantly, reduced system reliability is apparent since such types of systems are often subject to structural perturbations.

On the other hand, although decentralized controllers lead to suboptimal system performance, they need no transfer of information between subsystems, thus such an approach is economical, reduces communication overhead and increases system reliability.

Although constrained linear quadratic problems (LQP) have attracted many researchers in the last decades, few of them have tackled nonlinear constrained dynamical systems. Among the techniques developed to solve LQP are those falling into the class of model predictive control [1-2] and references therein, anti-wind up approach for which we quote [3] and references therein, coordinating based approach and time delayed systems [4-6] and references therein. A number of researchers have attempted to tackle constrained nonlinear optimization [7-8], however constraints were only applied on the input, with the states left unconstrained.

In this paper, an approach is suggested to design a suboptimal decentralized control structure for serially interconnected nonlinear dynamical systems with time delay and system constraints on the states and/or inputs. The main idea behind this approach is to consider time delayed coupling variables from proceeding subsystems as known inputs. Then, after introducing the coordinating variables, a decentralized algorithm is proposed to solve the problem at hand. This leads, at the end of convergence, to decentralized control strategy, which although suboptimal in its global sense, it guarantees the satisfaction of system constraints. Moreover, it allows parallel processing hence reduced computational time. The developed approach is then used to control the concentrations of the biochemical oxygen demand (BOD) and the dissolved oxygen (DO) in a three reach river system.

The rest of the paper is divided into the following. The problem is formulated in section 2. The developed decentralized approach is presented in section 3. In section 4, the water quality control problem is demonstrated while simulation results are given in section 5. Finally, the paper is concluded in section 6.

2. Problem Formulation

Let us consider the following nonlinear interconnected dynamical optimization problem with time delays, which comprises N-nonlinear interconnected subsystems, and is assumed to have a feasible solution:

$$\min J=\min\sum_{i=1}^{N} J_i=\sum_{i=1}^{N}\frac{1}{2}\int_{t_o}^{t_f}\left\|x_i(t)-x_i^d\right\|_{Q_{i1}}^2+\left\|u_i(t)\right\|_{R_{i1}}^2\,dt \tag{1}$$

subject to:

$$\dot{x}_i(t)=f_i(x,x(t-\tau_p),u,t)\ ;\ \text{with}\ \ x_i(t_o)=x_{oi} \tag{2}$$

$$\underline{x}_i\le x_i(t)\le \overline{x}_i \tag{3}$$

$$\underline{u}_i\le u_i(t)\le \overline{u}_i \tag{4}$$

where: $i\in\{1,2,....,N\}$, $n=\sum_{i=1}^{N} n_i$, $m=\sum_{i=1}^{N} m_i$, $x_i(t)\in R^{n_i}$, $u_i(t)\in R^{m_i}$ are the state and control variables of the i^{th} subsystem, respectively, τ_p is the time delay; with $p\in\{1,....,\theta\}$ and θ is a known positive integer representing the number of delays in the state vector, $x_i^d\in R^{n_i}$ is the desired steady-state value of the state vector, $f_i(x,x(t-\tau_p),u,t):R^{n_i}\to R^{n_i}$ is a nonlinear vector function, $\underline{x}_i$, $\overline{x}_i$, $\underline{u}_i$, $\overline{u}_i$ are the lower and upper bounds of the state and control vectors (component by component) and $Q_{i1}\ge 0\in R^{n_i\times n_i}$, $R_{i1}>0\in R^{m_i\times m_i}$ are the subsystem weighting matrices which are related to the original ones by the following:
$Q_1=diag\{Q_{11},Q_{21},....,Q_{N1}\}$; $R_1=diag\{R_{11},R_{21},....,R_{N1}\}$.

We assume in the rest of the paper that:

$$x(t-\tau_p)=x^d\quad \forall t-\tau_p<0;\quad p\in\{1,....,\theta\}$$

Since we are dealing with the special class of serially interconnected nonlinear dynamical systems with time delay, in this case any subsystem is coupled only with the preceding one(s). In our analysis, we consider the interconnection with the preceding subsystems, $x_j^*(t-\tau_p)$; $j\in\{1,2,...,(i-1)\}$, as known input signals. Hence, the system will be decoupled and it is possible to control its behavior through a decentralized control structure.

Moreover, in order to satisfy state constraints without violating the dynamics of the system and to be able to generate a partially closed loop control strategy, we introduce the coordinating variables $x_i^o(t)\in R^{n_i}$ and $u_i^o(t)\in R^{m_i}$. Accordingly, the above optimization problem can be rewritten in the following equivalent form:

$$J=\min\sum_{i=1}^{N} J_i=\sum_{i=1}^{N}\frac{1}{2}\int_{t_o}^{t_f}\left\|x_i^o(t)-x_i^d(t)\right\|_{Q_{i1}}^2+\frac{1}{2}\left\|x_i(t)-x_i^o(t)\right\|_{Q_{i2}}^2+\left\|u_i(t)\right\|_{R_{i1}}^2+\frac{1}{2}\left\|u_i(t)-u_i^o(t)\right\|_{R_{i2}}^2\,dt \tag{5}$$

subject to:

$$\dot{x}_i(t)=A_{oii}x_i(t)+B_iu_i(t)+f_i(x_i^o,x_j^*(t-\tau_p),u_i^o,t)-A_{oii}x_i^o(t)-B_iu_i^o(t)\quad ;\ \text{with: } x_i(t_o)=x_{oi} \tag{6}$$

$$x_i(t)=x_i^o(t) \tag{7}$$

$$u_i(t)=u_i^o(t) \tag{8}$$

$$\underline{x}_i\le x_i^o(t)\le \overline{x}_i \tag{9}$$

$$\underline{u}_i\le u_i(t)\le \overline{u}_i \tag{10}$$

$$A_{oii}=[\partial f_i(x_i^o,x_j^*(t-\tau_p),u_i^o,t)/\partial x_i]_{\substack{x=x_{ss}\\ u=u_r}}\quad ;\quad B_i=[\partial f_i(x_i^o,x_j^*(t-\tau_p),u_i^o,t)/\partial u_i]_{\substack{x=x_{ss}\\ u=u_r}} \tag{11}$$

where x_{ss},u_r are, respectively, the steady state response of the system and the reference input.

As will be shown below, the coordinating variable $x_i^o(t)$ will be obtained from a set of algebraic equations on which we can apply the inequality (9) to satisfy the state constraints. The Lagrange multiplier associated with (7) will force $x_i(t)$ -resulting from the solution of the state equation (6)- to approach $x_i^o(t)$ through the control signal.

3. The Developed Approach

Let us write the Hamiltonian for the i^{th} subsystem:

$$H_i(x_i,x_i^o,u_i,u_i^o,\lambda_i,\pi_i,\beta_i,t)\overset{\Delta}{=}\frac{1}{2}\left\|x_i^o(t)-x_i^d\right\|_{Q_{i1}}^2+\frac{1}{2}\left\|x_i(t)-x_i^o(t)\right\|_{Q_{i2}}^2+\frac{1}{2}\left\|u_i(t)\right\|_{R_{i1}}^2+\frac{1}{2}\left\|u_i(t)-u_i^o(t)\right\|_{R_{i2}}^2+\lambda_i^T(t)$$

$$[A_{o_{ii}}x_i(t)+B_iu_i(t)+f_i(x_i^o,x_j^*(t-\tau_p),u_i^o,t)-A_{oii}x_i^o(t)-B_iu_i^o(t)]+\pi_i^T(t)[x_i(t)-x_i^o(t)]+\beta_i^T(t)[u_i(t)-u_i^o(t)] \quad (12)$$

where $\lambda_i(t)\in R^{n_i}$ is the co-state vector, $\pi_i(t)\in R^{n_i}$ and $\beta_i(t)\in R^{m_i}$ are the Lagrange multiplies associated with the equality constraints (7) and (8) respectively. The necessary conditions of optimality lead to:

$$\frac{\partial H}{\partial u_i(t)}=0 \;\Rightarrow\; u_i(t)=\Re_i^{-1}[R_{i2}u_i^o(t)-B_i^T\lambda_i(t)-\beta_i(t)]=\Gamma_i(u_i^o,\lambda_i,\beta_i,t) \quad (13)$$

where $\Re_i=R_{i1}+R_{i2}$

However, to satisfy the constraints given by (10), we have [9]:

$$u_i(t)=\begin{cases}\underline{u}_i & if \quad \Gamma_i(u_i^o,\lambda_i,\beta_i,t)<\underline{u}_i\\ \Gamma_i(u_i^o,\lambda_i,\beta_i,t) & if \quad \underline{u}_i\le\Gamma_i(u_i^o,\lambda_i,\beta_i,t)\le\overline{u}_i\\ \overline{u}_i & if \quad \Gamma_i(u_i^o,\lambda_i,\beta_i,t)>\overline{u}_i\end{cases} \quad (14)$$

$$\frac{\partial H}{\partial\lambda_i(t)}=\dot{x}_i(t)\Rightarrow\dot{x}_i(t)=A_{o_{ii}}x_i(t)+B_iu_i(t)+f_i(x_i^o,x_j^*(t-\tau_p),u_i^o,t)-A_{oii}x_i^o(t)-B_iu_i^o(t)\;;\;\text{with}\;x_i(t_o)=x_{oi} \quad (15)$$

$$\frac{\partial H}{\partial x_i(t)}=-\dot{\lambda}_i(t)\;\Rightarrow\;\dot{\lambda}_i(t)=-Q_{i2}(x_i(t)-x_i^o(t))-A_{o_{ii}}{}^T\lambda_i(t)-\pi_i(t)\;;\;\text{with}\;\lambda_i(t_f)=0 \quad (16)$$

$$\frac{\partial H}{\partial u_i^o(t)}=0\Rightarrow\beta_i(t)=\frac{\partial f_i^T(x_i^o,x_j^*(t-\tau_p),u_i^o,t)}{\partial u_i^o(t)}\lambda_i(t)-R_{i2}(u_i(t)-u_i^o(t))-B_i^T\lambda(t) \quad (17)$$

$$\frac{\partial H}{\partial x_i^o(t)}=0\Rightarrow x_i^o(t)=\Theta_i^{-1}[Q_{i1}x_i^d+Q_{i2}x_i(t)-\sum_{\substack{p=1\\ \tau_1=0}}^{\theta}\frac{\partial f_i^T(x_i^o,x_j^*(t-\tau_p),u_i^o,t)}{\partial x_i^o(t)}\lambda_i(t+\tau_p)-A_{o_{ii}}^T\lambda_i(t)+\pi_i(t)] \quad (18)$$

$$=\Psi_i(x_i^o,x_j^*(t-\tau_p),u_i^o,\lambda_i,\pi_i,t)$$

where $\Theta_i=Q_{i1}+Q_{i2}$

As before, to satisfy system constraints given by (9), the coordinating vector $x^o(t)$, which minimizes the Hamiltonian, is given by:

$$x_i^o(t)=\begin{cases}\underline{x}_i & if \;\; \Psi_i(x_i^o,x_j^*(t-\tau_p),u_i^o,\lambda_i,\pi_i,t)<\underline{x}_i\\ \Psi_i(x_i^o,x_j^*(t-\tau_p),u_i^o,\lambda_i,\pi_i,t) & if \;\; \underline{x}_i\le\Psi_i(x_i^o,x_j^*(t-\tau_p),u_i^o,\lambda_i,\pi_i,t)\le\overline{x}_i\\ \overline{x}_i & if \;\; \Psi_i(x_i^o,x_j^*(t-\tau_p),u_i^o,\lambda_i,\pi_i,t)>\overline{x}_i\end{cases} \quad (19)$$

$$\frac{\partial H}{\partial\beta_i(t)}=0\Rightarrow u_i^o(t)=u_i(t) \quad (20)$$

Finally we have:

$$\frac{\partial H}{\partial\pi_i(t)}=x_i(t)-x_i^o(t) \quad (21)$$

This leads to the updating algorithm for $\pi_i(t)$ given by:

$$\pi_i^{k+1}(t)=\pi_i^k(t)+\varphi_i^k l_i^k(t) \quad (22)$$

where k is the iteration number, $l_i^k(t)$ can be specified according to the selected algorithm (conjugate gradient,..etc), while φ_i^k has to be positive to maximization of the dual function w.r.t. the dual variable $\pi_i^k(t)$.

In our procedure, and throughout this paper, we applied the gradient technique to update $\pi_i^k(t)$ with $l_i^k(t) = x_i^k(t) - x_i^{ok}(t)$.

Let $\lambda_i(t) = P_i(t)x_i(t) + \xi_i(t)$ (23)

$\Rightarrow \dot{\lambda}_i(t) = \dot{P}_i(t)x_i(t) + P_i(t)\dot{x}_i(t) + \dot{\xi}_i(t)$ (24)

Substituting (13) into (15), and replacing $\lambda_i(t)$ with the expression given in (23) while using (24) with (16), one gets, after simple mathematical manipulation:

$$\dot{P}_i(t) = -P_i(t)A_{o_{ii}} - A_{o_{ii}}^T P_i(t) + P_i(t)B_i\mathfrak{R}_\iota^{-1}B_i^T P_i(t) - Q_{i2} \tag{25}$$

where $P_i(t)$ is solution of the dynamic Riccati equation of the i^{th} subsystem with $P(t_f) = 0$, and:

$$\dot{\xi}_i(t) = (-A_{o_{ii}}^T + P_i(t)B_i\mathfrak{R}_\iota^{-1}B_i^T)\xi_i(t) + Q_{i2}x_i^o(t) - \pi_i(t) - P_i(t)[B_i\mathfrak{R}_\iota^{-1}R_{i2}u_i^o(t) - B_i\mathfrak{R}_\iota^{-1}\beta_i(t) + f_i(x_i^o, x_j^*(t-\tau_p), u_i^o, t)$$
$$- A_{oii}x_i^o(t) - B_iu_i^o(t)] \quad ; \qquad \text{with } \xi_i(t_f) = 0 \tag{26}$$

Finally, by substituting for $\lambda_i(t)$ from (24) into (14), we get:

$$u_i(t) = -\mathfrak{R}_\iota^{-1}B_i^T P_i(t)x_i(t) + \mathfrak{R}_\iota^{-1}R_{i2}u_i^o(t) - \mathfrak{R}_\iota^{-1}B_i^T\xi_i(t) - \mathfrak{R}_\iota^{-1}\beta_i(t) \tag{27}$$

It can be seen that the first term of the RHS of (27) is the closed loop component of the decentralized controller whilst the second gives the open loop part which will be used to satisfy system constraints given by (9). The satisfaction of the input constraints given by (10), is equivalent to satisfying the following:

$$\underline{v}_i(t) \le v_i(t) \le \overline{v}_i(t) \tag{28}$$

where $v_i(t) = -\mathfrak{R}_i^{-1}B_i^T\xi_i(t)$ (29)

$$\overline{v}_i(t) = \underline{u}_i + \mathfrak{R}_\iota^{-1}B_i^T P_i(t)x_i(t) + \mathfrak{R}_\iota^{-1}\beta_i(t) - \mathfrak{R}_\iota^{-1}R_{i2}u_i^o(t) \tag{30}$$

$$\underline{v}_i(t) = \underline{u}_i + \mathfrak{R}_\iota^{-1}B_i^T P_i(t)x_i(t) + \mathfrak{R}_\iota^{-1}\beta_i(t) - \mathfrak{R}_\iota^{-1}R_{i2}u_i^o(t) \tag{31}$$

This in turn gives:

$$v_i(t) = \begin{cases} \underline{v}_i(t) & if \quad v_i(t) < \underline{v}_i(t) \\ v_i(t) & if \quad \underline{v}_i(t) \le v_i(t) \le \overline{v}_i(t) \\ \overline{v}_i(t) & if \quad v_i(t) > \overline{v}_i(t) \end{cases} \tag{32}$$

Finally, substituting (23) into (17) and (18) gives:

$$\beta_i(t) = \frac{\partial f_i^T(x_i^o, x_j^*(t-\tau_p), u_i^o, t)}{\partial u_i^o}(P_i(t)x_i(t) + \xi_i(t)) - R_{i2}(u_i(t) - u_i^o(t)) - B_i^T(P_i(t)x_i(t) + \xi_i(t)) \tag{33}$$

$$x_i^o(t) = \Theta_\iota^{-1}[Q_{i1}x_i^d + Q_{i2}x_i(t) - \frac{\partial f_i^T(x_i^o, x_j^*(t-\tau_p), u_i^o, t)}{\partial x_i^o}(P_i(t)x_i(t) + \xi_i(t)) - A_{o_{ii}}^T(P_i(t)x_i(t) + \xi_i(t)) + \pi_i(t)]$$
$$= \Psi_i(x_i^o, x_j^*(t-\tau_p), u_i^o, P_i, x_i, \xi_i, \pi_i, t) \tag{34}$$

4. Water Quality Control

Water quality control in streams is usually achieved either through variable effluent flow rate with fixed BOD concentration or through fixed effluent flow rate with variable levels of BOD concentrations. In many practical applications, it may not be possible to get the desired water quality standards using any of the above two methods. Therefore, we may combine the two approaches to achieve our objective. This leads to the following nonlinear (bilinear) model for the i^{th} reach [10]:

$$\dot{z}_i(t) = -(k_{1i} + k_{3i})z_i(t) + \frac{Q_i}{V_i}\sum_{j=1}^{\theta} a_j z_{i-1}(t-\tau_j) - \frac{Q_i + \overline{Q}_{Ei} + \Delta Q_{Ei}(t)}{V_i}z_i(t) + \frac{\overline{Q}_{Ei} + \Delta Q_{Ei}(t)}{V_i}(\overline{m}_i + \Delta m_i(t)) \tag{35}$$

$$\dot{q}_i(t) = -k_{1i}z_i(t) - k_{2i}q_i(t) + \frac{Q_i}{V_i}\sum_{j=1}^{\theta} a_j q_{i-1}(t-\tau_j) - \frac{Q_i + \overline{Q}_{Ei} + \Delta Q_{Ei}(t)}{V_i}q_i(t) + k_{2i}q^s - \frac{k_{4i}}{A_x dx} \tag{36}$$

where , z_i and q_i are, respectively, the concentration of BOD (mg/l) and DO (mg/l), k_{1i} is the rate of decay of BOD, k_{2i} is the re-aerations rate, k_{3i} is the rate of loss of BOD due to settling, $(k_{4i} / A_x dx)$ is the removal of DO due to bottom sludge requirement and q^s is the concentration of DO at saturation level. $\overline{Q}_{Ei}$, $\overline{m}_i$ are respectively, the nominal flow rate and the nominal concentration of BOD in effluent to be discharged, while $\Delta Q_{Ei}(t)$ and $\Delta m_i(t)$ are the deviations around these values. Q_i and Q_{i-1} are the stream flow rates in reaches i and $i-1$ respectively, V_i is the water volume and θ is the number of delays.

Since the effluent flow rate to the river system is variable rather than constant, effluents from polluter stations have to be stored in tanks then discharged into the stream according to the derived control policy. This necessitates the introduction of a third state equation which describes the variation of the effluent volume in the storage tank with time:

$$\dot{\eta}_i(t) = F_i - (\overline{Q}_{Ei} / V_i) - (\Delta Q_{Ei}(t) / V_i) \tag{37}$$

where $\dot{\eta}_i(t) = \dot{V}_{Ti} / V_i$, $\dot{V}_{Ti}$ is the rate of change of the volume of the i^{th} tank, F_i is the inflow rate of effluent into the tank, assumed constant, and $(\overline{Q}_{Ei} + \Delta Q_{Ei}(t)) / V_i$ is the outflow rate of effluent from the tank. Assuming that the inflow rate equals the nominal outflow rate of the effluent, i.e. $F_i = \overline{Q}_{Ei} / V_i$, we get:

$$\dot{\eta}_i(t) = -\Delta Q_{Ei}(t) / V_i \tag{38}$$

Let $\Delta m_i(t) = u_{1i}(t)$, $\Delta Q_{Ei}(t) / V_i = u_{2i}(t)$, then by combining equations (35), (36) and (38), and to develop a completely decentralized control structure, we handle the interaction with the preceding reach as a known input signal. Therefore, the system can be described by the following state variable model for the i^{th} reach:

$$\dot{x}_i(t) = A_{o_{ii}} x_i(t) + B_i u_i(t) + f_i(x_i^o, u_i^o, t) + \varpi_i(t) + d_i \quad ; \text{ with, } x_i(t_o) = x_{oi}, \tag{39}$$

where $i \in \{1,2,...,N\}$, N being the number of subsystems, $u_i \in R^{m_i}$ is the control vector for which $u_i^T(t) = [u_{1i}(t) \quad u_{2i}(t)]$; $x_i \in R^{n_i}$ is the state vector for which $x_i^T(t) = [z_i(t) \quad q_i(t) \quad \eta_i(t)]$; $A_{oii} \in R^{n_i \times n_i}$, $B_i \in R^{n_i \times m_i}$ are the linear parts of the system matrices, $f_i(x_i^o, u_i^o, t) \in R^{n_i}$ is the nonlinear vector function given by $f_i(x_i^o, u_i^o, t) = [-x_{1i}^o(t) u_{2i}^o(t) + u_{2i}^o(t) u_{1i}^o(t), \ -x_{2i}^o(t) u_{2i}^o(t), \ 0]^T$; $d_i \in R^{n_i}$ is a constant vector; $A_{p_{ij}} \in R^{n_i \times n_j}$ are the coupling matrices associated with the j^{th} subsystem (reach); $p \in \{1,....,\theta\}$, θ is a known positive integer representing the number of delays in the state vector of the j^{th} reach; and finally:

$$\varpi_i(t) = \sum_{p=1}^{\theta} A_{p_{ij}} x_j^*(t - \tau_p) \qquad \text{with } j = i-1 \tag{40}$$

To satisfy both water quality standards and community needs, we have to insure that $z_i(t) \le \overline{z}_i$ and $q_i(t) \ge \underline{q}_i$, otherwise most of the aquatic life in the river body will die. Moreover, the volume of the effluent in the tank must satisfy $0 \le \eta_i \le \overline{\eta}_i$ where $\overline{\eta}_i$ is the maximum capacity of the tank. In addition, due to economical considerations Δm_i must not be less than a certain threshold $\Delta \underline{m}_i$,otherwise the treatment cost of the effluent will increase drastically. Finally, $\Delta Q_{Ei}(t) / V_i$ must not exceed the maximum capacity of the valves used as actuators in the system and must not be less than $-\overline{Q}_{Ei} / V_i$, which means zero discharge of effluent. These physical considerations add the following constraint equations to our model:

$$\underline{x}_i \le x_i(t) \le \overline{x}_i \tag{41}$$

$$\underline{u}_i \le u_i(t) \le \overline{u}_i \tag{42}$$

with $\underline{x}_i$, $\overline{x}_i$, $\underline{u}_i$,$\overline{u}_i$ being the lower and upper bounds of the state and control vectors (element by element), for the i^{th} subsystem.

Associated with each subsystem is a performance index J_i, to be minimized w.r.t. x_i and u_i, of the form:

$$\min_{x_i, u_i} J_i = \frac{1}{2} \int_{t_o}^{t_f} (\| x_i(t) - x_i^d \|_{Q_{i1}}^2 + \| u_i(t) \|_{R_{i1}}^2) dt \tag{43}$$

5. Simulation

The above water quality control problem is simulated using the following data:
Subsystem (1):

$x_{o1}^T = [10 \quad 7 \quad 0.01 \quad 5.937], x_1^{d^T} = [4.053 \quad 8 \quad 0.01], d_1^T = [5.35 \;\; 10.9 \quad 0], \underline{x}_1^T = [0 \quad 6 \quad 0.001], \overline{x}_1^T = [10 \quad open \quad 0.049],$

$\underline{u}_1^T = [-0.14 \quad -0.1], \; \overline{u}_1^T = [0 \quad 0.1].$

Subsystem (2):

$x_{o2}^T = [5.937 \quad 6 \quad 0.01], x_1^{d^T} = [5.937 \quad 6 \quad 0.01], d_2^T = [4.19 \;\; 1.9 \quad 0], \underline{x}_2^T = [0 \quad 5.6 \quad 0.001], \overline{x}_2^T = [6.55 \quad open \quad 0.045]$

$\underline{u}_2^T = [-0.35 \quad -0.1], \overline{u}_2^T = [0 \quad 0.1]$

Subsystem (3):

$x_{o3}^T = [5.707 \quad 4.5614 \quad 0.01], x_3^{d^T} = [5.707 \quad 4.5614 \quad 0.01], d_3^T = [2.19 \;\; 1.9 \;\; 0], \underline{x}_3^T = [0 \;\; 4.22 \;\; 0.001],$

$\overline{x}_3^T = [6 \;\; open \;\; 0.016], \quad \underline{u}_3^T = [-0.077 \quad -0.1], \overline{u}_3^T = [0 \quad 0.1]$

For the lack of space, Figs. 1,2 show, as an example, the concentration of BOD and DO in the second reach while Fig. 3 shows the volume of the effluent in the tank associated with this reach. The water treatment control action as well as the effluent discharge strategy are shown in Figs. 4,5 respectively.

6. Conclusion

In this paper, we considered constrained optimization problems of nonlinear serially interconnected dynamical systems with time delays. For this problem, a partially closed loop decentralized control structure is proposed that yields a suboptimal performance of the system. Simulation on a three reach river system shows the applicability and efficiency of the developed technique. It can be seen that the decentralized controller is capable of satisfying system constraints, as well as, achieving a satisfactory response of the system although globally suboptimal.

7. References

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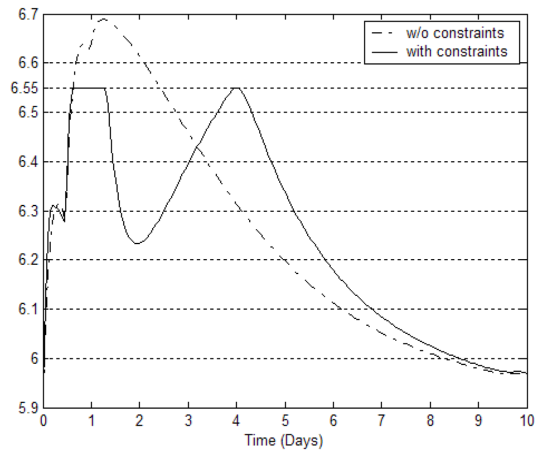

Fig. 1 BOD concentration in Subsystem (2)

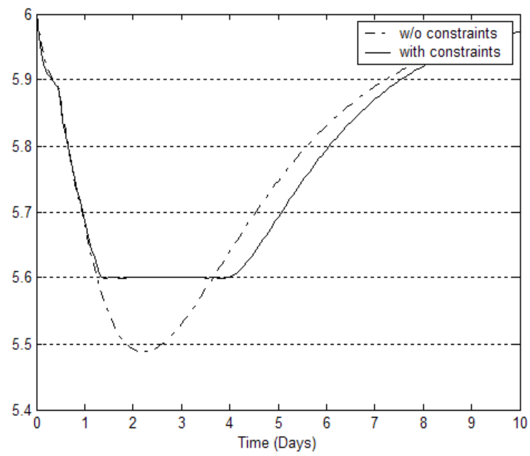

Fig. 2 DO concentration in Subsystem (2)

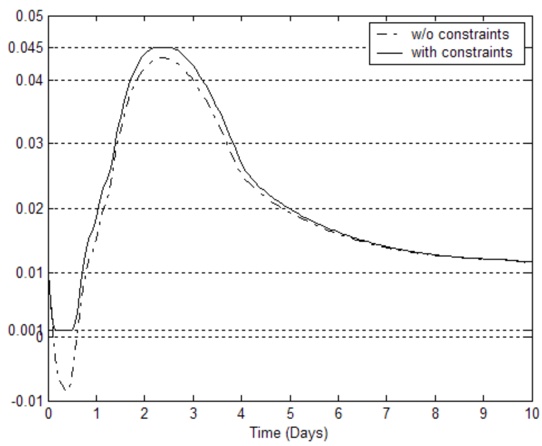

Fig. 3 Tank Volume of Subsystem (2)

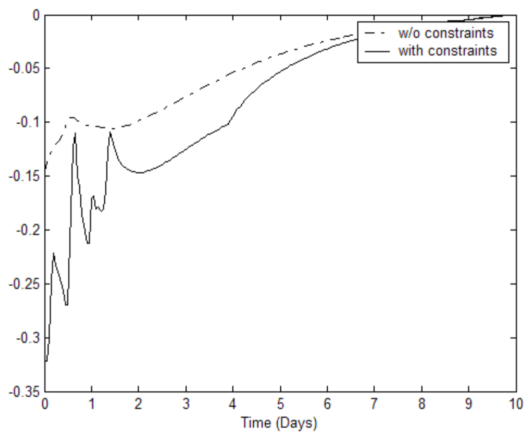

Fig. 4 Variable Treatment Control of Subsystem (2)

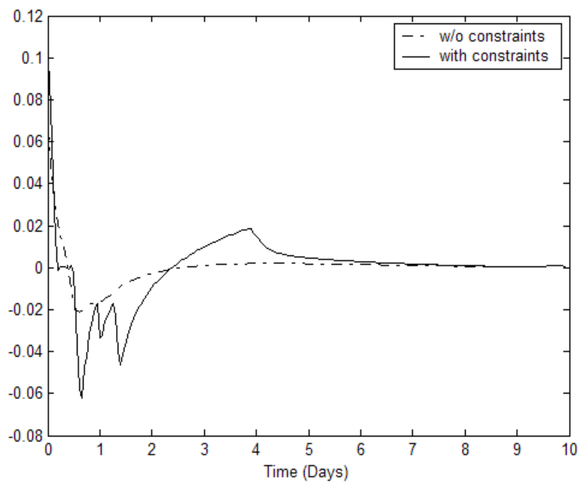

Fig. 5 Discharge Control of Subsystem (2)